COMMONWEALTH OF AUSTRALIA

DEPARTMENT OF NATIONAL DEVELOPMENT

BUREAU OF MINERAL RESOURCES, GEOLOGY AND GEOPHYSICS

RECORD No. 1963/102

TERRELLA MODEL EXPERIMENT, PROGRESS REPORT 1962

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SUMMARY

This Record is a progress report of the work being done on a terrella model designed to determine whether the conductivity of the water in the oceans controls the direction of the variation in the geomagnetic field as measured at observatories and temporary magnetic recording stations.

The model is far from complete. Sheets of copper to simulate the oceans and an aluminium sphere to simulate the conducting core still have to be installed. However the exciting and detecting circuits are complete and some preliminary measurements have been made of the direction of the primary field. These may be of some interest.

1. INTRODUCTION

For the past six years the Regional Magnetic Group, Geophysical Branch, Bureau of Mineral Resources has been investigating the directions of the vectors representing geomagnetic variations with a period between 10 and 100 minutes. This investigation developed out of an attempt to explain the correlation between the variations in declination and vertical intensity recorded at Watheroo. It was found that such correlations exist at most locations, but are rarely as distinct as at Watheroo. These correlations seem to depend on the position of the station relative to the coastline, or more accurately, to the edge of the continent (Parkinson, 1962a).

This effect could be explained qualitatively by electromagnetic induction in the highly conducting oceans owing to a primary field with a source in the upper atmosphere. The conductivity and size of the oceans appears to be adequate. For instance a flat circular disc 3 km thick with an area equal to that of the Indian Ocean and with the conductivity of sea water would have a time constant to induced eddy currents of the order of three hours (Ashour, 1950).

However, there are some stations at which the correlation between the components of variation cannot be caused by induction in the oceans. It is probable that the geomagnetic effects at these stations are due to non-uniform conductivity in the crust or upper mantle. This suggests the possibility that the effect found along coastlines may be due to some systematic effect deep underground. One possibility is that the mantle under the oceans is of a higher conductivity than that under the continents. This is supported by evidence that the temperature of the mantle under oceans is higher than that under continents at the same depth (Jacobs, 1960). If it can be shown that the correlation between magnetic variations in the three elements is a good index of electrical conditions deep in the earth, this will be a discovery of some geophysical importance. Before this can be established the effect of the conductivity of ocean water must be evaluated.

Some attempt has been made to calculate the effect of induced currents in the oceans, notably by Price (1949; 1950) and by Rikitake and his co-workers (e.g.Rikitake and Yokoyama, 1955). The problem has always been greatly idealised, however, and even so the mathematical difficulties are formidable.

2. PHYSICAL PRINCIPLES

Because the problem is difficult to solve by calculation, it is worth investigating the possibility of constructing a model that will contain the essential features of the problem. These essential features are:

(a) a sphere with a conducting core having a radius about 0.9 times that of the sphere and otherwise non-conducting except for thin sheets which occupy predetermined parts of its surface (the oceans),

(b) a varying primary magnetic field generated by a current sheet concentric with the sphere and of slightly greater radius (ionospheric currents).

We want to know the direction of the resultant magnetic field at all points on the surface of the sphere.

The construction of a satisfactory model depends on finding a system of practicable scale factors that will preserve the direction of the resultant field.

The dimensions of the earth are small compared with the wavelength of an electromagnetic wave with a period of tens of minutes. Therefore it is legitimate to neglect the displacement current term in Maxwell's equations. In this case the magnetic field ($\underline{B}$) varies according to the differential equation.

$$\mu . \sigma . (d\underline{B}/dt) = d^2\underline{B}/dx^2 + d^2\underline{B}/dy^2 + d^2\underline{B}/dz^2 \qquad (1)$$

(rationalised units). If we consider only sinusoidal time variations of angular frequency w, and write the length variables in terms of some constant length parameter of the system, for instance the radius of the sphere (a), so that

$$x = x'.a \qquad y = y'.a \qquad z = z'.a$$

equation (1) can be written

$$i.R.\underline{B} = d^2\underline{B}/dx'^2 + d^2\underline{B}/dy'^2 + d^2\underline{B}/dz'^2 \qquad (2)$$

where the dimensionless constant R

$$R = a^2.\mu.\sigma.w \qquad (3)$$

amounts to the magnetic Reynolds number.

All the variables and parameters in equation (2) are dimensionless except $\underline{B}$. Therefore, if the parameter R has the same value for the model as for the Earth, the essentials of the problem can be exactly duplicated, and the direction of the field should be the same in the model as in the corresponding position on the Earth.

3. THE TERRELLA

The model being constructed to solve this problem is called a 'terrella'. This name was coined by Birkeland (1908) who made a model of the Earth to test his theory of the aurora. It is used here to designate any model of the Earth design to test a geophysical hypothesis.

Scale factors

The physical properties of the terrella must be chosen so that R is equal to that of the earth. The factors of R will be discussed in the order in which they are stated in equation (3).

The size of the model is limited by practical consideration. It is desirable to measure the direction of the field at a point. Therefore the model must be large compared with the detector used. On the other hand it must be sufficiently small that it can be contained in a small room and handled without undue difficulty. The largest perspex sphere easily obtainable was of 56-cm radius and this sphere is used as a former to hold the 'oceans'. This sphere represents the surface of the Earth and so fixes the ratio of the length scale of the terrella to that of the Earth as:

$$\frac{0.56}{6.4 \times 10^6} = 0.88 \times 10^{-7}$$

so a^2 for the terrella is smaller than for the Earth by a factor of 0.77×10^{-14}.

The magnetic material in the crust of the Earth has only a very small effect on the world-wide field. Therefore we can consider the permeability, μ, of the Earth to be that of a vacuum. This is preserved in the terrella by making it of non-magnetic material.

A convenient material to represent the oceans is sheet copper. Its conductivity is about 0.8×10^8 mho/m. The conductivity of sea-water is about 3 mho/m, so the conductivity of the model is greater than that of the Earth by a factor of 2.67×10^7.

The frequency of the applied field must be adjusted so that R is unchanged. Using the subscripts 'e' for the Earth and 't' for the terrella this means that

$$W_e = w_t \times 0.77 \times 10^{-14} \times 2.67 \times 10^7$$

The intervals over which vector changes were scaled from magnetograms ranged from 5 to 50 min, but the interval most often used was 20 min, which corresponds to a sine wave of period 40 min, i.e. $w_e = 2.6 \times 10^{-3}$ radian/sec. Thus w_t is 12.6×10^3 radian/sec, i.e. 2000 c/s.

The values of physical quantities involved in the terrella and the Earth are summarised below:

	(Radius in metres)^2	Permeability	Conductivity of oceans (mho/m)	Frequency (c/s)
Terrella	$(0.56)^2$	μ_0	8×10^7	2000
Earth	$(6.4 \times 10^6)^2$	μ_0	3	4.2×10^{-4}
Ratio	$(0.88 \times 10^{-7})^2 = 0.77 \times 10^{-14}$	1	2.67×10^7	4.8×10^6

All the values which apply to the terrella can be easily achieved in the laboratory. Alternating magnetic fields with a frequency of a few kc/s can be easily detected with a small search coil.

Conductors

The thickness of the copper representing the oceans must be to scale, i.e. 0.88×10^{-7} times the actual depth of the oceans. The depth of most of the major oceans is about 5 km, and is remarkably uniform as shown, for instance, by Wilson (1954, p. 142). The thickness of the copper, therefore, should be:

$$5 \times 10^5 \times 0.88 \times 10^{-7} = 4.4 \times 10\text{-}2 \text{ cm}$$

The closest approach to this in commercially available sheet copper has a thickness of 0.043 cm (0.017 in.).

Rikitake (1961) has shown that the currents induced in the oceans are greatly influenced by the conductivity deep in the mantle, i.e. by the so-called 'conducting core'. Therefore if the terrella is to be a realistic model this conducting core must be represented. Lahiri and Price (1939) have calculated the effect of a number of conductivity distributions on the quiet-day diurnal variations. There is a range of distributions, any of which fits the observed facts, but in all of these distributions the conductivity increases very rapidly with depth at a depth between 0.095a and 0.13a ('a' being the radius of the sphere). In practice only a distribution in which the conductivity increases discontinuously can be incorporated in the terrella. 'Model E' of Lahiri and Price involves a discontinuous increase from zero to infinite conductivity at a depth of 0.095a. Although the calculations are made for infinite conductivity, any conductivity greater than 1 mho/m gives essentially the same result. Scaling up this conductivity by 2.67×10^7 we require a conducting core in the terrella of conductivity 2.67×10^7 mho/m and a radius of about 0.9a, i.e. 0.9 x 0.56 m.

The only practicable material to use for the conducting core is aluminium. Commercial aluminium for casting contains about 12 percent silicon and has a conductivity of only 2.17×10^7 mho/m. However, considering the range of conductivity distributions which agree with observed results, this is considered sufficiently close to 2.67×10^7 mho/m.

Fortunately it is not necessary to make the core solid. The skin depth at a frequency of 2 kc/s is 0.24 cm. This is much less than the radius and so a 'flat Earth' approximation can be used. With this approximation it can be shown that the current per cm for a lamina of thickness x is equal to A times the current due to an infinitely-thick conductor, and that the phase of the alternating current differs from that in an infinitely-thick conductor by b radians, where

$$A^2 = 1 - 2.e^{-(x/s)}.\cos(x/s) + e^{-2(x/s)}$$

$$\tan b = \sin(x/s) \,/\, \left[e^{(x/s)} - \cos(x/s)\right] \quad ,$$

s being the skin depth defined by

$$s^{-2} = \tfrac{1}{2}.w.\mu.\sigma$$

Provided x is greater than 4s, A is within a few percent of 1 and b is less than one degree. Therefore it is sufficient to use a spherical shell of thickness 4 x 0.24 cm (0.378 in.). A quote has been requested for a spherical shell of thickness 7/16 ± 1/16 in. The weight will be about 177 lb.

Primary coil

At many stations, (e.g. Watheroo), the correlation between the magnetic elements is very high and seems to be independent of the time of day, season of the year, and sunspot number. Therefore we can conclude that this correlation does not depend critically on the form of the primary field. Any reasonable current distribution in the primary coil should give essentially correct results. After a careful search through the literature it was decided to copy the current distribution given by Silsbee and Vestine (1942) with a modification suggested by Fukushima and Ono (1952). The modification is that the daylight circuits are reduced in amplitude by a factor of two relative to the night circuits.

The primary coil is wound on a masonite former of 30 masonite ribs, each curving over a little less than a quarter of the circumference of the sphere and with their ends meeting at a point, which can be taken as the geomagnetic pole. Two coils are necessary, one for the northern hemisphere and one for the southern hemisphere. They are mirror images of each other.

For the night circuits four wires were wound between adjacent lines of Figure 8 of Silsbee and Vestine (1942), and for the daylight circuits two wires between adjacent lines. Over the polar caps each line is represented by eight wires run together.

The masonite ribs are 1½ in. wide. This puts the primary current at an altitude of 435 km. This is higher than the ionospheric currents causing bays are thought to be, but to put them at a height corresponding to 100 km would not have left room to operate the search coil. At all locations of interest the current distribution is sufficiently uniform that the field should be almost independent of the distance from the coil.

Operating procedure

To determine the preferred plane at a location the search coil is kept in one position relative to the oceans, and the primary coil is rotated about its pole. This simulates a series of bays at different times of the day, from which points plotted on polar diagrams (Plate 2) are derived and from which the preferred plane is determined.

One natural phenomenon that cannot be simulated by the terrella is the rotation of the ionospheric current system by some tens of degrees during the occurrence of a bay. This gives rise to phase differences of magnetic elements. These are avoided if possible when scaling magnetograms.

4. EXCITING AND DETECTING EQUIPMENT

The alternating current for the primary coil is supplied by a Peekel 1510A amplifier driven by a Peekel 053A oscillator. The latter has an upper frequency limit of 3 kc/s. Sufficient field is generated by 100 mA output. This can be achieved with good sinusoidal waveform. A more irregular waveform is obtained by overdriving the oscillator. No difference in direction has been observed for sinusoidal and non-sinusoidal waveforms.

Voltage induced in a search-coil is displayed on an AWA A56031 oscilloscope. Using only the oscilloscope it is necessary to use about one ampere of primary current. A one-stage transistor pre-amplifier has therefore been installed before the oscilloscope. It uses an OC71 transistor. With the pre-amplifier it is possible to use only 100 mA of primary current. The oscilloscope is synchronised by a voltage obtained directly from the oscillator.

The search coil is shown in Plate 3. It consists of a small coil of 300 turns of 38 B. & S. gauge copper wire wound on a shaft of 1/16-in. diameter. Both the shaft and its holder are made of nylon. The outer diameter of the search coil is 3/16 in. Its total length is about 1/8 in. which is equivalent to 36 km on the Earth.

The coil can be turned in its holder through 90 degrees about an axle perpendicular to the shaft. The holder itself has a threaded stem which holds it in a 3/16-in. hole drilled 'vertically' into the perspex former of the terrella. This stem is perpendicular to the axle on which the coil can rotate. Thus the holder itself can be turned through 360 degrees about a 'vertical' axis (i.e. an axis pointing towards the centre of the terrella), and the axis of the search coil can be pointed to any direction in the 'upper' half-space.

The shaft on which the search coil is wound is extended half an inch to form a pointer. Onto this a small scale in the form of a curved strip has been glued. The strip occupies slightly more than 90 degrees of a half-inch diameter circle concentric with the axle about which the coil turns in its holder. It is graduated every 15 degrees. With this scale the tilt of the coil can be estimated to five degrees.

To determine the direction of the field, the pointer is put in a 'horizontal' direction and the holder rotated about the 'vertical' stem until the induced voltage is a maximum. The coil is then turned about the axle until a null is detected. The direction of the field is then in the same 'vertical' plane and 90 degrees from the direction of the pointer. The 'horizontal' direction is read to an accuracy of $22\frac{1}{2}$ degrees (i.e. N, NNE, NE, ...) and the angle of tilt in the 'vertical' plane to an accuracy of five degrees.

Leads to the search coil are shielded. They have been carefully twisted and so arranged that any loop presented to the alternating magnetic field is much smaller than the total turns-area of the search coil itself, which is about 4 in.^{2}.

5. PRELIMINARY TESTS

The first feasibility tests were made with a flat primary coil consisting of 45 circular loops with a common tangent. Each loop differed by half an inch from its neighbours in diameter which ranged from 2 in. to 24 in. The coil approximated a current sheet over most of its area and duplicated the concentration of current forming the auroral electrojet near the common tangent. Tests were made that indicated the kind of search coil and the magnitude of the primary exciting current necessary. The model described above was based on these tests.

Tests were made on the accuracy with which the direction of the field could be determined. A primary coil consisting of a single circular loop was made. The field from this coil can be calculated. The direction of the field was determined with the search coil at seven positions from the centre of the loop to a distance of $1\frac{1}{2}$ radii. The results are shown in Plate 1. The full line is the calculated tilt of the field, and the circles are the observed tilts. The observed results are within the five degrees required.

6. RESULTS

At the time of writing (November 1962) a few preliminary measurements have been made on the terrella model. Some of these have been made with no conductors present, i.e. measuring the primary field only. The results are shown in the polar diagram in Plate 2. There is some tendency for the directions of the field to lie in a plane tilting upwards to the north-north-west at about 50 degrees. These were measured at a latitude of 30° N (taking the centre of the primary coil as the pole). This is a much steeper tilt than is found on the Earth (except at a very few stations) and indicates the importance of the conducting core. The fact that the tilt was to the west and towards the pole may explain the observed fact that the coastal effect is much more definite on a west coast than on an east coast (Parkinson, 1962a). It might also help to explain the very strong correlation found on the south coast of Western Australia.

No attempt has been made yet to represent the oceans with copper sheeting, but a copper disc has been beaten into a segment of a sphere with a radius of curvature equal to that of the terrella. Its diameter is about 40 degrees, so it represents a circular ocean about 4400 km in diameter. A few measurements have been made on the 'coasts' of this 'ocean'. The unexpected result obtained so far is that the copper disc has no measurable effect on the direction of the resultant field. It introduces a small out-of-phase component so that the field becomes elliptically polarised, but the eccentricity of the ellipse is almost unity. This phenomenon is often present to a greater or lesser extent in the geomagnetic field.

Another unexpected result was obtained when the search coil was put in the centre of the copper disc, simulating the conditions on an oceanic island. The field was then found to be exactly horizontal. This is not always observed at islands, even in the largest oceans (Parkinson, 1962b).

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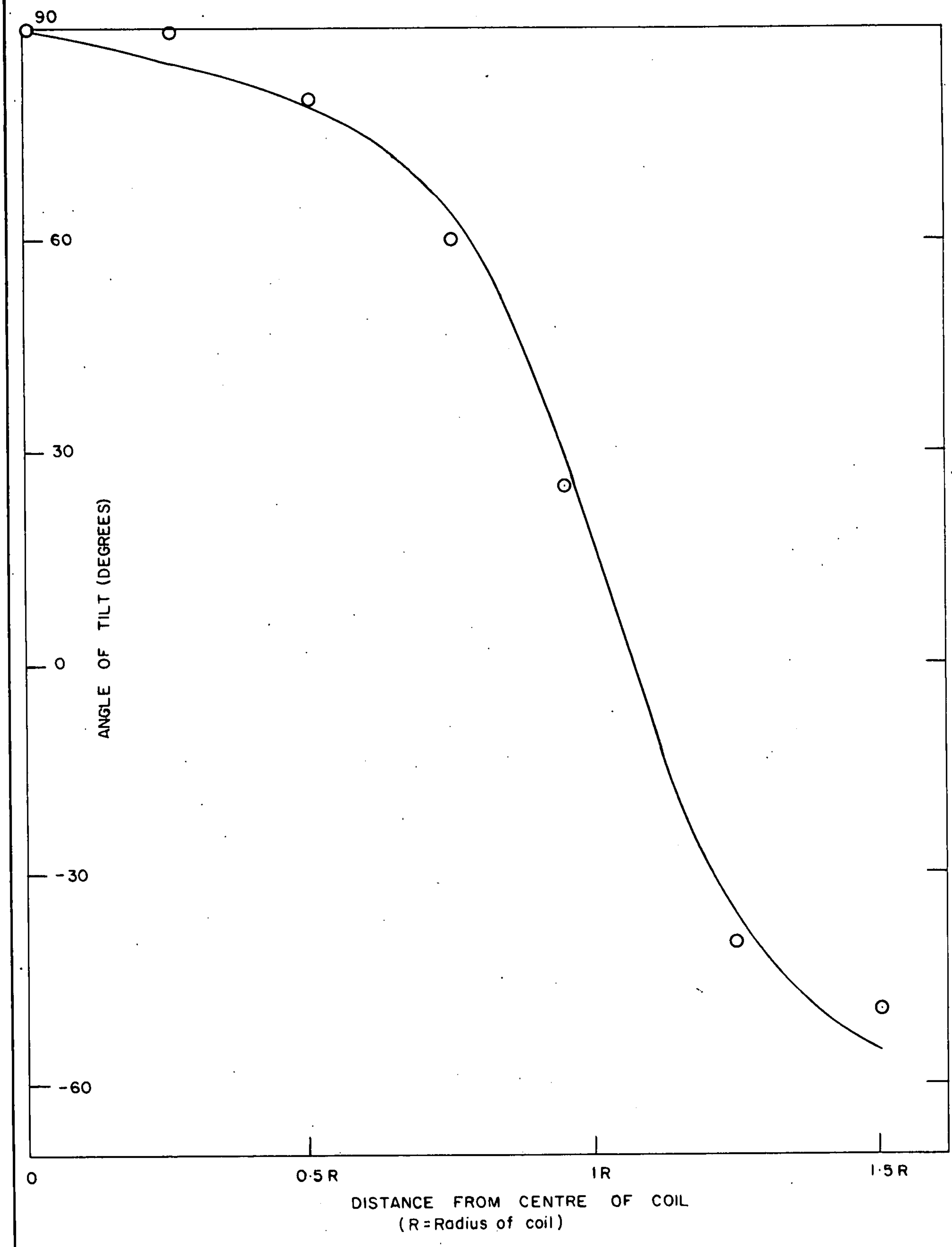

MEASURED AND CALCULATED ANGLES OF TILT

——— Calculated

O Measured

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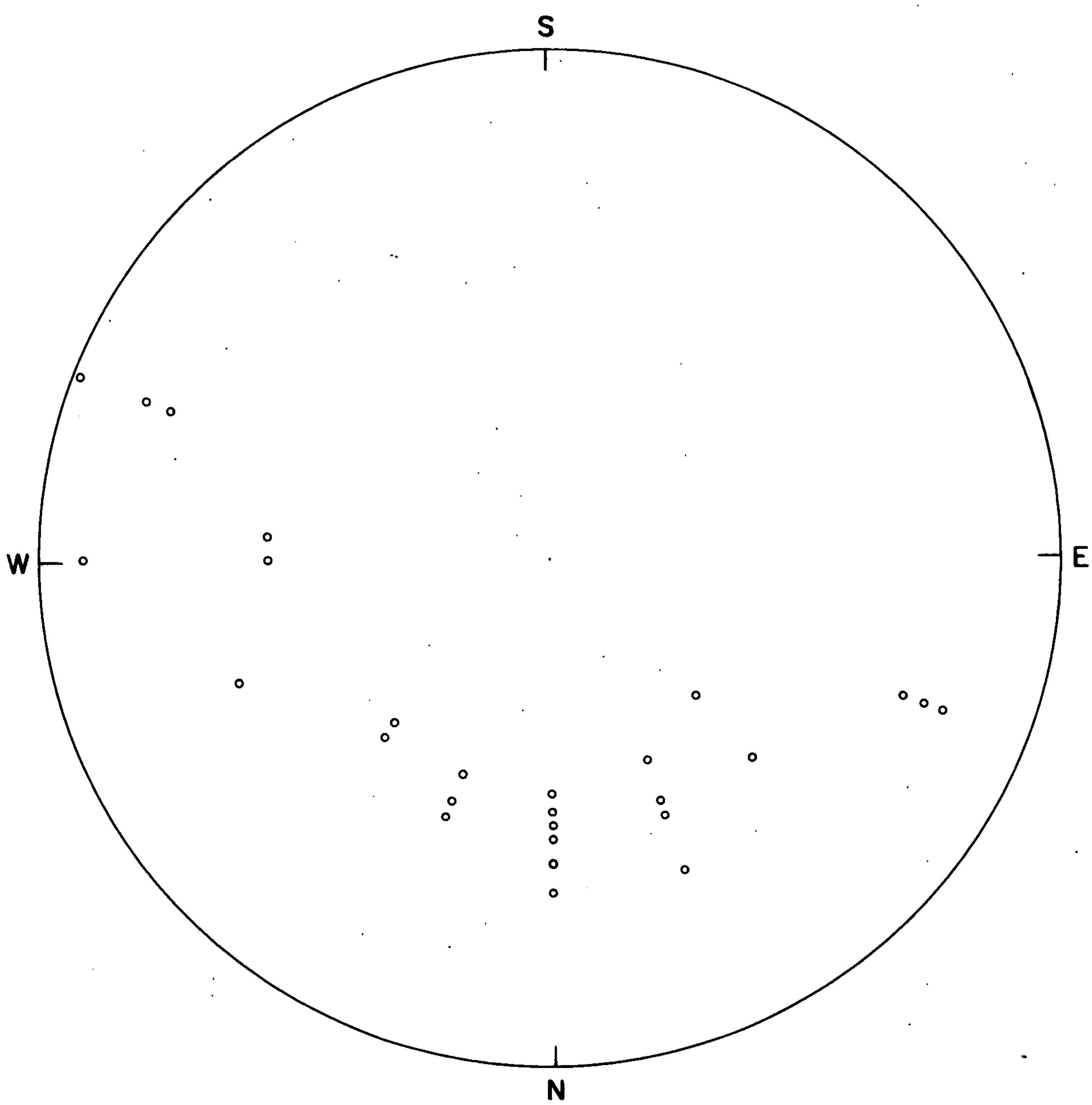
To accompany Record No. 1963/102

PLATE 2



POLAR DIAGRAM OF UPWARD DIRECTIONS OF ALTERNATING MAGNETIC FIELD AT LATITUDE 30°N

EACH POINT CORRESPONDS TO A DIFFERENT LONGITUDE OF THE PRIMARY COIL RELATIVE TO THE SEARCH COIL

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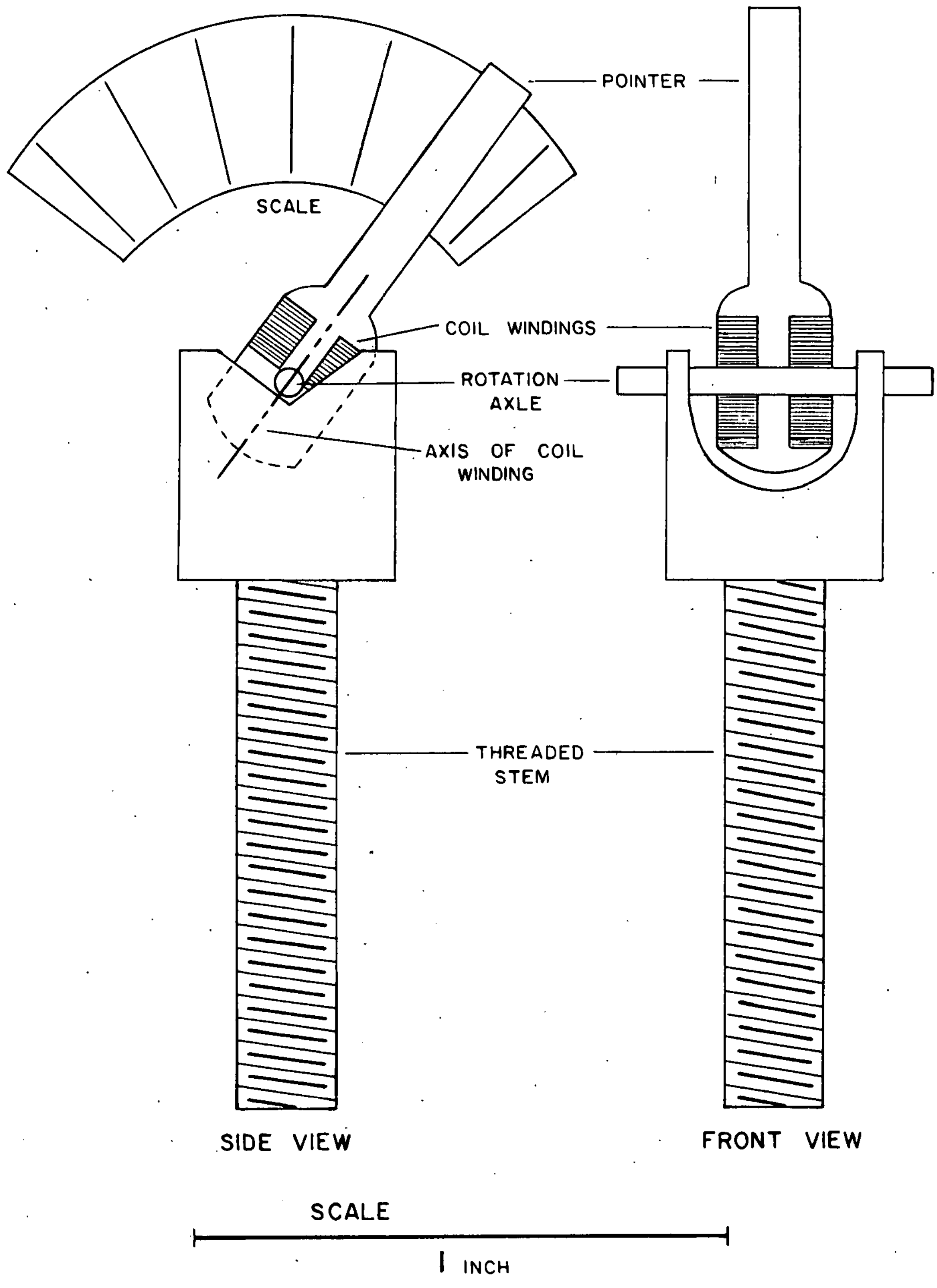
PLATE 3



SEARCH COIL

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