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A STUDY OF SOME ASPECTS OF RADIOMETRIC BORE LOGGING

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ABSTRACT

Results of a series of test radiometric bore logs have been made available to the Bureau by Territory Enterprises Pty. Ltd. Analysis of the results of the tests has led to the development of a simple theory which is capable of predicting the performance of a continuously recording radioactive logger under any desired set of conditions.

1. INTRODUCTION

Radiometric bore logging plays an important part in the exploration of deposits of radioactive minerals and has been constantly used by Territory Enterprises Pty. Ltd. at Rum Jungle. In order to test the accuracy with which the width and grade of a radioactive formation could be determined from a log obtained using a continuously recording logger, the company ran a series of test logs under known conditions and submitted the results to the Bureau for mathematical investigation.

The present report deals with this mathematical examination and the conclusions which can be drawn therefrom.

2. GENERAL THEORY OF RADIOMETRIC BORE LOGGING.

The advantages of a radiometric logger to investigate the radioactivity along a drill hole are obvious. From a radiometric log, the most strongly radioactive portions of the hole can be selected immediately. This information may then be used to select portions of the core for assay, and for geological correlation with adjacent holes.

However, it is desirable that the interpretation of radiometric logs be taken considerably further than this. Core drilling is a slow and expensive process, and in bad ground requires highly skilled drillers. Even when all possible precautions are taken, core recovery in crumbling ground is frequently low, and it is commonly found that the recovery is lowest in the mineralised portions of the hole, with the result that the core is often a very imperfect record of the geological formations intersected in the hole. For these reasons core drilling is unpopular both with the management and the drillers. A great gain is obtained if it is possible to use the log as a means of assaying, and thereby avoid the necessity for taking cores. This process is more or less essential in percussion or wagon drill holes, which are particularly convenient in the exploration of some types of deposits.

Such a process requires stringent control. The usual difficulties of radiometric assaying, due to disturbance of radioactive equilibrium, and the possible presence of thorium, are very likely to be encountered, particularly in the oxidised zone. These matters are discussed by Daly, Urquhart and Gibson (1957), together with precautions which may be taken in the laboratory to overcome them. Such precautions are impossible in logging. It is therefore desirable that the method of interpretation of the logs be based on core assays from several holes, and that it be checked continually during the drilling programme by coring a percentage of the holes. It is also necessary to revise the technique as soon as there is evidence of any change in the geological environment.

The simplest form of logger is a Geiger tube, which can be lowered down the hole, and a means of recording the count rate of the tube, located at the top of the hole. In principle, the construction of such a logger is no more difficult than that of any other type of Geiger counter, except for the difficulty of transmitting the signal from the Geiger tube up the hole against the capacitative load of a long cable. This difficulty can be overcome in several ways.

The functioning of the various types of registering circuit which may be coupled to a Geiger tube is described by Daly et al (1957). All radiometric loggers are ratemeters, the time constant of which is chosen as a compromise between the steadiness of the reading and the time required to obtain it. The major problems in the design of such loggers are practical ones. Although the loggers constructed by Austronic Engineering Laboratories Pty.Ltd. to the Bureau's specification are not in question in the present report, some discussion of their design may be helpful, as giving an indication of the considerations involved.

The design adopted for these loggers was governed by the following considerations:-

(1) The holes to be logged were EX holes at Rum Jungle. This meant that the holes would be reasonably shallow, but the diameter of the probe could not exceed $1\frac{3}{8}$ inch.

(2) The only Geiger tubes available were glass tubes more than $\frac{3}{4}$-inch diameter. These tubes had to be enclosed in a metal probe, which meant that the probe diameter could not be less than about $1\frac{1}{4}$ inch.

(3) On the basis of previous mining experience at Rum Jungle, it was expected that the ground in general would be extremely bad; this was fully confirmed by later experience. With the probe fitting the holes so closely, it seemed very likely that probes would stick, and a rigid connection with the surface was thought necessary, to give a reasonable chance of recovering the probes. This involved the use of push rods.

(4) A further reason for the use of push rods was that many of the holes to be logged were horizontal ones, drilled from underground workings.

Based on these considerations, the manufacturer adopted a design which requires that the push rod be used in all holes. This enables the push rods to be used as an electrical connection, making possible an ingenious circuit which overcomes the difficulty caused by the capacity of the cable. The use of push rods means that the equipment has to be raised and lowered by hand. The method of reading is to have the probe at each reading point for a time long enough for the reading to build up to almost its full value.

The disadvantages of this equipment are as follows:-

(1) The push rods are cumbersome, although no better system of push rods has been seen.

(2) The logging is slow, due to the necessity for stopping at the reading points for a time depending on the time constant of the ratemeter circuit.

(3) The log is recorded as a series of readings, and has to be plotted afterwards.

It has the following advantages:-

(1) Switches for varying the time constant and scale value can be set at the best positions for each individual reading. The time constant can thus be set so as to give the fastest reading at each point, and it is never necessary to run a log more than once.

(2) As the probe is rigidly connected with the surface it can often be forced past obstructions which would halt a probe falling under its own weight. Also, distances down the hole are measured along the push rods, thus eliminating any uncertainty due to cable stretch.

(3) A relatively low cost and low capacity cable with little mechanical strength can be used. Cables with high mechanical strength are difficult to obtain in Australia, are expensive, have generally much higher capacities, and in any case, cannot be as strong as the push rods.

(4) The final reading at any point is independent of the time constant.

The logger used by Territory Enterprises Pty. Ltd. in the series of tests under discussion is of a different type. The logger is known as a "Deedlebug" and was designed and constructed by the U.S. Bureau of Mines. It uses a Geiger tube with a stout metal wall, about $\frac{3}{4}$-inch diameter. As the wall of the tube is of metal, it is not necessary to enclose it in a probe. The diameter of the tube is considerably less than that of the hole, thus reducing the risk of jamming. It is therefore unnecessary to use push rods. This means that the probe need not be raised and lowered by hand, and the logger is designed to be moved at constant speed by means of a winch. It is then possible to record the log on an Esterline Angus Pen Recorder, the paper drive of which is coupled to the winch. The log is thus directly plotted on any desired scale. This equipment cannot be used in horizontal holes. When used in vertical or near-vertical holes, it has the following advantages:-

(1) The log is plotted directly (neglecting the minor disadvantages arising from the curved scale of the Esterline Angus Recorder).

(2) The operation is convenient, and the logging relatively fast.

Its disadvantages are the following:-

(1) The scale value and time constant cannot be conveniently changed during a run. It may therefore be necessary to repeat a run using a different scale value.

(2) If the probe sticks, the strength of the cable is inadequate to recover it.

(3) The reading obtained at any point is a function, not only of the degree of radioactivity at the point, but also of the logging speed, and the time constant of the ratemeter. This is a major difficulty and is the main subject of the present report.

3. NATURE OF THE TESTS

In order to construct what might be termed an artificial drill hole, several standard samples were prepared by Territory Enterprises Pty.Ltd. These samples were in the form of concrete discs, about 2 feet in diameter and 1 foot thick, loaded with concentrations of uranium ore, varying from 1 lb. per ton (about 0.05 percent) to 20 lb. per ton (about 1 percent). Each disc had a central hole, so that the discs could be threaded on to a string of AX casing in any desired order, with blank discs interspersed as required. The logger was then run down each assembly of discs at various logging speeds.

4. MATHEMATICAL FORMULATION OF THE BORE LOGGING PROBLEM.

The ideal log would be a profile, the ordinate of which would be exactly proportional to the intensity of radioactivity at the corresponding point in the hole.

There are three reasons which prevent such a record being obtained, namely:-

(1) The probe is affected, not only by the radioactivity of the part of the hole adjacent to it, but also by strongly radioactive portions at a distance. No account of this is taken in what follows, mainly because there is no obvious means of making an explicit allowance for it.

(2) The finite length of the probe. The base point for the measurement of distances in the hole is the centre of the probe. However, due to the length of the probe, the tube begins to count before its centre reaches the boundary of the radioactive layer, and does not attain the count rate proportional to the intensity of radioactivity of the layer until the full length of the probe is exposed to the radiation. This does not occur until the full length of the probe has passed the boundary of the layer.

(3) The time constant of the ratemeter. A definite time is necessary before the ratemeter will give a true reading of the count rate of the tube. As the tube is moving continuously, it will not in general be exposed to the radiation from a particular layer for a sufficient time to enable a true indication to be obtained.

All other time constants in the equipment may be neglected in comparison with the time constant of the ratemeter tank circuit. The operation of the equipment therefore consists of feeding a current proportional to the count rate at any instant into a resistance-capacity circuit, and recording the voltage developed across the circuit, as shown in Fig.1.

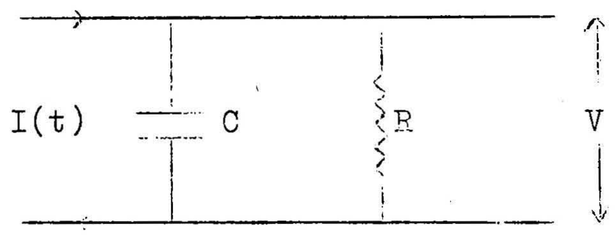


Fig. 1

If the current is I(t), a function of t, then the voltage (V) is given by -

$$\frac{CdV}{dt} + \frac{V}{R} = I(t)$$

or, putting the equation in the more usual form:-

$$\frac{RCdV}{dt} + V = R.I(t).$$

RC is the time constant of the tank circuit.

This is a differential equation of the simplest type and can be solved immediately, provided I(t) can be expressed as an analytic function of t. An example of the use of the analytic solution is given in a later section, in connection with a particular problem. However, for the purpose of obtaining actual profiles, a graphical method of solution is more convenient for the following reasons:-

(1) The analytic solution of the equation, though elementary, is rather tedious. Plotting of the solution is another elementary, but tedious, operation.

(2) The work requires tables of exponential functions which may not be readily available.

(3) Graphical methods can be applied to any case in which I(t) can be plotted as a graph, whether or not it can be conveniently expressed in analytic form.

The accuracy of graphical methods is inherently limited, but is sufficient for the present application. These methods are discussed by Bailey and Somerville (1938), and the present equation is easily solved by the process described by them.

5. DETAILS OF FORMULATION OF TEST CONDITIONS FOR GRAPHICAL SOLUTION.

(1) The test logs were recorded on a scale of 1 inch to the foot, and this scale was used in preparing the calculated logs. The first process is to represent the various discs by rectangles, the thickness being plotted horizontally, on a scale of 1 inch to the foot, and the uranium content vertically on a scale which makes the amplitude of the calculated log about the same as that of

the observed one. This plot may be considered as the ideal log,

(2) The effect of the finite length of the probe is to change the vertical sides of the rectangles on the ideal log to sloping lines. If the length of the probe is ℓ, the logger will begin to record when the centre of the probe is a distance $\frac{\ell}{2}$ above the boundary of the disc.

When the centre of the probe is level with the boundary of the disc, half of the tube will be counting, and the full count rate will be attained when the centre of the probe is a distance $\frac{\ell}{2}$ below the edge of the disc. When the probe is passing from a radioactive disc to blank material, the reverse order will hold. The trapezoidal plot obtained in this way gives the input current to the ratemeter. The length of the probe used in the tests is stated as 8 inches.

(3) The effect of logging speed and time constant of the ratemeter is introduced in the graticule used for the graphical integration. The time constant is converted to a distance, using the logging speed. Thus at a logging speed of v feet per minute, a time constant of t seconds is equivalent to $\frac{vt}{60}$ feet. This is inserted in the graticule at the same scale as that used for the plotting mentioned above.

6. DETAILS OF TESTS

Details of the test discs used in the experimental logs are shown in Table 1 (information supplied by Territory Enterprises Pty.Ltd).

TABLE 1.

Disc No.	Thickness	Uranium Content (lbs U_3O_8/ton)
1	10 inch	20.12
2	1 foot	12.49
3	1 foot	9.01
4	1 foot	5.09
5	1 foot	3.9
6	1 foot	2.60
7	1 foot	2.54
8	1 foot	1.55

Thirteen runs were made using discs in the following orders.

Run 1 Disc 7, 1 foot blank, disc 6.

Run 2 Disc 6, disc 3, disc 4.

Run 3 Disc 7, disc 4, disc 6.

Run 4 Disc 7, disc 6.

Run 5 Disc 1, disc 6, disc 2.

Run 6 Disc 1, disc 2, disc 3, disc 4, disc 5, disc 6, disc 8.

Run 7 Disc 8, disc 6, disc 5, disc 4, disc 3, disc 2, disc 1.

Run 8 Disc 1, disc 2, disc 5, disc 4, disc 3, disc 6, disc 8.

Run 9 Disc 8, disc 6, disc 3, disc 4, disc 5, disc 2, disc 1.

Run 10 Disc 7, disc 3, disc 4, disc 1,disc 5, disc 2, disc 6, disc 8.

Run 11 Disc 8, disc 6, disc 2, disc 5, disc 1, disc 4, disc 3, disc 7.

Run 12 Disc 1, 3 foot blank, disc 2, 3 foot blank, disc 3, 3 foot blank, disc 4, 3 foot blank, disc 5, 3 foot blank, disc 6, 3 foot blank, disc 8.

Run 13 Disc 7, disc 2, disc 6.

Each run had a 3 foot blank disc at beginning and end.

7. RESULTS

Results of the logging, together with calculated logs, are shown on Plate 1. Logs were run at various speeds, and using both of the scale factors shown on the logger as "X1" and X10". Only one log of each run (that taken at 10 feet per minute) has been reproduced. The scale factor "X1" has been used where possible, but on most of the logs the recorder went off scale on this range when opposite the discs containing the higher uranium values. In these cases, the "X10" log was used. The vertical scale of uranium concentrations was chosen so as to make the calculated logs roughly equal in amplitude to the observed ones. The values found suitable were 1 inch equals 1 lb. U_3O_8 per ton for the "X1" logs, and 1 inch equals 5 lb. U_3O_8 per ton for the X10" logs. Plate 1 shows the ideal log, input current wave form, calculated and observed log for each run. As there is no means of relating the horizontal scale of the observed log precisely to the position of the block, the observed and calculated logs have been matched by eye.

8. DISCUSSION OF RESULTS

(1) The agreement between observed and calculated logs is surprisingly good, considering the crude nature of the assumptions made and the very simple process of graphical integration employed. The general shape and dimensions of the logs are faithfully reproduced, the main discrepancy being that the observed log is considerably

more sensitive when passing the discs with high uranium content than would be expected from the calculations. A factor which would improve the general agreement in a minor way is that the length of the probe has been taken as 8 inches. This is the physical length, but the sensitive length of the Geiger tube would certainly be less than this. A shorter length would improve the fit. However, this is insufficient to account for the discrepancies which appear in logs involving Disc No.1, for instance.

These discrepancies are difficult to explain, as the most obvious explanations which can be suggested for them would be expected to give errors in the other direction. The following causes appear possible.

(a) Insufficiency of the basic assumptions made in the calculations. It is difficult to see how such an insufficiency could cause an error of this type.

(b) Inaccuracy of the assay values for discs 1,2 and 3. This does not seem likely.

(c) Non-linear behaviour of the electronic portion of the logger. Normally it would be expected that any error due to this would be in the other direction. A similar logger, belonging to the Bureau, was tested in the Geophysical Laboratories. It was found that its behaviour was linear, but that its sensitivity was very low. It is understood that the logger used in the tests at Rum Jungle had previously been modified in order to increase its sensitivity, and it seems possible that this modification may have introduced non-linearity such as overshooting.

(2) The fact that the logs taken on scales marked "X1" and "X10" respectively can be fitted closely on uranium scales with a ratio of 5 to 1 indicates that the stated ratio of the scales on the logger used is in error.

9. GENERAL RESPONSE OF CONTINUOUSLY RECORDING LOGGER

The close agreement between observed and calculated logs indicates that the present calculations can be used as a basis for predicting the response of the loggers to various conditions with reasonable confidence. Neglecting the effects which are apparently peculiar to the logger used in the tests, and which are discussed in the next section, the general performance of a logger of the type under consideration may be described as follows:-

(1) If the probe is passing a formation in which the radioactive intensity increases or decreases gradually, the log will follow the level of radioactivity fairly well.

(2) If the logger passes a formation with uniform radioactivity bounded by non-radioactive formations, the response may be seriously in error.

(3) The nature of the response to a discrete band of uniform radioactivity varies with the width of the band. If the band is of sufficient width, the response is delayed when the probe reaches the band, attains a maximum when the probe is just leaving the band, and thereafter falls away at a rate depending only on the electrical constants of the logger and not on the presence or absence of radioactivity. The effect of the finite probe length is to modify the shape of the log in a minor way. As the width of the band decreases, the probe length has more and more influence on the nature of the results. The sloping sides of the trapezoid representing the current input come closer together. A critical point is reached when the width of the band is exactly equal to the length of the probe. In this case, the current input to the ratemeter appears as an isosceles triangle, whose base is twice the probe length, and whose altitude is equal to the amplitude of the ideal log. As the width of the band decreases still further, the width of the base decreases, the sloping sides steepen, and the altitude decreases below the amplitude of the ideal log, the ratio $\frac{\text{amplitude of current input wave form}}{\text{amplitude of ideal log}}$ being equal to the ratio $\frac{\text{width of radioactive band}}{\text{length of probe}}$.

In the limiting case of a very narrow band, the width of the current input wave form becomes equal to the length of the probe, and the sides become vertical. It is obvious that no radioactive band, however small in width, can give a current input wave form whose width is less than the length of the probe.

Plate 2 shows ideal logs, current input wave forms, and calculated logs for the cases of a layer 1 foot wide containing 1.0 percent U_3O_8, a layer 6 inches wide containing 2.0 percent U_3O_8, and a layer $\frac{1}{4}$ inch wide containing 48.0 percent U_3O_8. It will be seen that the current input wave form in the third case is practically identical with the ideal log in the second case and the calculated logs are not very different. The log gives no indication of the actual width of a radioactive band whose width is less than the length of the probe.

It is apparent that the most serious possibility of obtaining erroneous information from the logs would be to allow too much weight to the falling portion of the log, which depends only on the constants of the equipment. Some detailed calculations of the possible error are given later. In general, this error is to some extent compensated by the fact that the log considerably underestimates the grade of the layer. A particularly misleading case would be one in which the grade of the layer increases slowly, and cuts off rapidly (i.e. the ideal log is in the shape of a right-angled triangle). Under such circumstances the log would follow the gradual increase fairly faithfully, and the maximum value attained would be close to that corresponding to the maximum grade, whereas the decay side of the log would correspond to no mineralisation at all.

10. POSSIBILITIES OF ERROR DUE TO THE LOGGER USED IN THE TESTS

As mentioned earlier, the comparison of observed and calculated logs indicates that the logger used possesses certain peculiarities, which offer opportunities for deriving wrong information. These are -

(1) Possible overshoot at high counting rates; this could give rise to over-estimation of grade.

(2) The ratio of the scale values, shown as 10 to 1, appears to be about 5 to 1. If the instrument were calibrated using the X10 scale, and this calibration used as a basis for interpreting results obtained on the X1 scale, on the assumption that the ratio of the scales was 10 to 1, grade would be over-estimated by a factor of 2.

11. INVESTIGATION OF SOME BORE LOGGING PROBLEMS

As an example of the possibilities of the method of calculation described, it will be applied to three typical problems.

(1) Response to Disseminated Mineralisation.

Plate 3 shows calculations bearing on this problem. Fig.1 shows the log to be expected from a layer 3 feet wide, uniformly mineralised. Figs. 2,3,4 and 5 show logs calculated on the assumption that the average grade of mineralisation is the same over three feet as in Fig.1, but the mineralisation is concentrated over half the length, distributed in 1,2,3 and 6 layers respectively. The log in the 6-layer case is practically indistinguishable from the log of Fig.1. It appears that no serious over-estimation of width and grade is likely in any of these cases.

(2) Possibility of Improving Performance.

Improvement in performance is to be sought by decreasing the probe length and decreasing the time constant. Plate 4 shows logs calculated for time constants of 4, 3 and 1.2 seconds, and probe lengths of 6 inches and zero respectively. It is apparent that the logger with a probe of zero length, with a time constant of 1.2 seconds, gives a fair representation of a layer 10 inches thick at a logging speed of 10 feet per minute. A scintillation logger could be constructed which would approach this. The disadvantage of such a logger would be that the probe would be very expensive.

(3) Possibility of Error in the Interpretation of the Logs.

The most natural method of interpreting the logs would be to calibrate the scale of the log in uranium content, thence to select the ordinates of the log corresponding to the minimum grade of ore which can be economically mined, and measure the spacing of these ordinates on the log. This spacing is taken as the width of ore of this grade. It appears from the foregoing discussion that this method is not likely to be seriously in error if the band of ore is wide enough, but requires justification before it can be used for narrow layers. The analytic method of solution is used below to estimate the error caused in one particular case.

The case chosen is one, which is critical in some respects, in which the width of the radioactive layer is exactly equal to the length of the probe. The equation to be solved is:-

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{I(t)}{C}$$

The general solution of this equation is obtained by the standard methods as:-

$$V = e^{\frac{-t}{RC}} \int_{-\infty}^{t} \frac{I(t)}{C} e^{\frac{s}{RC}} ds + Ke^{\frac{-t}{RC}}$$

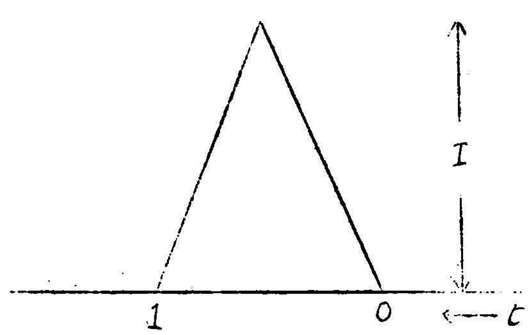
where K is an arbitrary constant.

If the origin of time is taken as the point at which the end of the probe just reaches the radioactive layer, the initial condition is V=o when t=o. Then K=o, and the solution may be written:-

$$V=e^{\frac{-t}{RC}} \int_{0}^{t} I(t) \; e^{\frac{s}{RC}} ds$$

where I(t) has absorbed the constant multiplier $\frac{1}{C}$.

The form of I(t) is as shown below:-



where I is proportional to the grade of the layer, and the base of the triangle is proportional to twice the width of the layer. This base has been taken as the unit of time.

The analytical expression of this form is:-

$$0 \leq t \leq \tfrac{1}{2}, \qquad I(t) = I.2t.$$

$$\tfrac{1}{2} \leq t \leq 1, \qquad I(t) = I.2(1-t)$$

$$t \geq 1, \qquad I(t) = 0$$

Substituting these values in the solution above, it becomes:-

$$0 \leq t \leq \tfrac{1}{2}, \quad \frac{V}{I} = 2e^{\frac{-t}{RC}} \int_0^t s\, e^{\frac{s}{RC}}\, ds.$$

$$\tfrac{1}{2} \leq t \leq 1, \quad \frac{V}{I} = 2e^{\frac{-t}{RC}} \int_0^{\frac{1}{2}} s e^{\frac{s}{RC}}\, ds + 2e^{\frac{-t}{RC}} \int_{\frac{1}{2}}^t (1-s) e^{\frac{s}{RC}}\, ds.$$

$$t \geq 1, \quad \frac{V}{I} = 2e^{\frac{-t}{RC}} \left(\int_0^{\frac{1}{2}} s e^{\frac{s}{RC}}\, ds + \int_{\frac{1}{2}}^1 (1-s) e^{\frac{s}{RC}}\, ds \right)$$

Under the conditions used in the test, the unit of time is twice the time required for a probe 6 inches long to pass a given point at a speed of 10 feet per minute. This equals 6 seconds.

The time constant of the equipment

= RC = 5 seconds = 0.83 in the above unit

Thus $\frac{1}{RC} = 1.2$, and the solution becomes

$$0 \leq t \leq \tfrac{1}{2}, \quad \frac{V}{I} = 2e^{-1.2t} \int_0^t s e^{1.2s}\, ds$$

$$\tfrac{1}{2} \leq t \leq 1, \quad \frac{V}{I} = 2e^{-1.2t} \left(\int_0^{\frac{1}{2}} s e^{1.2s} ds + \int_{\frac{1}{2}}^t (1-s) e^{1.2s}\, ds \right)$$

$$t \geq 1, \quad \frac{V}{I} = 2e^{-1.2t} \left(\int_0^{\frac{1}{2}} s e^{1.2s}\, ds + \int_{\frac{1}{2}}^1 (1-s) e^{1.2s}\, ds \right)$$

Integrating by parts, it is immediately proved that -

$$\int x\, e^{ax}\, dx = \frac{e^{ax}}{a^2} \left(ax - 1 \right)$$

The integrals may now be evaluated, and the solution becomes (to slide rule accuracy):-

$$0 \leq t \leq \tfrac{1}{2}, \qquad \frac{V}{I} = 1.39e^{-1.2t} + 1.67t - 1.39.$$

$$\tfrac{1}{2} \leq t \leq 1, \qquad \frac{V}{I} = 3.05 - 1.67t - 3.66e^{-1.2t}$$

$$t \geq 1, \qquad \frac{V}{I} = 0.76e^{-1.2t}.$$

For the method of interpretation used, we require values of t such that $V = \frac{I}{n}$, where ne is a fairly large constant.

Two such values will be obtained, one in the first range of the solution, and one in the third.

Considering the first range, we have to solve the equation:-

$$1.39e^{-1.2t} + 1.67t - 1.39 = \frac{1}{n}$$

$$\text{or} \quad e^{-1.2t} + 1.2t = 1 + \frac{1}{1.39n}$$

Putting $1.2t = x$ and $\frac{1}{1.39n} = \delta$,

we require solutions of

$x + e^{-x} = 1 + \delta$, where δ may be considered as small.

Expanding e^{-x} and retaining the first three terms of the expansion, the equation becomes

$$x + 1 - x + \frac{x^2}{2} = 1 + \delta$$

$$\text{i.e.} \quad \frac{x^2}{2} = \delta$$

so that a first approximation to the solution is

$$x = \sqrt{2\delta}$$

The solution could be improved by iteration methods, but since this value of t is ultimately required for subtraction from another larger value, this solution is close enough for our purposes.

$$\text{We have } 1.2t = \sqrt{\frac{2}{1.39n}}$$

$$\text{i.e. } t = \frac{1}{\sqrt{n}} \quad \text{approximately}$$

In the third range, we require solutions of

$$0.76e^{-1.2t} = \frac{1}{n}$$

$$e^{1.2t} = 0.76n$$

$$t = \frac{\log_e n}{1.2} - 0.23$$

Calling the solution in the first range t_1, and that in the third range t_2, we can tabulate the results as follows (for n = 5, 20, 50, 100).

n	t_1	t_2	t_2-t_1
5	0.45	1.11	0.66
20	0.22	2.26	2.04
50	0.14	3.02	2.88
100	0.1	3.61	3.51

As the width of the radioactive band is equal to the probe length of 6 inches, and the unit of time is equivalent to twice this length, the table shows that, with this method of interpretation, a band 6 inches wide containing say 10 percent U_3O_8 could be interpreted as:

a band 8 ins. wide containing 2.0% U_3O_8

a band 2.04 feet wide containing 0.5% U_3O_8

a band 2.88 " " " 0.2% U_3O_8

a band 3.51 " " " 0.1% U_3O_8

These values considerably underestimate the total amount of uranium present. However, they assume that the value of I is that appropriate to the actual uranium content of the radioactive band; in other words, that the logger has been calibrated by holding the probe steadily against the radioactive band until the reading has had time to reach its full value. If the logger scale were calibrated in other ways, the situation could be less favourable.

For example, suppose the logger has been calibrated by running a log of a hole containing a band 6 inches wide containing 10 percent U_3O_8, assaying the core, and associating the maximum value attained on the log with the assay value of the core. The log will attain its maximum value in the second range of the solution, where

$$\frac{V}{I} = 3.05 - 1.67t - 3.66e^{-1.2t}$$

Differentiating with respect to t, and equating the value of $\frac{dV}{dt}$ to zero, the maximum height is reached where -

$$3.66 \times 1.2e^{-1.2t} - 1.67 = 0$$

$$e^{1.2t} = \frac{3.66 \times 1.2}{1.67}$$

$$1.2t = 0.967$$

$$t = 0.805$$

Substituting this value, we find that the maximum value of $\frac{V}{I} = 0.33$. On this basis, the grades equivalent to the values of n used will be multiplied by 3, and the band 6 inches wide containing 10 percent U_3O_8 will be equivalent to -

a band	8 inches wide	containing	6% U_3O_8
" "	2.04 feet wide	"	1.5% U_3O_8
" "	2.88 " "	"	0.6% U_3O_8
" "	3.51 " "	"	0.3% U_3O_8

This is still a considerable underestimate of the total amount of uranium. Cases still more unfavourable could be encountered if the log were calibrated in this way, and if the band of active material were considerably less than 6 inches wide and correspondingly higher in grade. As mentioned earlier, this fact would not be visible from the log. However, it would be unsound to base the calibration of the log on such evidence. If the log were calibrated using the set-up of test blocks about 1 foot thick, the indicated grades would be less than those for the case last calculated by a factor of about 2.

It appears, then, that such a method of interpretation may be expected to underestimate the total amount of uranium present, in general. Any considerable overestimate of total uranium content based on interpretation of logs by this method as compared with core assays can only be caused, either by instrumental peculiarities such as may be present in the logger used for the present tests, or by the probe being affected by radioactive material which is not represented in the core.

12. CONCLUSIONS

(1) The results of the theory presented agree with the results of the test logs apart from certain discrepancies which are attributed to instrumental peculiarities.

(2) It is considered that this theory can be used to predict the performance of a continuously recording bore logger for any desired set of conditions with reasonable confidence.

(3) It appears that, although any reasonable method of interpretation applied to logs obtained using a continuously recording logger is unlikely to overestimate the total amount of uranium present, such a logger gives no information as to the true width or grade of narrow formations.

13. ACKNOWLEDGMENTS

This study has been made possible by the communication of the results of test logs made by Territory Enterprises Pty.Ltd. The results are used as the basis of this publication by permission of the Company.

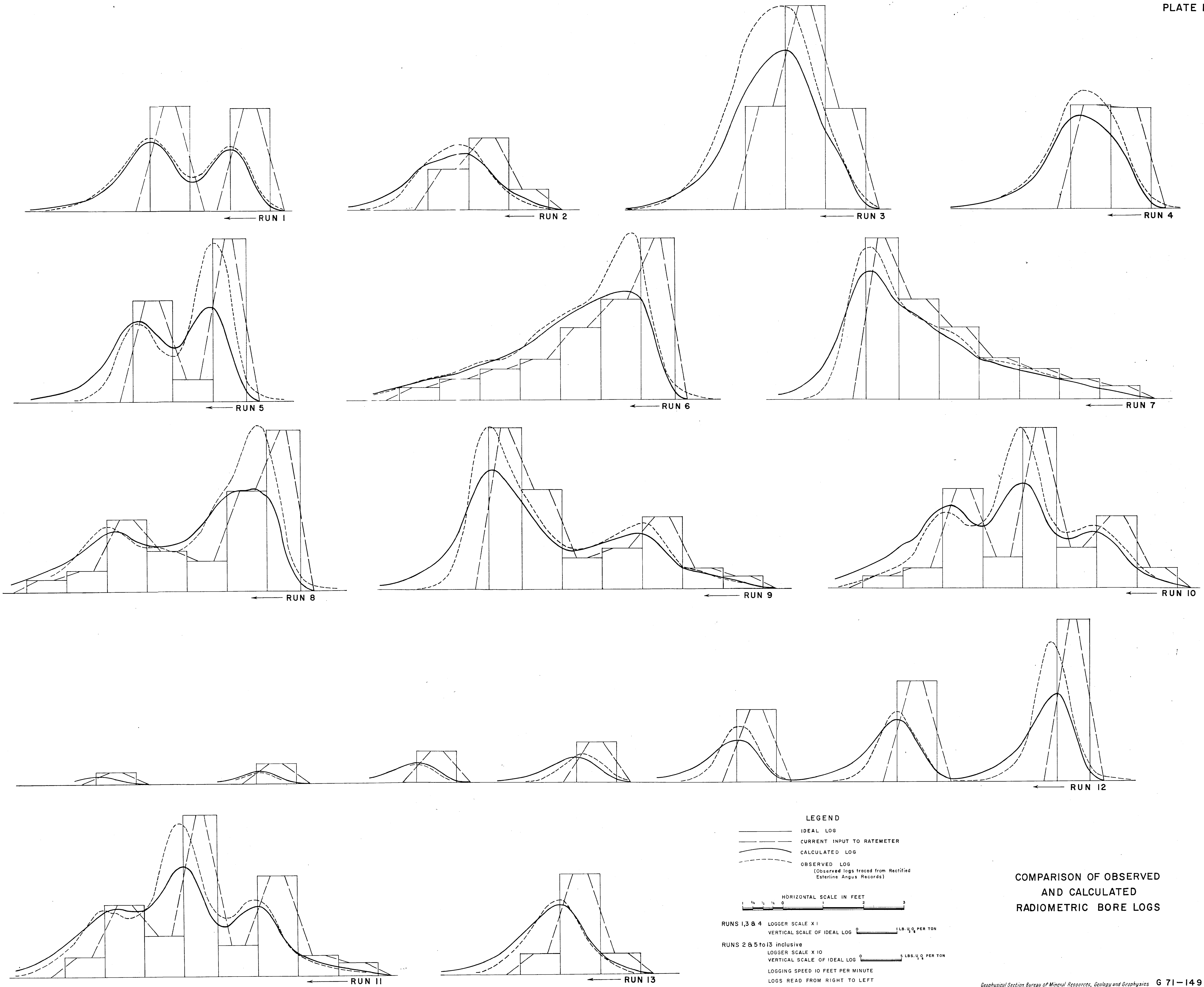
It is desired to acknowledge the assistance gained from discussions on mathematical points with Dr. W.D.Parkinson of the Bureau.

14. BIBLIOGRAPHY

Bailey, V.A. and Somerville J.M., 1938. - The Graphical Solution of Ordinary Differential Equations. Phil.Mag.Series 7, Vol. 26.

Daly, J., Urquhart, D.F. and Gibson M.R., 1956. - Assaying of Radioactive Rocks and Ores. Bur.Min. Resour.Aust., Bull. 31.

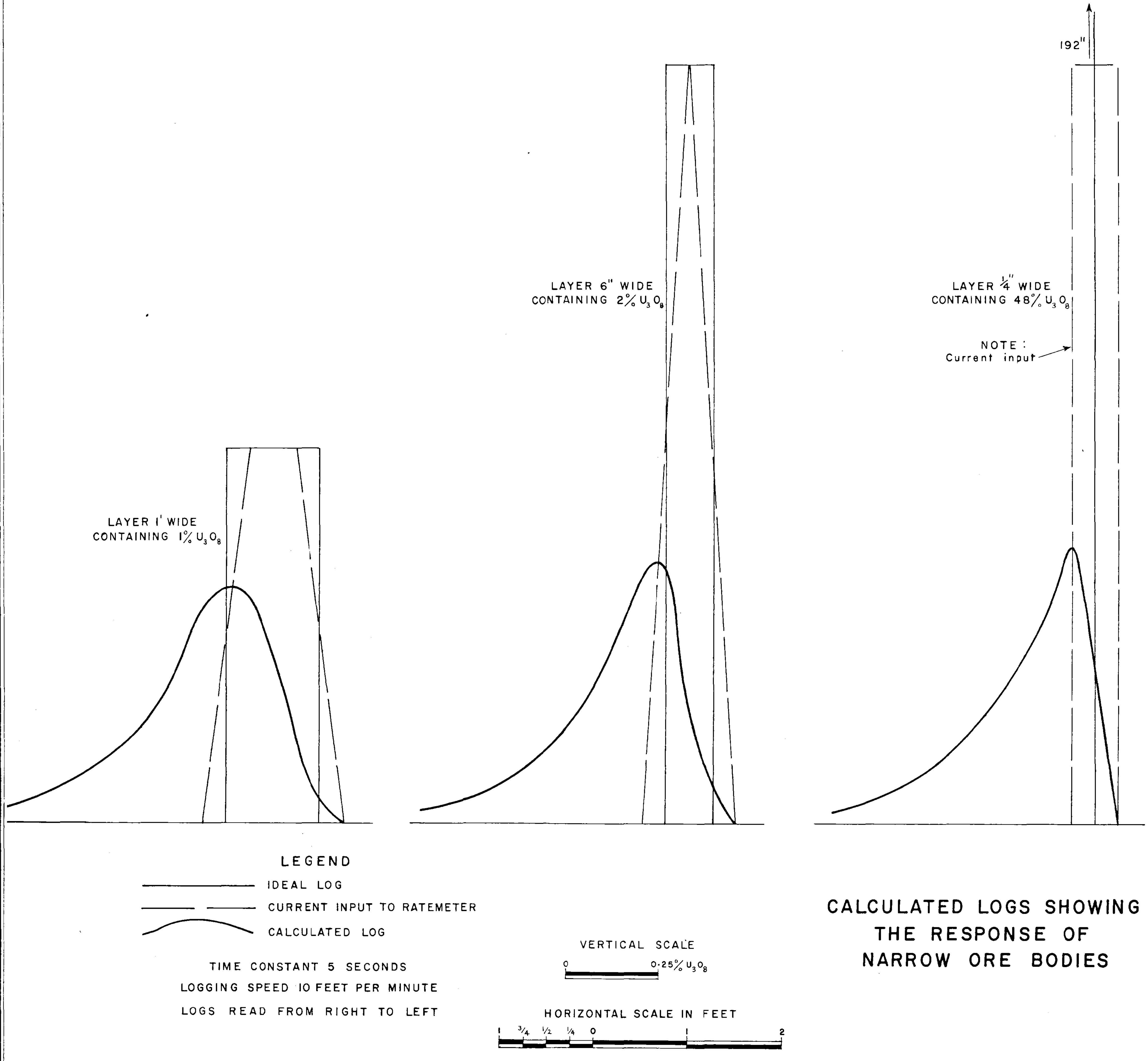
PLATE I



COMPARISON OF OBSERVED AND CALCULATED RADIOMETRIC BORE LOGS

Geophysical Section, Bureau of Mineral Resources, Geology and Geophysics G 71–149

PLATE 2



CALCULATED LOGS SHOWING THE RESPONSE OF NARROW ORE BODIES

Geophysical Section, Bureau of Mineral Resources, Geology and Geophysics G 71—152

PLATE 3

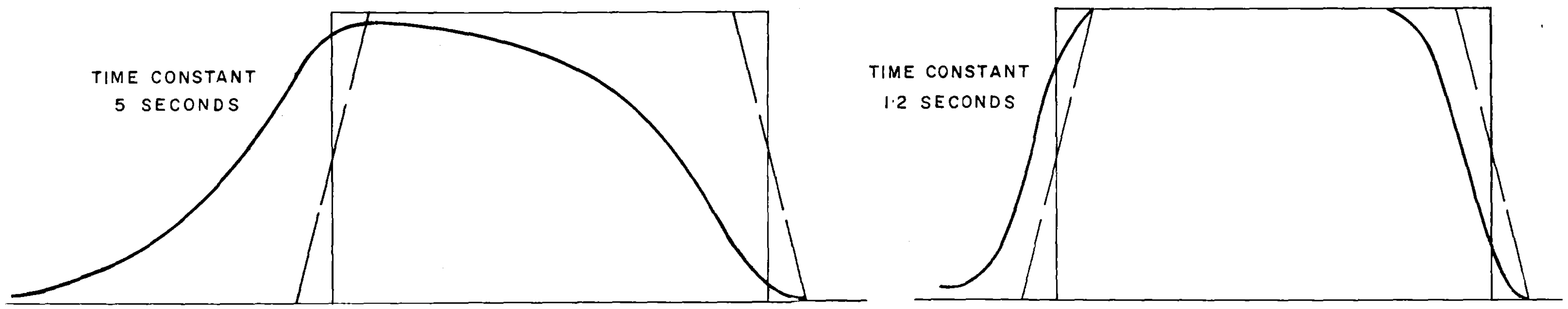


FIG. 1

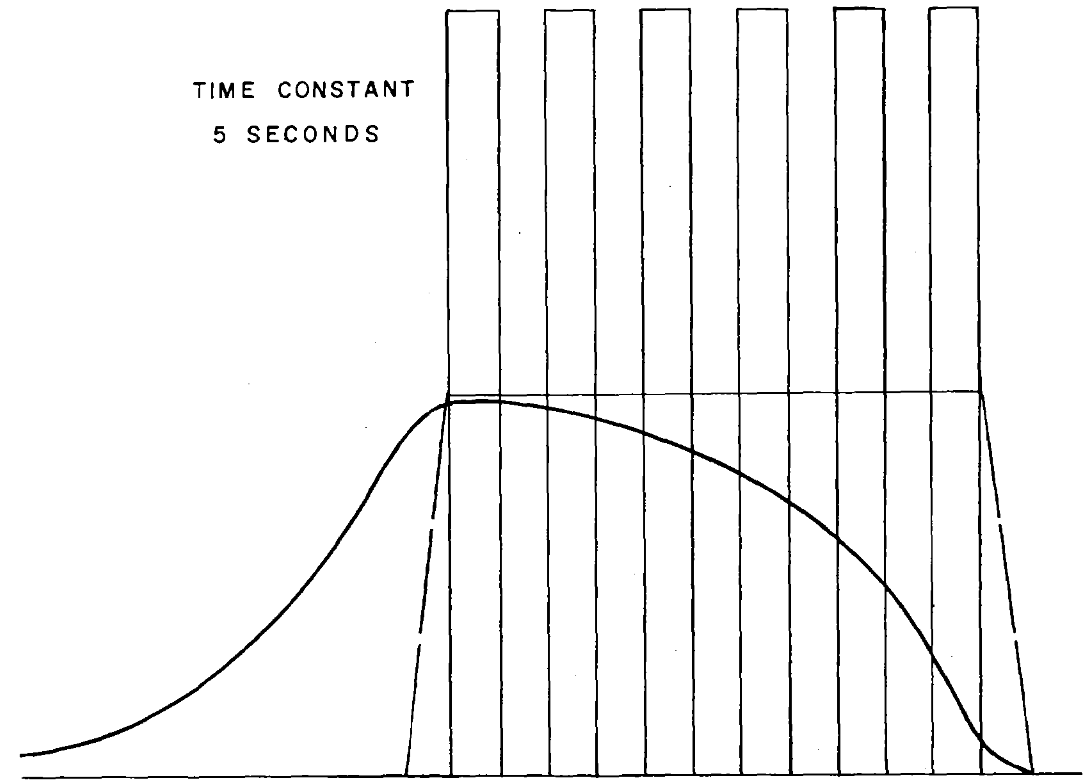


FIG. 5

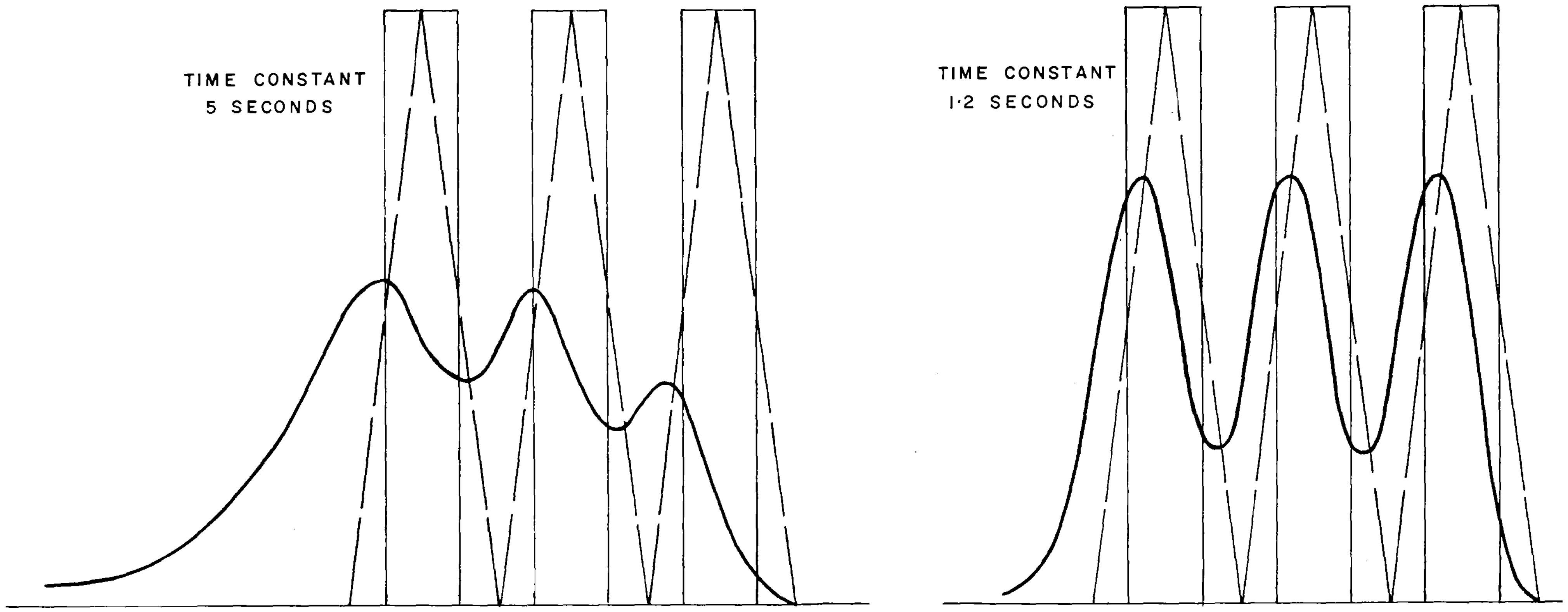


FIG. 4

LEGEND

IDEAL LOG

CURRENT INPUT TO RATEMETER

CALCULATED LOG

PROBE LENGTH 6 INCHES

HORIZONTAL SCALE IN FEET

1 ¾ ½ ¼ 0 1 2 3

VERTICAL SCALE OF IDEAL LOG

0 1 LB U_3O_8 PER TON

LOGGING SPEED 10 FEET PER MINUTE

LOGS READ FROM RIGHT TO LEFT

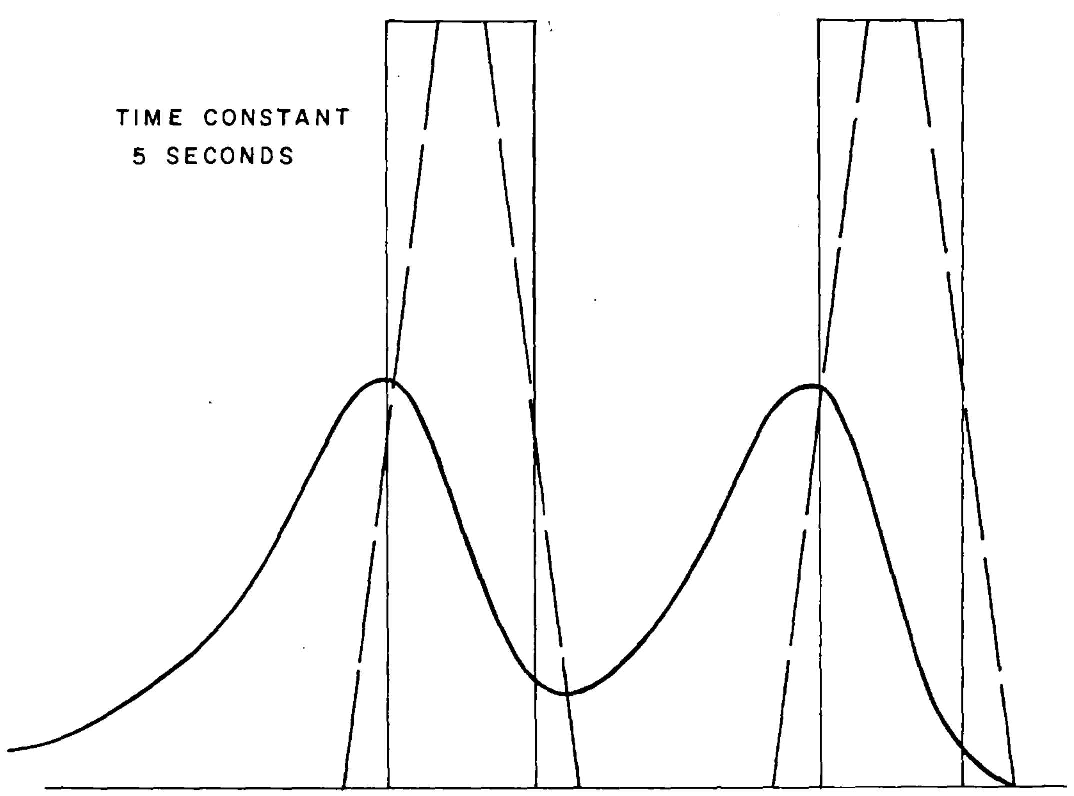


FIG. 3

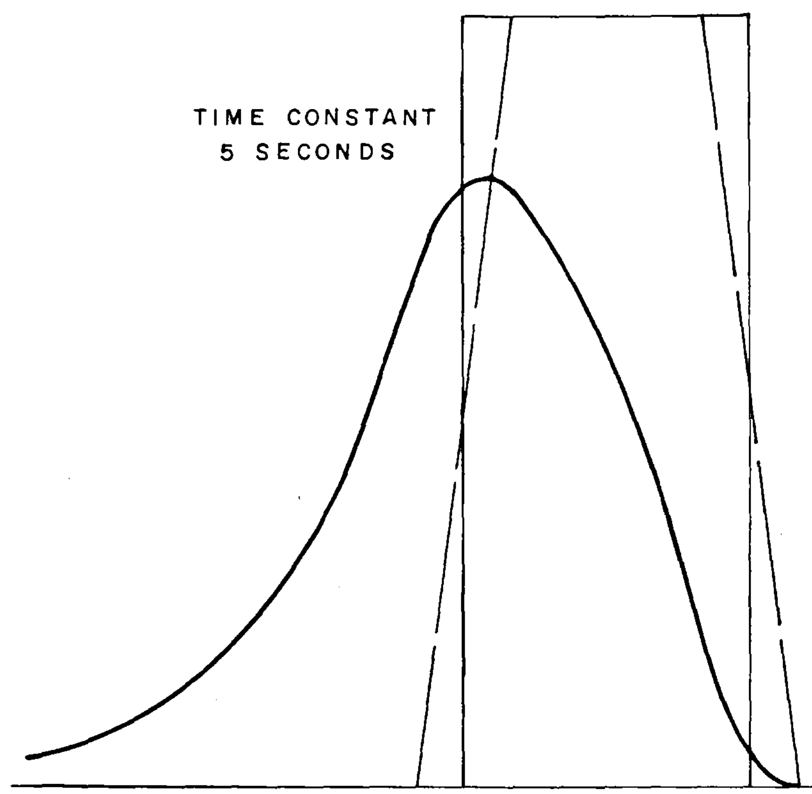


FIG. 2

CALCULATED RADIOMETRIC BORE LOGS SHOWING EFFECT OF DISSEMINATED MINERALISATION

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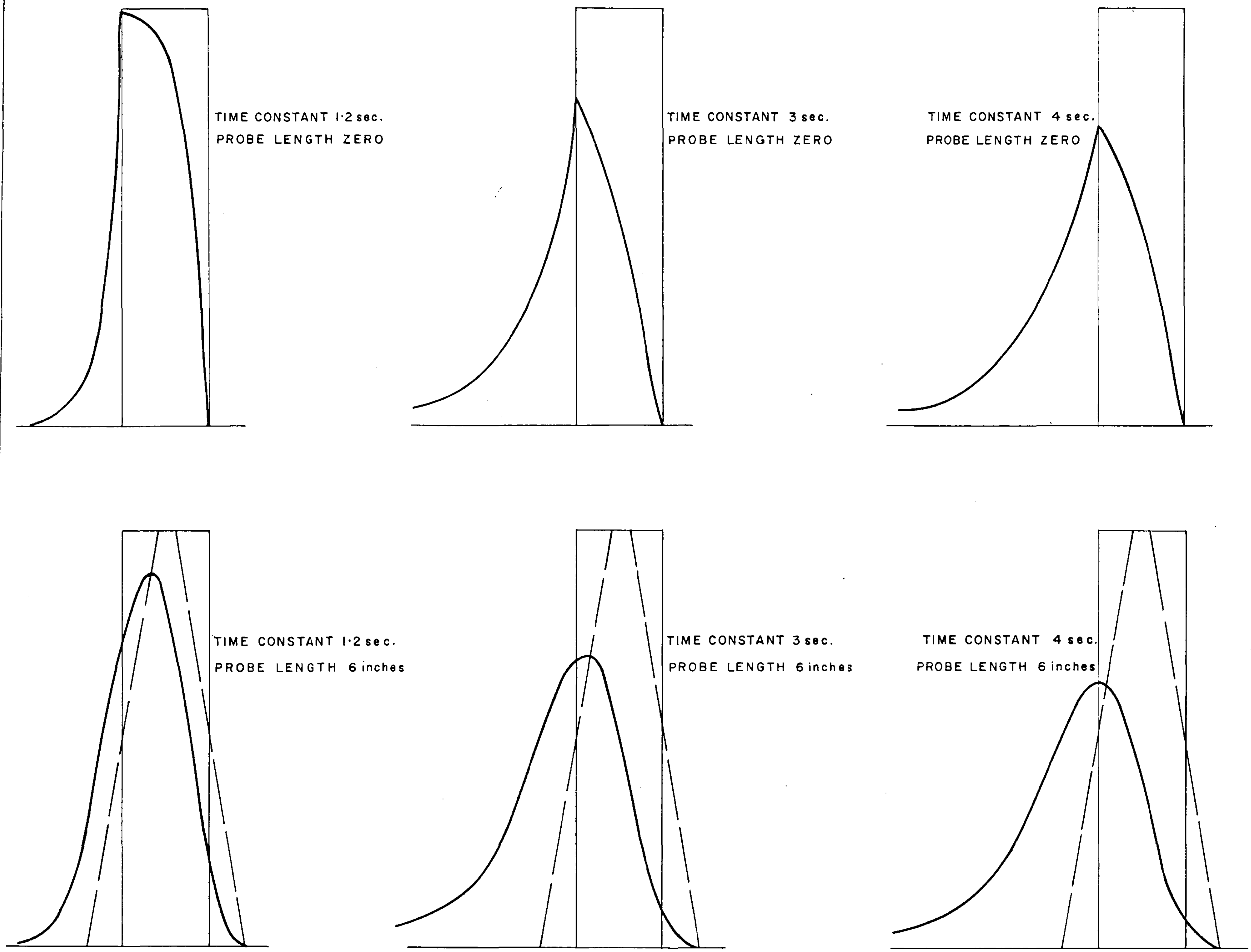
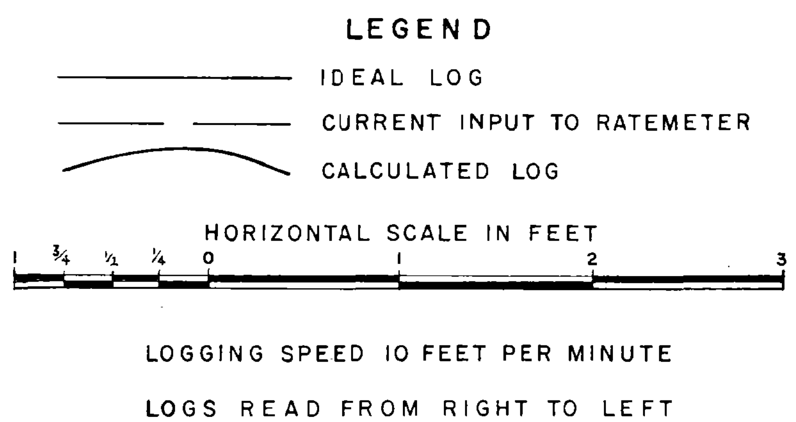
PLATE 4





CALCULATED RADIOMETRIC
BORE LOGS FOR VARIOUS
PROBE LENGTHS AND
TIME CONSTANTS

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