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AN ATTEMPT TO MEASURE THE NON-POTENTIAL FIELD, 1959.

by

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SUMMARY

An attempt was made to measure directly the line integral of the geomagnetic field around a closed circuit with an area of 3620 km^2. The intensity and direction of the horizontal component of the magnetic field were measured at 146 sites spread around the circuit, which was located near Carnarvon (Western Australia), where local irregularities due to crustal magnetism are small.

In theory the line integral should be almost zero. The result obtained is

$$\oint \underline{B}.d\underline{R} = -15 \pm 450 \text{ gil}$$

(1 gil = 1 gilbert = 1 gauss-cm).

$$\text{or curl } \underline{B} = -0.0041 \pm 0.125 \text{ gammas per km}$$

the upward direction being considered positive for curl $\underline{B}$. The quoted uncertainty is the standard deviation. The result is not significantly different from zero, but the uncertainty is larger than the average magnitude of curl $\underline{B}$ over the Earth, according to the recent world magnetic charts.

1. INTRODUCTION

The magnetic field as measured at the surface of the Earth may have three causes:

(1) Electric currents or magnetic material within the Earth.

(2) Electric currents or magnetic material outside the Earth.

(3) Electric currents flowing through the surface of the Earth.

It has been well established that the major part of the field is due to (1), and that transient fields are principally due to (2). Whether or not part of the main field is due to (3) has been a matter of controversy for some years.

The combined field from (1) and (2) is irrotational in the immediate neighbourhood of the Earth's surface; that is to say its curl vanishes. Now the curl of a vector can be defined as a line integral of that vector around a closed circuit, per unit area enclosed. Hence if the curl vanishes, the line integral must vanish when taken around any closed circuit. If this is so, a scalar potential can be defined in terms of line integrals, and the field can be expressed as the gradient of that scalar potential. The field from (3), however, will have a non-zero curl on the surface and therefore cannot be expressed as the gradient of a scalar potential. Therefore it is usually called the 'non-potential' field.

One of the objects of atmospheric electric research is the detection and measurement of the electric current flowing vertically through the surface of the Earth. This current varies somewhat with position, time of day, and weather. A reasonable average fair-weather value for locations remote from cities (e.g. at Watheroo, Western Australia) is of the order of 10^{-6} e.s.u. (Wait, 1943). The resulting values of curl $\underline{B}$ and the line integral are given in Table 1. They are too small to be detected by magnetic field survey.

Historical background

World magnetic charts for early epochs suggested that the non-potential field was a considerable fraction of the total field. As a result of a detailed study of the 1922 world magnetic charts, Bauer (1923) concluded that almost three per cent of the measured field cannot be derived from a scalar potential. He says: 'Whatever the proper interpretation of the non-potential effects may be found to be we apparently cannot escape the probable existence of a non-potential magnetic system'. He concluded that there were downward currents in the tropics and upward currents near the poles. Dyson and Furner (1923) say: 'Though there is some evidence for Prof. Bauer's results the existence of vertical electric currents is not indicated with any great certainity'.

Sir Arthur Schuster (1926) examined the question in detail, using a survey of Great Britain. He concluded that the evidence for the existence of a non-potential field was strong and states: 'Were it not for the extreme difficulty of finding a rational explanation, it would probably be accepted as decisive'. The survey on which this conclusion was based consisted of about 200 stations throughout the British Isles. They were chosen to fulfil the requirements of a normal magnetic survey and not to settle the question of the existence of a non-potential field.

As the quality of magnetic charts has improved, the existence of a non-potential field has become less likely rather than more likely. An extensive analysis of the main field was made by Vestine and his co-workers for epoch 1945 (Vestine *et al.*, 1947; Vestine *et al.*, 1948). In the former publication there is a table of the vertical component of curl $\underline{B}$ for every five-degree square. For many squares the values are over 100 gammas per degree (0.9 gammas per km), but most of these are in the polar regions or over oceans, where large errors in the magnetic charts are likely. There is a tendency for line integrals around parallels of latitude to be negative between 50°S and the south pole, and positive between 50°S and the equator, but there is no such tendency in the Northern Hemisphere, where the field is better determined.

Table 1 indicates some of the previous estimates of the magnitude of the non-potential field for various epochs and locations, including the neighbourhood of Carnarvon, Western Australia.

TABLE 1

Location	Epoch	Vertical current (mA/km^2)	curl B (γ/km)	Line integral around 3620 km^2 (gil)	Authority
Overall average	1885	167	0.21	760	Schmidt, quoted by Bauer (1920)
SE England	1918	+240	+0.302	1094	Schuster (1926)
45°N to 45°S	1920	10	0.0126	46	Bauer (1923)
American Zone	1885 1922 1945	+36.7 +20.6 -0.6	+0.046 +0.026 -0.0008	162 94 3	Vestine *et al.* (1947, 12)
Eurasian Zone	1885 1922 1945	-81.6 -35.0 +2.4	-0.102 -0.044 +0.003	369 159 11	ditto
20°S to 30°S 90°E to 120°E	1945	+57	+0.071	257	ditto
22½°S to 27½°S 112½°E to 117½°E	1945	+158	+0.198	717	Vestine *et al.* (1948, 407)
Typical Atmospheric Electric Observation	-	3×10^{-3}	4×10^{-6}	0.014	Wait (1943)

Line integral survey

In spite of the evidence against the existence of a non-potential component of the magnetic field, a direct measurement free of many of the errors in magnetic charts is worth attempting.

The relation between the line integral, the vertical component of curl $\underline{B}$, and the vertical current density is

$$A\,(\mathrm{curl}\ \underline{B})_Z = \oint \underline{B}.d\underline{R} = 4\pi \underline{AJ}_Z = 4\pi I$$

where the line integral is taken around a closed horizontal circuit enclosing an area A through which the average current density is J and the total vertical current is I. The subscript Z indicates the vertical component of the vector. The only one of these quantities measurable directly by magnetic means is the line integral.

To be of scientific value, curl $\underline{B}$ must be determined with a standard deviation of about 0.01 gammas per km. This is equivalent to 36 gil in the line integral around a circuit of area 3620 km^2.

Units and notation

As is the custom in almost all geomagnetic surveying, the unit of magnetic intensity used in this report is the gamma (γ), which is defined as 10^{-5} gauss.

Magnetic potential and line integrals of the magnetic field have the same dimensions. The unit in the e.m.u. system is the 'gilbert' (gil) which is defined as one gauss-cm. It is also equal to one gamma-km, which is a more convenient form for field surveying.

All distance measurements made by the surveyors were in chains. The computations were carried out in units of 'hectachain-gamma'. This is a convenient unit, because it is very nearly 2 gil. The following notation is used in this report (vector quantities are underlined):

$\underline{B}$	magnetic field
H, X, Y, Z	horizontal, northward, eastward, and vertical (downward) components of the magnetic field.
D	declination
L	line integral of $\underline{B}$ around a closed circuit
$\underline{R}$	position vector of a point on the circuit with respect to an arbitrary origin, so that $d\underline{R}$ is a vector element of the perimeter of the circuit.
A	area of the circuit
s	distance between adjacent sites
e, E, δ	errors (explained in text)
x, y, z	north, east, and vertical components of the vector relative position of adjacent sites
λ	latitude
G	geomagnetic constant
r	radius of the Earth

2. SURVEY DESIGN

The quantity to be measured is the line integral

$$L = \oint \underline{B} \cdot d\underline{R}$$

$\underline{B}$ being the vector magnetic field and $d\underline{R}$ the vector element of the perimeter of the closed circuit about which the integral is taken. In practice the field must be measured at a finite number N of sites, so that the circuit becomes a polygon with N sides, and the integral defining L is approximated by the sum

$$L = \sum \underline{B} \cdot d\underline{R}$$

$$= \sum_i (X_i x_i + Y_i y_i + Z_i z_i) \quad \ldots\ldots (1)$$

in which X_i, Y_i, Z_i are the north, east, and vertical components of the mean field between sites i and (i + 1), and x_i, y_i, z_i are the distances in north, east, and vertical directions between sites i and (i + 1).

Choice of circuit

Suppose observations were made at N sites around a circular circuit and space s km apart. If the individual errors are uncorrelated, the error in L is $es\sqrt{N}$ where e is the standard deviation of an individual observation. The area of the circuit is

$$A = N^2 s^2 / (4\pi)$$

We require that the error in curl $\underline{B}$ is about 0.01 gammas per km, i.e.

$$0.01 = e\, s\sqrt{N}/A \quad \ldots\ldots\ldots\ldots (2)$$

$$= 4\pi e\, s^{-1} N^{-3/2}$$

Taking e = 3 gammas, $s\,N^{3/2} = 3750$ km

It is not practicable to make N more than about 150.

Hence $s = 2$ km

and $A = 7000\ \text{km}^2$

In order that the sum (1) shall be a good approximation to the integral defining L, the average value of the field over each side of the polygon must not differ by more than a few gammas from the mean of the values obtained at the two adjacent sites. In other words the gradient of each component of the field must not vary by more than a few gammas per km over a distance of 2 km. In most places in Australia this is certainly not true; the problem is to find a sufficiently large area over which it is true. The Bureau of Mineral Resources has made aeromagnetic surveys over most of the Carnarvon Basin, which is a deep sedimentary basin and therefore inherently smooth magnetically. Even there, however, it is necessary to avoid the steeper magnetic gradients which occur in some places even far from the surface contact with magnetic rocks. It is not sufficient that the location be a deep sedimentary basin; the details of its magnetic field must be known.

The circuit finally chosen is shown on Plate 1. It is more or less circular in shape with a radius of about 18 miles, the centre being about 45 miles east-south-east of the town of Carnarvon. Its exact shape was dictated largely by roads, fences, etc. Its area is 3620 km^2 and perimeter 266 km so that the area is only 64 per cent of that of a circle with the same perimeter. Also shown on Plate 1 are base stations and contours of total magnetic intensity derived from an airborne survey.

To summarize the advantages of this location :

(a) It is known to be free of local geological magnetic interference, so a site separation of 2 km is practicable.

(b) It is almost flat, so the value of the line integral is insensitive to the value of the vertical component. In fact the contribution of Z can be shown to be negligible (see '<u>Time variations</u>,' below).

(c) The district is generally free of artificial fields. The only possible sources of interference are wire fences and iron telegraph poles, and these can easily be avoided.

(d) The district is sufficiently sparsely populated for interference with local inhabitants to be negligible. On the other hand there is enough habitation to provide roads and tracks. Two-wheel-drive vehicles could be used on all but one small portion of the circuit.

(e) Vegetation is thin, although most of it is higher than eye level. Very little clearing had to be done for sighting.

(f) The town of Carnarvon is within half a day's driving from any part of the circuit.

(g) The country is similar to that at Watheroo (some 500 miles to the south) where air-earth currents were measured for many years. There is no reason to believe that air-earth currents near Carnarvon are appreciably larger.

(h) Throughout the circuit the declination is numerically less than one degree. This makes some of the calculations easier.

It is interesting to compare the accuracy aimed at (standard deviation of 36 gil) with the value of the line integral half way around the circuit. This is about 1,800,000 gil. The object is to determine L with an accuracy of 2 parts in 10^5.

<u>Accuracy of geometric survey</u>

As well as accurate values of the magnetic field, accurate values of geometric quantities are required for the determination of L.

The North and East components are related to the declination and horizontal intensity by

$$H = H.\cos D$$

$$Y = H.\sin D$$

When D is a small angle, X is almost independent of D, and Y is almost independent of H. The standard deviation of an individual reading of H is expected to be about 3 gammas and that of D about 0.4', so that the expected error in either X or Y is about 3 gammas.

The errors in the first two terms of (1) are

$$\delta(X_x) = x.\delta X + X.\delta x$$

$$\delta(Yy) = y.\delta Y + Y.\delta y$$

The values of x and y depend on the direction of the side of the polygon, but the larger can vary, only between 2 km and 1.4 km (for a site separation of 2 km). Hence $x.\delta X$ or $y.\delta Y$ will be about 6 gil. It is desirable that errors due to $X.\delta x$ or $Y.\delta y$ shall be no greater than 6 gil.

The north component X has a value of about 28,000 gammas in the neighbourhood of Carnarvon. In order that $X.\delta x$ shall be 6 gil or less, δx must be no more than 21 cm.

This requires surveying of high precision. However, the requirement can be greatly relaxed by dividing the field into two parts, one of which approximates the average value of H but is known to be irrotational. If we write

$$\underline{B} = \underline{B}_o + \underline{B}'$$

where $\underline{B}_o$ is a centred dipole field, the remainder $\underline{B}'$ can be made much smaller than $\underline{B}$. In fact $\underline{B}_o$ can be so chosen that the greatest value of X (the north component of B) is 169 gammas. The accuracy required in x is then

$$\delta x = \frac{6}{169} \text{ km} = 35 \text{ m}$$

which is easily obtained by rapid survey methods.

It would have been possible to use an inclined dipole field for $\underline{B}_o$ which has a non-zero east component. However, the declination near Carnarvon is so small that the maximum value of Y is 407 gammas, and the extra complication is not worthwhile. With a value of Y of 407 gammas the necessary accuracy in y is

$$\delta y = \frac{6}{407} \text{ km} = 15 \text{ m}$$

which again is easily achieved.

Contribution of the vertical component

Unless the circuit is completely level it can not be assumed that the contribution of $\sum Z.z$ to L is negligible. However, since z is always very much smaller than $\sqrt{(x^2 + y^2)}$ it is clear that L is not critical to values of Z. Therefore Z was measured at only 26 sites around the circuit.

Elevations around the circuit were derived from a map showing results of the gravity survey of the Carnarvon Basin (BMR drawing No. 98-38). They range from 70 to 345 ft above sea level.

Altitudes and values of Z were plotted and averages estimated for groups of 10 consecutive sites. The resulting value of $\sum Z.y$ (in a counter-clockwise sense) is -13,600 gamma ft = -4.2 gil.

This is negligible compared with the errors involved in L, so the contribution of the vertical component can be ignored.

Although this figure was derived rather approximately, it could hardly be in error by as much as a factor of two.

Time variations

The magnetic field varies with time as well as with position. Therefore control measurements must be made to eliminate the time variation. It is assumed throughout this work that the time variation applies uniformly over the whole circuit, that is to say
$\partial^2 \underline{B} / \partial x . \partial t = 0$.

This assumption is almost certainly justified over distances of up to a hundred miles in the case of H and D. It is less certain regarding Z, but this element is not used in the survey. Details of the method of allowing for time changes will be described under 'Field operations' and 'METHOD OF ANALYSIS'.

3. SURVEY OPERATIONS

Geometric survey

The geometric and geodetic data were obtained by a party from the Lands and Survey Branch (Western Australia) of the Department of the Interior, who collaborated with the BMR party. The data supplied by the Survey branch are:

(a) The true azimuth to a reference mark from each site; this is required for calculating declination.

(b) The northward and eastward distances between adjacent sites (x and y).

(c) The latitude and longitude of each site; these are required for calculating values of $\underline{B}_o$.

Field operations

Most of the field work was done between 4th August and 30th September 1959. The complete party consisted of:

Bureau of Mineral Resources :	W.D. Parkinson (until 22nd August) R.G. Curedale J. van der Linden
Lands and Survey Branch:	A. Rochford D. Moor J. Terry
Field hands:	B. Hack A. Roberts

Messrs Curedale and van der Linden revisited the location from 15th to 30th June 1960 (see '<u>Resurvey of 1960</u>', below).

Instruments

Two Askania horizontal torsion magnetometers (HTM Nos. 158 and 704) were used for measuring the horizontal intensity and magnetic azimuth of the reference mark. The declination is the difference between true and magnetic azimuths.

A La Cour magnetic zero balance (BMZ) was used to measure the vertical intensity. A continuous record of the time variations of the magnetic field was obtained with an Askania geomagnetic variograph. Azimuths at field sites were supplied by the surveyors, but at base stations an Askania midget theodolite was used for solar azimuth determinations.

The HTM is designed to measure the horizontal intensity. The magnetic azimuth is obtained as a by-product of the horizontal intensity determination. Preliminary tests indicated that the instrument was suitable for high-accuracy declination measurements. In order not to involve too many instruments and to keep the observing technique as simple as possible, it was decided to use HTM for declination as well as horizontal intensity measurements. This was an unfortunate choice. Declination, as determined by an HTM, is sensitive to torsion in the fibre, and it was found that this can change erratically within a few days. Frequent intercomparisons are necessary, and complicated corrections must be made.

Survey procedure

Camps were established at five base stations: Brick House, Callagiddy, Marron, Yalbalgo, and Doorawarra (See Plate 1). An adjacent section of the circuit was surveyed from each camp. Two different techniques were used. The first is a 'leap-frog' technique; e.g. observer A observes at site 1 and observer B observes at site 2 simultaneously, then observer A moves to site 3 while observer B remains at site 2 and they observe simultaneously again, then observer B moves to site 4 etc. Thus a sequence of site differences is obtained by simultaneous observations. This is not a suitable technique. Errors are cumulative and coherent. The error in L, using this method, is proportional to $N^{3/2}$ (see '<u>Systematic errors</u>', Chapter 5). Errors can be minimized by using frequent ties spanning several sites.

The leap-frog technique was used from sites 1 to 36. Thereafter, one HTM was kept at the base station while the other was used simultaneously at sites along the circuit. At the same time, the variograph was operating at the base station. There are thus two methods of correcting for time variations, directly from the base HTM or from variograph scaling. The latter appears to give more consistent results and was used except when the variograph baseline values were not steady. The base station HTM values were used to standardize the variograph and for time control when the variograph was unsteady.

Sites 37 to 42 were controlled from Callagiddy base station (C), sites 44 to 83 from Marron (M), sites 85 to 105 from Yalbalgo (Y), and sites 107 to 146 from Doorawarrah (D). Sites 43, 84, and 106 were tied to both base stations; in this way the base stations themselves were tied together.

Both of these methods have the disadvantage that coherent errors can occur. For instance the easterly distance between sites 43 and 84 is 48.6 km. An error of 0.2' in declination at Marron base station involves an error of 78 gil in L. Thus the line integral is very critical to the relative values at base stations.

To try to make a consistent set of values for base station differences, independent ties were made between the following sets of sites : 1-18, 18-33, 33-M, 35-M, 1-M, D-33, D-27, D-23, D-18, D-1.

Remarkably good closures were obtained from the mean values of the ties between base stations as determined in 1959. For declination, after adjusting two ties (D-33 by 0.5' and 1-M by 1.1') station values could be adopted which agreed with every tie value within 0.1'. For horizontal intensity all original tie values agreed with adopted values within 4 gammas except those tying D to sites 23, 27, and 33. These indicated that H values between sites 23 and 33 were too low, but they were not consistent and were rejected in favour of the values that gave a small misclosure.

Resurvey of 1960

The 1959 survey yielded the result

$$L = +300 \text{ gil.}$$

An analysis of sources of errors indicated that errors due to base station ties were far larger than those from any other source. Therefore it was decided to resurvey the base station differences in 1960.

Doorawarrah was used as a base, for simultaneous observations at Callagiddy, Marron, Yalbalgo, and site 1. Two sets of simultaneous observations were made at each pair of stations, the instruments being interchanged between sets. Each set consisted of 16 observations. Generally the scatter was small, so the reading error in the mean must be considered to be very small in every case.

Table 2 summarizes the base station differences.

TABLE 2

Base Station	Values relative to Site 1: Horizontal Intensity (gammas) 1959 Individual observations	1959 Adopted	1960	Declination (minutes) 1959 Individual observations	1959 Adopted	1960
23	-103, -95	-103	-81			
28	-131, -128	-131	-122			
31	-159, -147	-159	-148			
Callagiddy	no direct	-173	-165	no direct	-8.1	-8.2
Marron	-259, -260	-259	-250	-11.4, -11.5	-12.9	-13.6
Yalbalgo	-61, -64	-59	-57	+ 1.9, + 2.2	+ 0.7	+ 2.5
Doorawarrah	+73, +73, +74	+73	+73	+16.0, +16.1 +15.7	+15.9	+15.9

The 1960 survey disagrees with the 1959 adopted values in two regions;

(a) the adopted values of H appear too low between site 23 and Marron by about 9 gammas

(b) the adopted values of D at Marron and Yalbalgo both appear to be wrong.

A change in the value at a base station δX causes a change in the value of L of $\delta X\,(x_f - x_i)$ where $(x_f - x_i)$ is the distance, in a northerly direction, between the initial and final sites of the group for which this base station is used, e.g. for Marron between sites 43 and 84. Table 3 shows the effect of adopting the 1960 base station values. For convenience westerly declination Y was considered positive.

TABLE 3

DECLINATION

Base station	Group of sites	Change in base station value (minutes)	(gammas)	Westing (km)	δL (gil)
('leap-frog')	1–37	0	0	+10.7	0
Callagiddy	37–43	+0.1	+1	- 0.8	-1
Marron	43–84	+0.7	+5.7	-48.6	-277
Yalbalgo	84–106	-1.8	-14.8	-17.3	+256
Doorawarrah	106–116	0	0	+57.8	0
			Total		-22

HORIZONTAL INTENSITY

Base station	Group of sites	Change in base station value (gammas)	Northing	δL
-	1–28	0	-16.3	0
Callagiddy etc.	28–43	+9	-31.0	-279
Marron	43–84	+9	- 6.8	-61
Yalbalgo	84–106	+2	+30.0	+60
Doorawarrah	106–1	0	+24.1	0
		Total		-280

This table shows how sensitive the value of L is to changes in base station values.

4. METHOD OF ANALYSIS

The value of L is almost independent of the absolute values of H and D. (See 'Uniform errors', Chapter 5). Therefore site 1 was chosen as a reference, and all subsequent observations were corrected to the first observations made at site 1.

The value of L is also insensitive to components of the field almost perpendicular to the direction of $d\underline{R}$. Therefore not all computations of observations were checked. For stretches of the circuit travelling north or south, D was left unchecked, and for stretches of the circuit travelling east or west, H was left unchecked. Otherwise all observations were checked.

After a careful consideration of all intercomparison data, it was decided to apply uniform corrections to HTM No. 158 of +4.0' for D and +6 gammas for H. The intercomparisons were not always consistent, but these corrections seemed to give no systematic errors.

Slightly different methods of analysis had to be used for the results of the 'leap-frog' style of observation and those referred to base stations. In the former case, the observations yield a sequence of site differences between adjacent sites. Every sequence of 20 or fewer sites was controlled by simultaneous observations at the first and last (tie observations), for instance at sites 1 and 18. Individual site values were graphed as a function of latitude, and adjustments were made to reconcile them with the tie values.

For sites referred to a base station, the value at the site with respect to the base station, and hence to site 1, was determined by simultaneous observations.

These processes yielded the values of D and H at each site at the time of the first observations at site 1. These values of D and H had to be converted into Y and X. The computation of Y presented no problem. For each site D was multiplied by H and rounded off to one gamma. Values of H changed sufficiently for this rounding off process to be random. The value of y for each side of the polygon was multiplied by the mean value of Y for the two sites at the ends of the side, and the resulting values of yY were tabulated and summed.

For computation of X an interesting difficulty arises. The declination is so small that in no case is the difference between H and X as much as one gamma. However, if H is used instead of X at all sites there is a systematic error, because the declination is generally greater (westerly) where the circuit is running southward (when taken in a counterclockwise sense). This difficulty was overcome by using H instead of X for all computations and then applying a correction to the final result.

To reduce the sensitivity of L to errors in distance measurements, an irrotational field that approximates the actual field was substracted from all measured values of H.

Let X_o be the north component of the irrotational field. Then writing the residual as X',

$$H = X = X_o + X'$$

$$\oint X.dx = \oint X_o dx + \int X'dx$$

$$= \int X'dx$$

The irrotational field used was that of an axial dipole of moment $0.31026r^3$ c.g.s. units, where r is the radius of the Earth. This field has no east component, so north and horizontal components are both

$$X_o = 31026 \cos\lambda \text{ gammas,}$$

where λ is the latitude. Values of X were calculated for each site (latitudes had been supplied by Lands and Survey Branch), and residual north components

$$X' = H - X_o$$

were also calculated. These were slightly in error because H was used instead of X.

The correction necessary because H was used can be calculated as follows:

$$H = H \cos D = H - H\,D^2/2 \qquad \text{(D in radians)}$$

$$\sum xX = \sum xH - \sum xHD^2/2$$

H has a total variation of only three per cent; therefore a good approximation to the second term on the right is

$$\tfrac{1}{2}\,\bar{H} \sum x\, D^2$$

where $\bar{H}$ is the mean value of H.

This was calculated by dividing the circuit into groups of eight consecutive sites, tabulating an average value of D, and deriving x from the latitude change. The result is

$$\tfrac{1}{2}\,\bar{H} \sum xD^2 = -\,76 \text{ gil}$$

Any error due to using an average value for H in this expression can be only a few gil.

5. DISCUSSION OF ERRORS

Errors in the quantities which determine L can be divided into three types :

(a) uniform errors

(b) random errors

(c) systematic errors

Both magnetic and geometric survey results could be subject to all of these types of errors.

Uniform errors

Magnetic errors of type (a) arise due to faulty absolute calibrations. If all the H values are uniformly too high or too low, this amounts to a spurious field consisting only of a uniform north component. Such a field is irrotational, so its presence would not affect L.

If all D values are uniformly in error by D', the error introduces a spurious field consisting of an east component only of the form

$$Y = H \sin D'$$

To a good approximation, H can be written

$$H = G \cos\lambda$$

where λ is the latitude and G is the geomagnetic constant

Then

$$Y = G \cos\lambda \sin D'$$
$$\doteqdot GD' \cos\lambda$$

The vertical component of the curl of such a field is

$$2\ GD'\ (\sin\lambda)/r$$

(r is radius of the Earth), so the line integral is

$$\delta L = 2AGD'\ (\sin\lambda)/r$$
$$\text{or} \quad \delta L/D' = 4.3 \text{ gil/min}$$

The HTM instruments used to measure D were standardized at Toolangi Observatory just before leaving for Carnarvon, and after return. Taking into account the different torsion error, the adopted value at site 1 appears to be in error by 1.2'. Therefore the error in L due to uniform magnetic errors is about 5 gil, which is negligible. The same considerations apply to uniform errors in azimuths of reference marks. They are certainly less than one minute, so their influence on L is negligible also.

Random errors

At every site there will be a reading error, which should not be correlated with the reading error at adjacent sites. From a consideration of baseline values and repeated site differences, the standard deviation of each reading is estimated to be 3 gammas in both X and Y. In summing the 146 sides of the circuit, the error in L due to this is

s

$$e\ s\sqrt{N} = 3 \times (266/146) \times \sqrt{146} = 66 \text{ gil}$$

(the perimeter is 266 km so s = 266/146)

A similar random error can occur due to errors in azimuths. Probably all azimuths were accurately determined, but in some cases the reference mark was too close to the magnetic site and in some cases there was the possibility of movement of the reference mark. These errors probably apply only to a few sites. They are difficult to assess, but they are most likely to show up in a plot of declination as a function of longitude. Thirteen sites were found at which the D values departed abnormally from the smooth curve. The average departure was only 7 gammas. An estimate of this type of error is therefore

$$7 \times \sqrt{13} \times (266/146) = 46 \text{ gil}$$

Systematic errors

The type of error that has most affect on L is the sytematic error. This applies coherently to large portions of the circuit. The resulting error in L is not reduced by the large number of sites, nor by the closure of the circuit. Any error in a secondary base station is of this type, because it applies coherently to all sites referred to that base station. An error in H at a base station must be multiplied by the total northing of all sites referred to the base station, and an error in Y by the total westing; this has been done in the section headed 'Resurvey of 1960' (Chapter 3). We shall now try to estimate the error of the base station values.

For horizontal intensity eight simultaneous observations, with an individual standard deviation of about 3 gammas, were made to establish the station differences. Instruments were exchanged between sets, and instrument differences were constant throughout the series. It is probably safe to allow a standard deviation of 2 gammas for the H station differences as determined in 1960. This will give an error in L of 28, 14, 60, and 48 gil for sites based on Callagiddy, Marron, Yalbalgo, and Doorawarrah respectively. These are independent, so the overall error is estimated as 83 gil.

For declination the same procedure was used to redetermine the base station differences in 1960. The magnetic meridians therefore are determined with an accuracy of about 0.25' and so errors in L due to errors in magnetic azimuth at Callagiddy, Marron, Yalbalgo, and Doorawarrah are 2, 97, 35, and 116 gil respectively, giving an overall error in L of 155 gil.

However, in the case of declination, errors in the determination of azimuths at the base stations must be taken into account. For the 1960 resurvey results to be applicable, azimuths at base stations in 1959 and in 1960 must both be accurate. Azimuths are determined by the mean of either 4 or 8 individual observations, the standard deviation of each of which is about 0.4'.

However, the means of groups often disagree by as much as a minute. One minute is probably a reasonable standard deviation of a declination determination, including azimuth.

According to this the uncertainty in azimuths introduces a further error of (standard deviation) 232 gil.

However, the disagreement between the 1959 and 1960 station differences suggests much larger errors in declination. For instance the adopted station difference between Marron and Yalbalgo from the 1959 survey is 13.6'. This is based on two measurements between Marron and site 84 (11.8 and 10.8) and two between site 84 and Yalbalgo (2.0 and 2.7). According to the 1960 survey it is 16.1', based on two sets of 16 simultaneous measurements between each station and Doorawarrah.

In spite of the large changes made to the adopted declination measurements at some base stations, it is interesting to note that the 1959 and 1960 values of ΣYy differ by only 22 gil. This, of course, may be fortuitous. Another source of systematic error is the 'leap-frog' method. If the value of L is derived from the (n-1) observed differences between n sites, adjusted so that their sum equals a known difference between sites 1 and n, and e is the standard deviation of each observed difference, and s is the distance between adjacent sites (assumed uniform), then it is shown in the appendix that the standard deviation of L (for large n) is

$$E = (e\ s\ n^{3/2})/3^{\frac{1}{2}} \quad \ldots\ldots \quad (3)$$

Two sequences of sites were surveyed by this method, namely sites 1 to 18 and 19 to 33. Taking e as 3 gammas and s as 2 km the standard deviations for the two sequences are 264 gil and 201 gil respectively. This overestimates the error slightly, because sites 1 to 36 have been included in the random reading errors above.

Summarizing the sources of error, and the contributions to the variance :

Type of error	Standard deviation	Variance
reading errors	66 gil	4356 gil^2
azimuth errors	46	2116
'leap-frog' errors	334	110,400
base station H	73	6889
base station magnetic azimuths	155	24,000
base station true azimuths	232	53,800
Total variance		201,561

The total variance is approximately 200,000 gil^2 giving a standard deviation of 447 gil. This estimate is not very well determined. The uncertainty in declination base station differences suggests that the estimate may be low.

6. OBSERVED VALUE OF LINE INTEGRAL

The uncorrected value of

$$\sum (xH + yY)$$

from the 1959 survey was +211 gil. The circuit was traversed counter-clockwise, so a positive value of L indicates an upward flow of electric current. To this must be applied a correction of +76 gil because H instead of X was used in the computations. Rather than recalculate the field at every site on the basis of the 1960 survey, a correction for the difference between the 1959 and 1960 base station differences was applied. From '<u>Resurvey of 1960</u>' (Chapter 3) they are -22 for D and -280 for H. Hence the final result for the line integral around circuit is

$$\begin{aligned} L &= +211 + 76 - 22 - 280 \\ &= -15 \pm 450 \text{ gil}, \end{aligned}$$

for the average vertical component of curl of field

$$(\text{curl } \underline{B})_z = -0.0041 \pm 0.125 \text{ gammas per km}$$

and for the average current density

$$J = -3.3 \pm 99 \text{ mA/km}^2.$$

The final value is certainly not significantly different from zero but, because of the large uncertainty, the result is of doubtful value. Referring to Table 1, the above result is not inconsistent with any of the values except Schuster's figure for south-eastern England, Vestine's figure for 115°E, 25°S, and the overall average obtained by Schmidt. Modern values over extended areas are less than the standard deviation obtained here.

7. RECOMMENDATIONS

The experience gained in carrying out and analysing this survey indicates that the method could be improved in many ways. The following suggestions are offered in case similar work should be attempted in the future:

(a) A fibre declinometer should be used for determining magnetic azimuths instead of an HTM. This should eliminate much of the instrumental uncertainity in measuring D.

(b) Reference marks should be more definite and should always be at sufficient distance from the magnetic site. It would be preferable if the geophysicists were to choose the magnetic sites and arrange reference marks, leaving the surveyors to determine angles and azimuths.

(c) All sites on the circuit should be referred to one base station. A variograph should be set up at that base station, and the same instruments should be used for field observations and to standardize the variograph. Progress would be much slower with this technique; probably about four months would be required to complete the survey.

With a method based on these recommendations it should be possible to decrease the standard deviation of the result to about 70 gil around this circuit, i.e. 0.026 gammas per km in Curl B.

8. LINE INTEGRAL FROM MAGNETIC CHARTS

It is worth while to compare the accuracy of this survey with that obtainable from isomagnetic maps. Using Equation (2), the error in curl B is

$$E = e\, s \sqrt{N}/A$$

Parkinson, Curedale and van der Linden (1962) give a frequency distribution of the difference between the values of H observed at about 700 locations in Australia, and the values read at those locations from the contour lines of the map. The standard deviation of the difference is 154 gammas. The standard deviation for the corresponding quantity in Y is also about 150 gammas. This, then, can be taken as the standard deviation of the error involved in reading a value of H or Y from an isomagnetic map. This slightly underestimates the value of the standard deviation, because the contours themselves are based on the same values, but since they are smoothed over groups of about five stations, the true standard deviation would be larger only by a factor of about 5/4.

A further relation can be used to eliminate N. If the circuit is circular its area is

$$A = N^2 s^2 / 4\pi$$

so that (2) can be written

$$E = e\, s^{\frac{1}{2}} (4\pi)^{\frac{1}{4}} A^{-\frac{3}{4}}$$

In this formula e is about 150 gammas. Suppose we take A to be 10^6 km^2. To avoid systematic errors, s must not be less than the average spacing between magnetic stations, i.e. about 100 km.

Substituting these values, E = 0.089 gammas per km. This is about the accuracy actually achieved and considerably more than the probable error if the recommendations of Chapter 7 were put into effect.

Since E depends on $A^{-\frac{3}{4}}$, it could be decreased by increasing A. However, if A is increased to a value much more that 10^6 km^2, regions of sparse magnetic stations will be included in the survey and s must be correspondingly increased.

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APPENDIX

Consider a series of n sites between which differences d_i are observed each with a standard deviation e. Assume that the errors are mutually independent. Let the distance between sites be s and the value of the magnetic element at the first site be h_1, which is exact. At the second it is

$$h_2 = h_1 + d_1$$

and at site i it is

$$h_i = h_{i-1} + d_{i-1}$$

$$= h_1 + \sum_{j=1}^{i-1} d_j$$

The line integral L is calculated from the formula

$$L = \tfrac{1}{2}\, s\,(h_1 + h_2) + \tfrac{1}{2}\, s\,(h_2 + h_3) + \ldots\ldots\ldots + \tfrac{1}{2}\, s\,(h_{n-1} + h_n)$$

$$= \tfrac{1}{2} \sum_{j=1}^{n-1} s\,(h_j + h_{j+1})$$

$$= \tfrac{1}{2}\, s \left[h_1 + 2 \sum_{j=2}^{n-1} h_j + h_n \right]$$

$$= \tfrac{1}{2}\, s \left[h_1 + 2 \sum_{j=2}^{n-1} (h_1 + \sum_{i=1}^{j-1} d_i) + h_1 + \sum_{i=1}^{n-1} d_i \right]$$

Only the sums containing d_i will contribute to the variance of L. The double sum can be converted by

$$\sum_{j=2}^{n-1} \sum_{i=1}^{j-1} d_i = \sum_{i=1}^{n-2} (n - i - 1)\, d_i$$

so that

$$L = s \sum_{i=1}^{n-2} (n-1-i) d_i + \tfrac{1}{2}\, s \sum_{i=1}^{n-1} d_i + \text{exact terms}$$

$$= s \sum_{i=1}^{n-2} (n-\tfrac{1}{2}-i)\, d_i + \tfrac{1}{2}\, s\, d_{n-1} + \text{exact terms}$$

Now the d_i are statistically independent; hence the variance of L is

$$E^2 = e^2 s^2 \left[\sum_{i=1}^{n-2} (n-\tfrac{1}{2}-i)^2 + \tfrac{1}{4} \right]$$

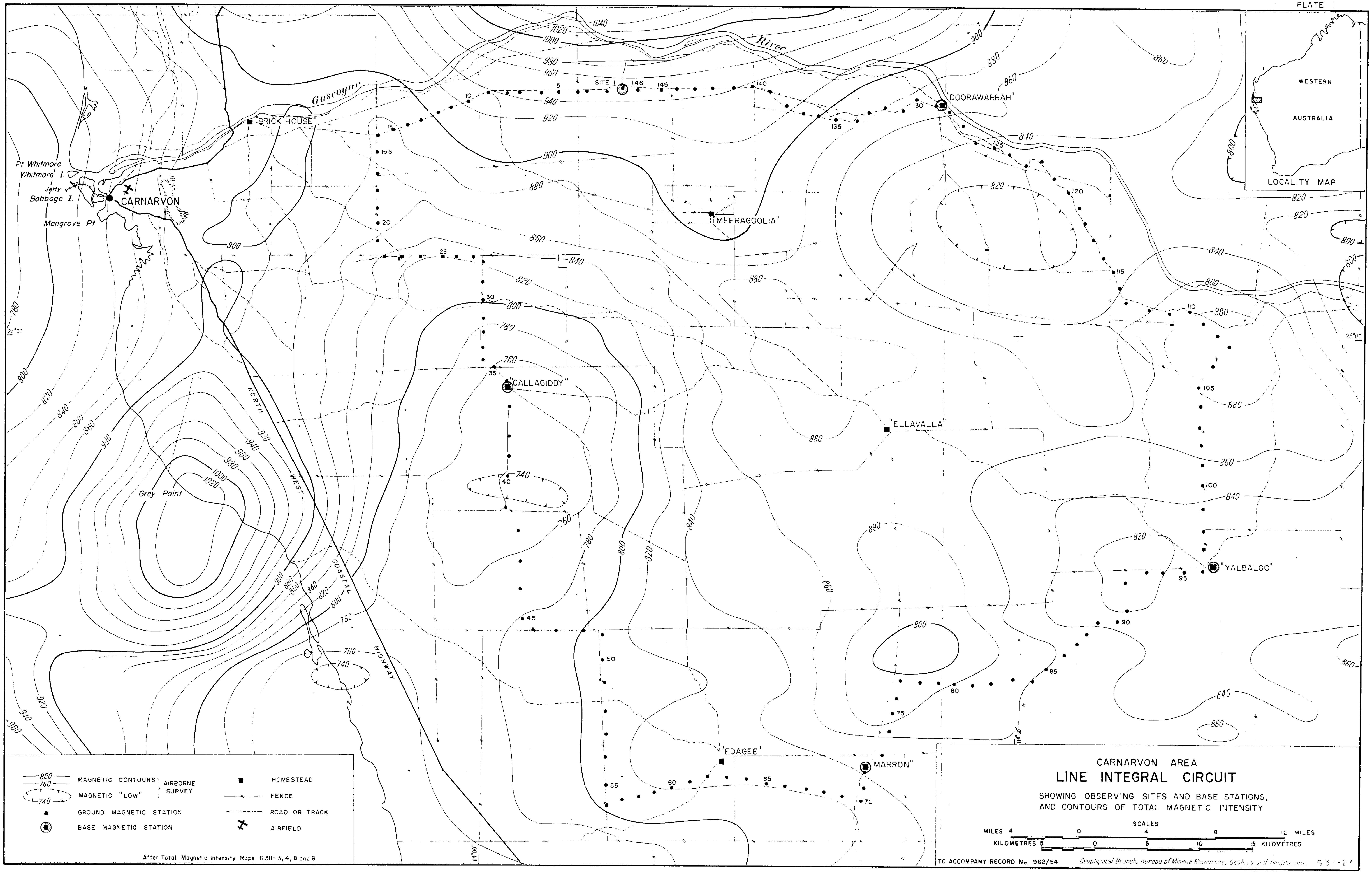
$$= e^2 s^2 \quad (4n^3 - 12n^2 + 11n - 3)/12$$

For large n this approximates

$$E = e\ s\ n^{3/2} / 3^{\frac{1}{2}}$$

If the d_i are adjusted so that their sum equals an exactly determined tie value $h_n - h_1$, the individual standard deviation e should be replaced by $e(1-1/n)^{\frac{1}{2}}$. This changes the coefficient of n^2 in the variance, but not the coefficient of n^3 and so does not affect the asymptotic formula for large n.

PLATE 1



CARNARVON AREA

LINE INTEGRAL CIRCUIT

SHOWING OBSERVING SITES AND BASE STATIONS, AND CONTOURS OF TOTAL MAGNETIC INTENSITY

TO ACCOMPANY RECORD No 1962/54

Geophysical Branch, Bureau of Mineral Resources, Geology and Geophysics. G 31-27