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RECORD No. 1964/129

THE "VIBROSEIS" METHOD OF SEISMIC EXPLORATION

by

E. SCHWING

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Bureau des Etudes Geologiques, Paris)*

The opinions and views expressed in this Record are those of the author and are not necessarily those of the Bureau of Mineral Resources.

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APPENDICES

1. INTRODUCTION

The object of this note is to present the data and information on the "Vibroseis" method collected by the author from the following sources: a visit to a Seismograph Service Ltd. "Vibroseis" party in the Carnarvon Basin on the 27th, 28th, and 29th August 1963; South Africa Patent 62/1852; various published articles; discussions with geophysicists having had some experience with the method. The note is incomplete because very little has been published on this subject and because, as certain phases of the method are confidential, most discussions with Seismograph Service Ltd. representatives were not very informative. It must also be considered provisional and will be completed or revised when further information is obtained.

2. RESULTS FROM LINEAR FILTER THEORY

The basic ideas and results from linear filter theory needed to understand the "Vibroseis" method are briefly discussed. The mathematical definitions and derivations may be found in any standard textbook on filter theory or in notes by C. Chenon (in preparation).

In seismic exploration, if the earth is considered as a filter, and the explosion as a unit impulse function (it is of very short duration and its frequency spectrum is very rich), the seismic record will represent the characteristic or weighting function of the earth. The object of all seismic methods is to obtain the filter characteristics of the earth. It will be shown how the "Vibroseis" does it.

A linear filter may be defined as a device modifying an input function f (t) with a weighting function k (t) to give an output x (t), where

$$x(t) = \int_{-\infty}^{+\infty} f(\tau)\, k(t-\tau)\, d\tau \quad \ldots . \quad 1.$$

The weighting function k (t) is the filter response to a unit impulse function $\delta(t)$ (known as a Dirac), so that

$$k(t) = \int_{-\infty}^{+\infty} \delta(\tau)\, k(t-\tau)\, d\tau \quad \ldots . \quad 2.$$

The filtering process may be explained intuitively thus: the input signal is sub-divided into impulses at equal time intervals and the summation of all the responses to these impulses gives the output. This is expressed mathematically in Equation (1) and is illustrated in Figure 1.

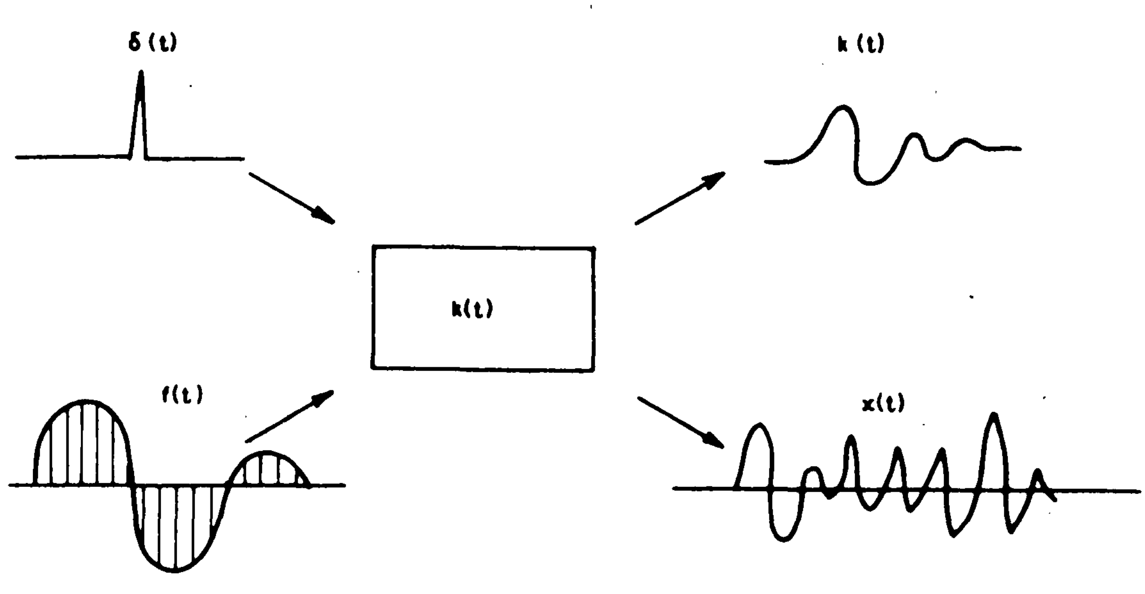

Figure 1. The filtering process - schematic representation

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The filtering process can be transposed in the frequency domain by a Fourier transformation, that is, the functions will be expressed in terms of their frequency distribution.

The input signal becomes: $F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$

The output signal becomes: $X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$

The weighting function will become the frequency transfer function:

$$K(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} k(t) e^{-j\omega t} dt$$

with inverse transform: $k(t) = \int_{-\infty}^{+\infty} K(\omega) e^{j\omega t} d\omega$

It can be shown that $F(\omega) \times K(\omega) = X(\omega)$

which is the equivalent of Equation 1 in the frequency domain.

Since the transform of a Dirac is one, then

$$1 \times K(\omega) = K(\omega)$$

corresponding to equation 2 in the frequency domain. Thus the response of a filter to a given function may be computed when the weighting function is known. This may be done either in the time domain or in the frequency domain. These results are shown in Figure 2.

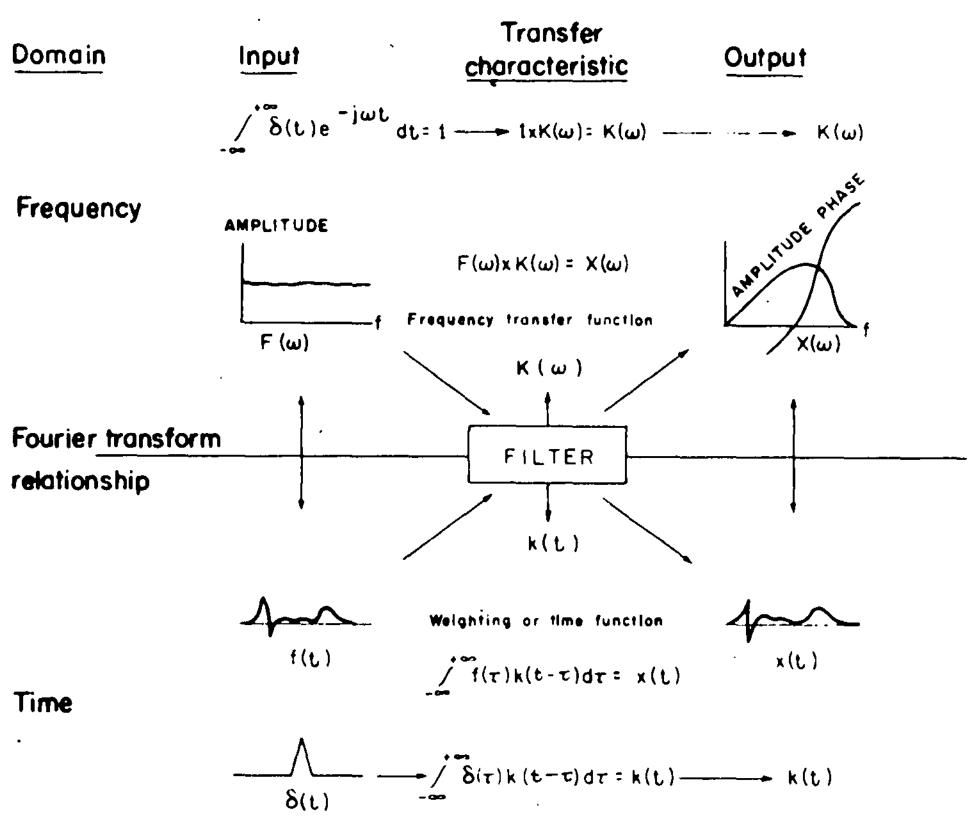


Fig. 2 Filter response characteristics

It can be shown that if the auto-correlation of an input function is a Dirac, the cross-correlation between the input and the output yields the weighting function, i.e., if

$$f(t) * f(t) = \delta(t)$$

then $f(t) * x(t) = k(t)$

which may be written $k(t) = \int_{-\infty}^{+\infty} f(\tau)\, x(\tau - t)\, d\tau$

This result is very important and is the basis for the "Vibroseis" method. It can be explained intuitively as follows.

Consider an input signal of constant amplitude, varying linearly in frequency, and lasting for seven seconds. Such a signal is used in "Vibroseis". An important property of the signal is that nowhere does it repeat itself. If this signal is transmitted into the ground, then at each reflecting surface a fraction of the original signal will be reflected. The signal received at the geophones will be the summation of all these reflected signals and as such will be meaningless to the geophysicist.

If two identical signals are inspected by eye, it will be found that the highest degree of coherence will be obtained when the two signals are in coincidence. Mathematically it is done by an auto-correlation; one signal is made to slide against the other at equal time intervals, and for each interval the summation of the products of the corresponding amplitudes is made. Obviously the sum will be a maximum when the two signals are in coincidence.

Now if a similar process is performed with the reflected signal, that is, if the original signal is cross-correlated with the received reflected signal, every time the original signal coincides with a reflected event a maximum will appear, corresponding to a reflection on the conventional trace, and a conventional trace will be obtained. This "Vibroseis" trace is identical with the conventional record and effectively is the same thing. All the seismic phenomena are the same, the seismic paths are the same, only the signal is different.

3. SWEEP SIGNAL

The signals used in "Vibroseis" are frequency modulated signals of constant amplitude. They have a duration of seven seconds. The range of frequencies that can be used varies from 10 c/s to 120 c/s, but usually the signal used has a frequency range of two octaves. A choice is available of various signals recorded on master tapes, which are used in the field to actuate the vibrators by radio. The sweep signal is of the form

$$e(t) = E \sin(\omega_c t + \tfrac{1}{2} \mu t^2)$$

where $-T/2 < t < T/2$, ω_c is the centre frequency, E the amplitude, and μ the rate of change of frequency.

It is to be noted that no part of the signal is repeated.

Under ideal conditions the auto-correlation of the signal should yield a Dirac. The mathematical computation has not yet been done to determine the theoretical result of the auto-correlation, but the auto-correlation performed by the correlator unit yield a wavelet not unlike the one described by Ricker (1940).

In the field the sweep signal to be used is determined by experimenting under known field conditions. The frequencies prevalent in surface noise and those attentuated by the earth are not utilised. The sweep signal may be used beginning with either the high or the low frequencies, but it is found that if the sweep begins with the high frequencies, the vibrator settles better in the ground and the harmonic distortion associated with low frequencies are recorded later on the record and do not interfere with the signal.

4. RECORDING TRUCK

The functions of the recording truck are to send the sweep signal by radio to the vibrators, and to record the seismic signal.

It is equipped with the following main instruments, a block diagram of which is shown in Figure 3.

A frequency modulated radio

A microtrack two drum magnetic recorder TR 415

S.S.L. amplifiers type A.A.S. 20 channels

A control sweep amplifier

A monitor

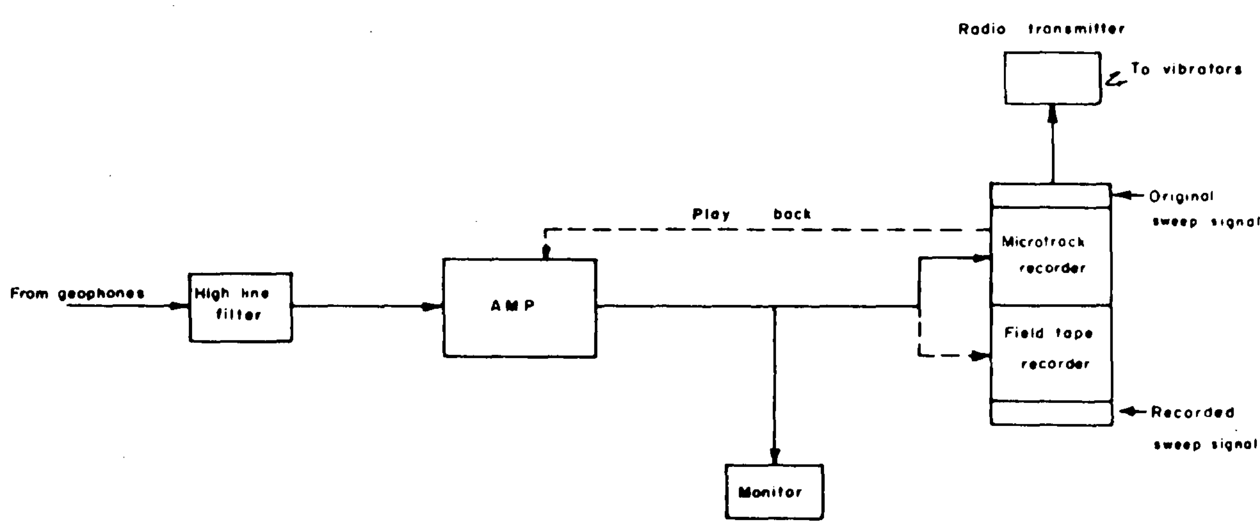


Fig.3 Instruments in recording truck

The microtrack recorder consists of two drums, on each of which 20 channels can be recorded. The drums rotate independantly at a speed of 13 seconds per revolution and are mounted on a single axis. The microtrack heads are moveable miniature magnetic heads, recording on tracks one tenth the width of the standard one. Direct recording (bias modulation) is used. The heads are moved laterally after each recording until 10 microtracks have been recorded. The ten microtracks are then read off by one fixed standard magnetic head and the signal resulting from the compositing of these ten microtracks is recorded on the other drum. A second compositing may be accomplished in the same manner from the second drum. Thus a single channel resulting from the compositing of 100 recordings may be obtained. In practice any number of sweeps can be recorded in groups of ten, and then composited into a single trace.

The selected master sweep signal is read off from the magnetic tape, amplified, and sent, on a phase modulated carrier of 27.12 Mc/s, to the vibrators, from which it is transferred into the ground. At the same time the signal is recorded on the field tape where it can be used later in determining the time origin of the record. The signals received from the geophones are recorded by the microtrack heads on tape without A.V.C. or filtering, direct recording being used. The gain is set so that the received signal is recorded at 100% tape modulation. The duration of the sweep signal is seven seconds, and the duration of the seismic trace is thirteen seconds, yielding, after cross-correlation, about six seconds of information.

5. VIBRATOR UNIT

The vibrator consists of a two-way hydraulic piston that is driven via a valve controlled by an electrical signal. The hydraulic energy is provided by a high pressure pump. The active part of the vibrator is a 2000 lb mass, which is driven by the hydraulic piston, and which activates by reaction a base plate coupled to the ground. A drawing of the vibrator is shown in Figure 4.

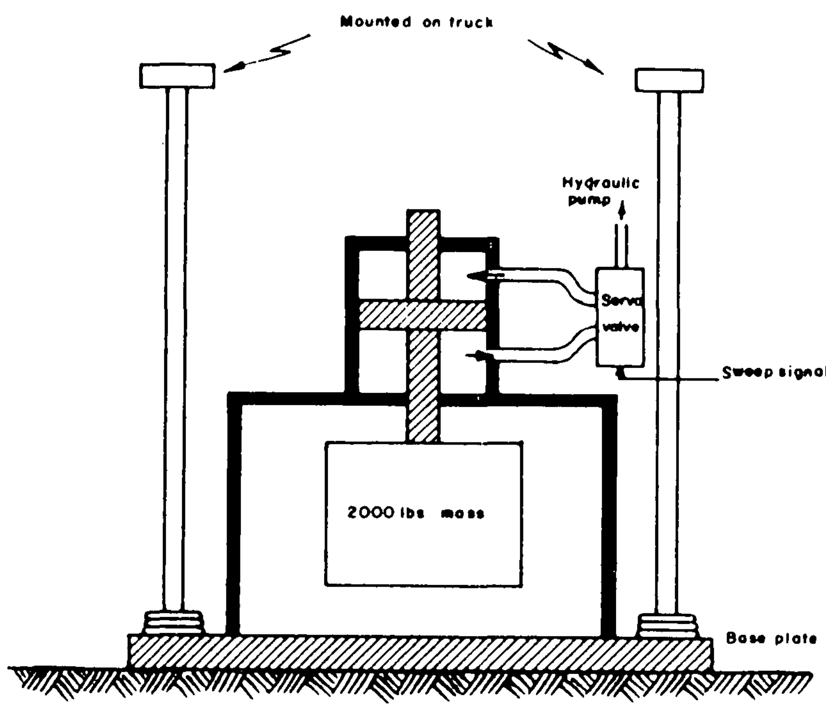


Fig. 4 Vibrator

The characteristics of the vibrator unit are such that any signal received is reproduced by the base plate without distortion and is transmitted into the ground. When the sweep signal is received from the instrument truck it is demodulated and amplified. The amplified signal drives a torque motor, which activates a hydraulic valve. This valve, which is a mechanical amplifier, controls a flow of oil, having a pressure of some 3000 psi, to the main hydraulic cylinder. Thus the piston and the mass are fully controlled by the hydraulic valve and move according to the sweep signal. A generator is mounted on the piston and generates a signal according to the motion of the shaft. Part of this signal is sent back to the servo amplifier through a feed-back circuit to prevent distortion. A block diagram of the unit is shown in Figure 5.

The unit is mounted to the rear of a 6 x 6, 1850 Ford Truck. The base plate is 3'6" x 4'0". When in operation, the rear of the truck is jacked up so as to add the weight of the truck to that of the unit. The total weight is about six tons. The pressure in the hydraulic circuit is some 3000 psi. The difference in pressure across the cylinders is about 2000 psi. The piston area is 5 sq. in. so that the total force available is 5 x 2000 = 10,000 lb.

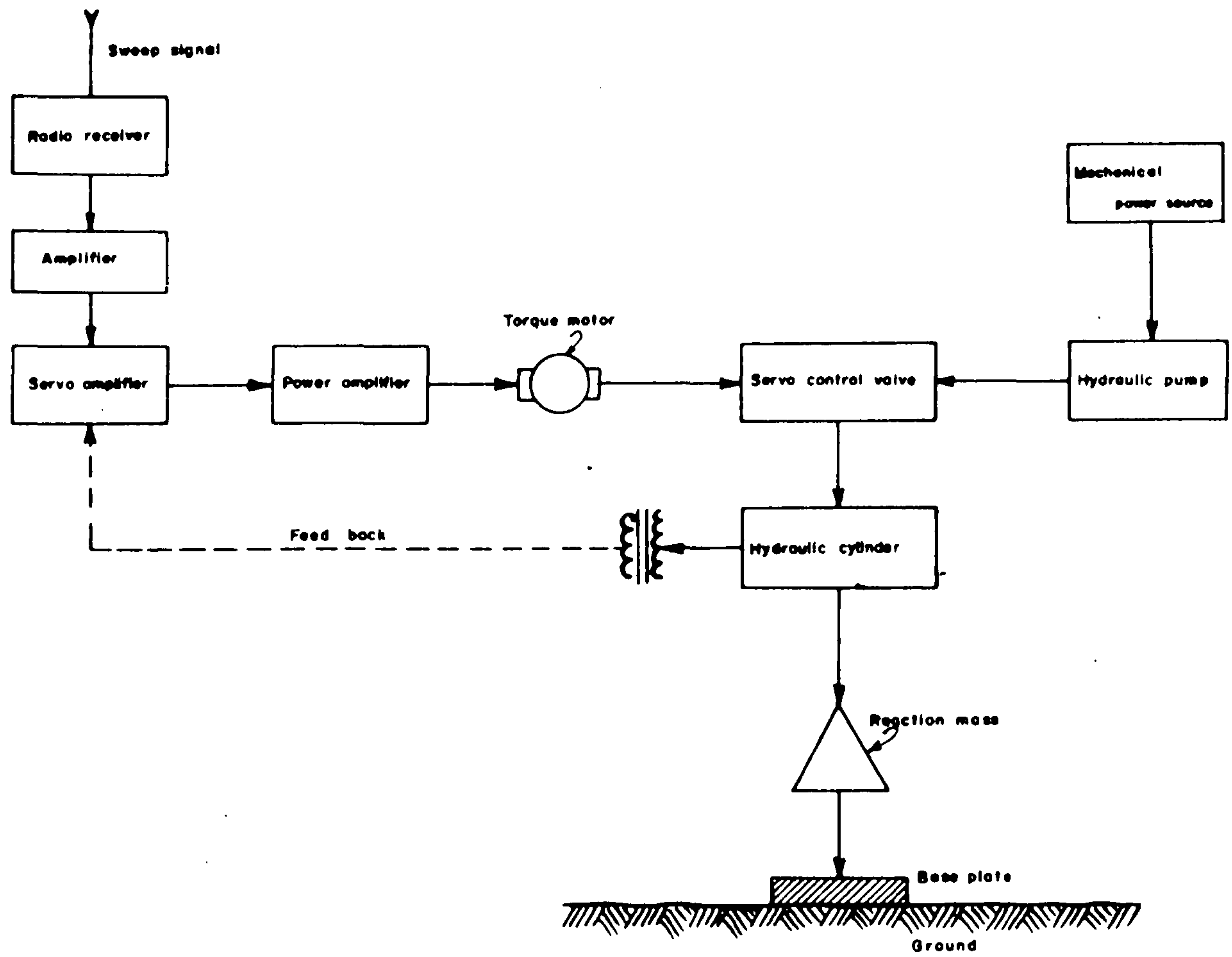


Fig.5 Block diagram of vibrator unit

The sweep signal is of the type

$$e(t) = E \sin(\omega_c t + \frac{1}{2}\mu t^2),$$

where e (t) is in volts and E is the maximum amplitude. The power amplifier transforms the voltage into mechanical force and the expression becomes

$$f(t) = F \sin(\omega_c t + \frac{1}{2}\mu t^2),$$

in which f (t) is in units of force.

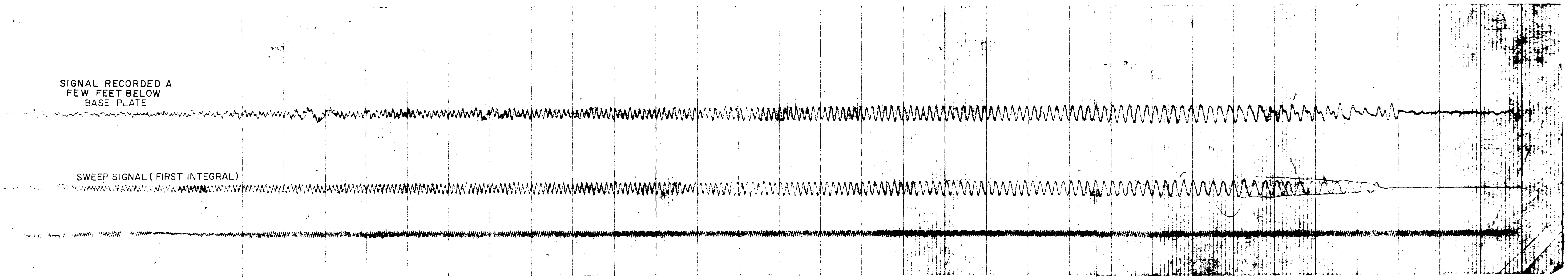
It is desired to transmit energy into the ground with constant average force. From the fundamental expression

$$f(t) = m \frac{d^2 x}{d t^2} ,$$

as m is a constant, the acceleration $\frac{d^2 x}{d t^2}$ will also be a constant.

The main problem at present is coupling the base plate to the ground and avoiding the resonant frequency of the vibrator-to-ground system. The mathematical theory of the "Vibroseis" is based on the assumption that the coupling is perfect. However, this is not the case; the nature of the coupling varies widely with the type of ground, and it is felt that, at present, the problem has not been thoroughly studied. High frequencies are attenuated by the ground more than the low frequencies. This is treated empirically in the field by using the first integral of the sweep signal so that the amplitudes vary inversely with the frequencies. A test of signal transmission has been performed by burying a geophone a few feet below the base plate to detect the transmitted signal. Figure 6 shows the record on which the sweep signal and the transmitted signal were recorded. With the exception of saturation in the high frequencies, probably due to the proximity

FIGURE 6



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of the geophone to the source, it can be seen that there is a remarkable correspondence between the two signals. The synchronisation of the vibrators is good enough for several of them to be used together and the recordings of many independent sweeps may be composited prior to cross-correlation. The energy transmitted by the ground is very small, so that it has become standard practice to use several vibrators. Three are considered a minimum, but as many as twelve have been used.

6. CORRELATOR UNIT

The function of the correlator unit is to produce conventional records from the field "Vibroseis" records, from which information cannot be directly extracted by the geophysicist. This is done, as explained above, by cross-correlating the sweep signal and the received signal. The unit is part of a playback centre, from which fully corrected electrostylus amplitude records and variable-area sections may be produced. The instruments are small and compact enough to be installed in a trailer that will operate in the field. It is essential to process the records in the field in order to control, on the spot, the quality of the results. An uncorrelated "Vibroseis" record does not yield any information and does not allow the geophysicist to judge its quality.

The playback equipment consists of :

- A two drum magnetic recorder (similar to the field recorder) with microtrack facilities
- A correlator unit
- A corrector unit
- A variable-area cross-section recorder S.S.L. Type VAX
- A translator unit, comprising a playback amplifier and a paper record oscillograph.

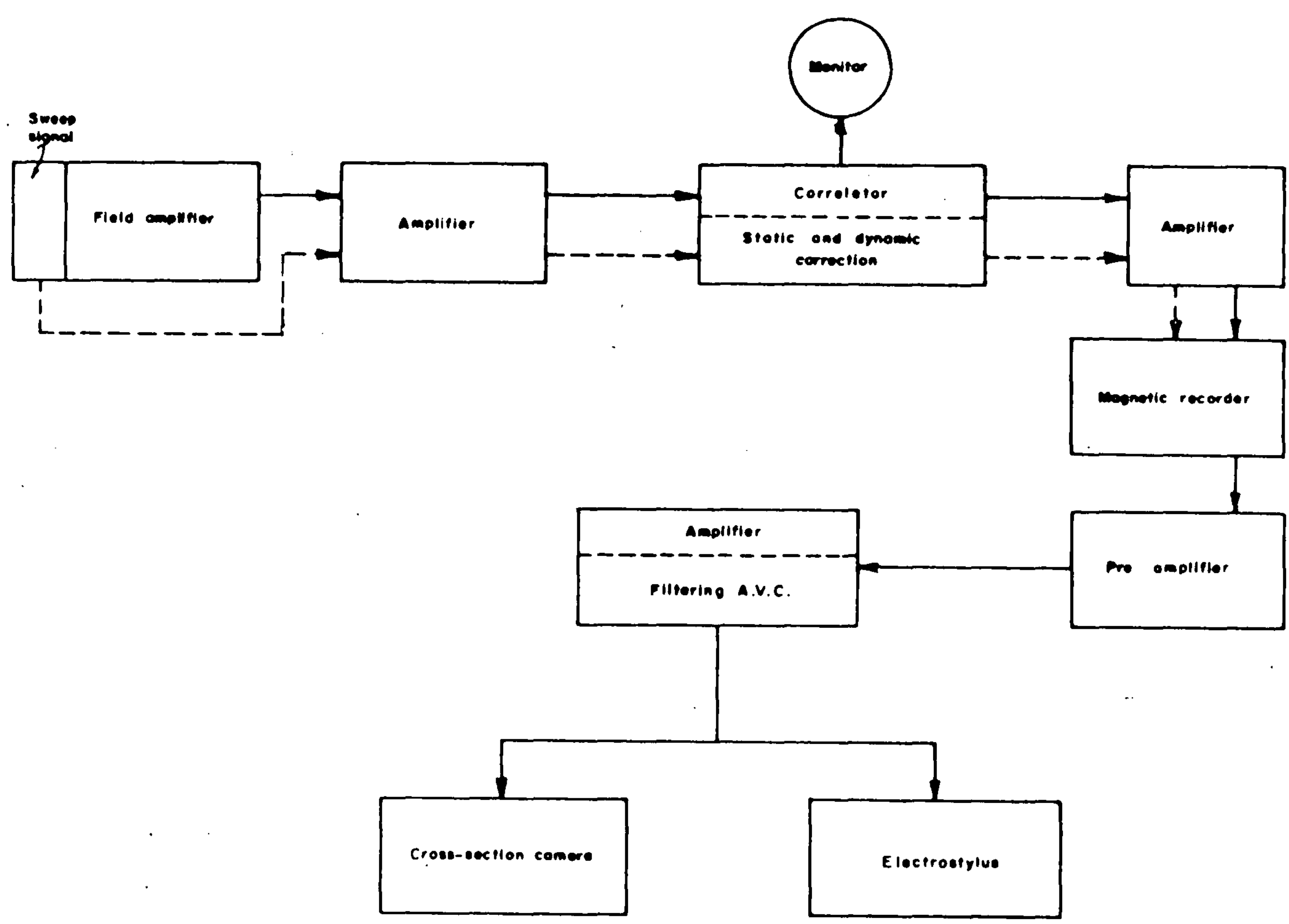

Fig.7 Block diagram of a play-back centre

The sweep signal recorded on the field tape during vibration is auto-correlated to yield the time origin. (It was mentioned above that a condition was that the auto-correlation of the sweep signal must be a Dirac). Then, the operation being sequential, all the traces from the field record are read one by one in the reproducer drum, which rotates at a speed of 13 sec per revolution. The field traces are amplified and cross-correlated with the sweep signal in the correlator unit. The correlated signal produced by the correlating head is amplified, monitored, and recorded on a magnetic recorder drum, which rotates at a standard speed of 6.5 sec per revolution. Dynamic and static corrections are applied during cross-correlation by moving the correlating magnetic head, which is controlled by a servo motor. The corrected tape is played back sequentially, with any filtering and A.V.C. desired, to produce an electrostylus record or a continuous variable-area cross-section. Bias modulation is used throughout for recording on tape. A block-diagram of the playback centre is shown in Figure 7.

The correlator unit is confidential and no information concerning its principle or its operation was revealed during the visit to the S.S.L. field party. However, the principle and specifications as described in South Africa Patent 62/1852 are briefly discussed below.

The dimensions of the unit are small, it being about 4 ft long, 1 ft wide, and 9 in. high.

Some of its characteristics are:

(a) It performs cross-correlation between any two given signals

(b) It has a correlating head (see Appendix II) whose output signal is

$$x''(t) = \int_0^T \frac{df(\tau)}{d\tau} \frac{\partial k(\tau - t)}{\partial t} d\tau$$

(c) It has a memory of 13 sec. The signal is recorded on a tape in the form of an endless loop having a speed of 3.75 in. per sec.

(d) It operates sequentially.

(e) Static and dynamic corrections are made sequentially by moving the correlating head. Static corrections of up to ± 100 ms, and dynamic corrections of up to a maximum of 450 ms with a maximum rate of change of 360 ms/sec can be applied.

The principle of operation is as follows.

One of the signals to be cross-correlated (the sweep signal) is reproduced in the form of a printed circuit by an etching process. The conducting material is copper and the base is fibre-glass, and the scale is such that the length will match a signal recorded on tape at a speed of 3.75 in. per second. The width of the printed signal matches exactly that of the tape. This element is called the correlating head (see Figure 8).

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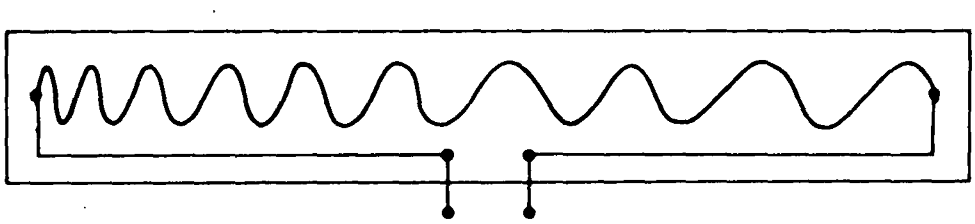

Fig.8 Correlating head

The tape, on which the field signal has been recorded over the full width, is made to pass over the correlating head. The magnetic field of the tape induces a current in the conducting element of the correlating head and an output signal is generated at the terminals of the conductor. It is important to control very accurately the speed of the tape to obtain a true correlation. In this operation a sinusoidal signal is matched against a signal that is in variable-density (magnetic) on the tape, as shown in Figure 9.

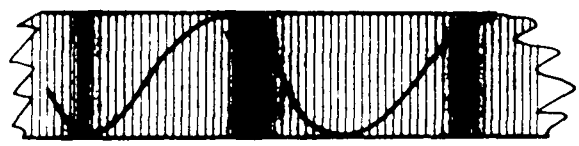

Fig.9 Magnetic field recorded on tape matched against recording head

It is seen that every part of one signal sees all parts of the other. This leads us to think that the result is bound to be related to a correlation. Intuitively, it can be seen that when a sweep signal recorded on the tape is coincident with a similar signal on the correlating head, a maximum output voltage is obtained; this is the method used to find the time break. Similarly, when the sweep signal is coincident with a reflected signal on the field record a maximum is obtained.

The Patent specification states that the output voltage of the correlating head represents the cross-correlation of the first differentials of the two signals. The mathematical relation is derived in Appendix I. The final seismic signal is obtained from the second integral of the output signal of the correlator. The validity of this operation is demonstrated in Appendix II.

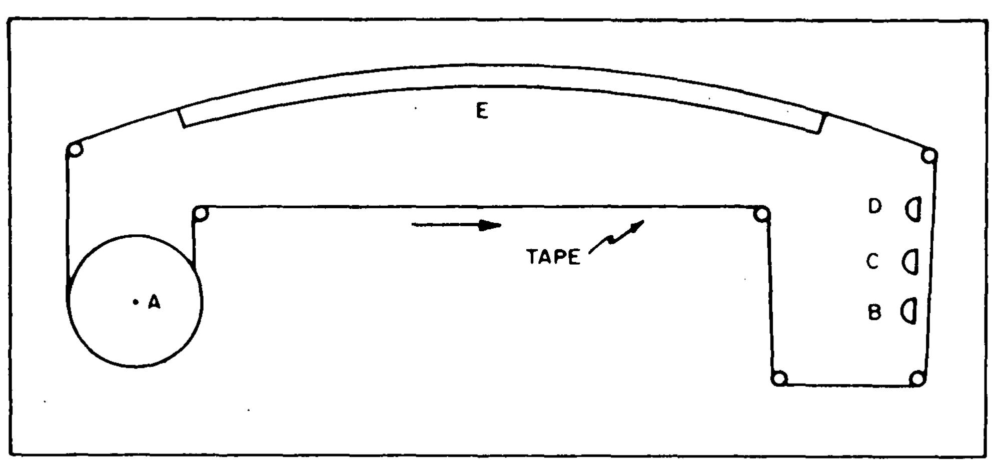

Fig.10 Cut-away view of correlator unit

A cut-away view of the correlator unit is shown in Figure 10. The tape in an endless loop is driven by the drum A past an erasing head B, a recording head C for recording the field signal, a reading head D for monitoring, and the correlating head E.

The specifications claim that it is an easy matter to engrave the printed circuit on the correlating head. Of course a correlating head must be available for each type of sweep signal used.

Filtering of certain frequencies can be performed by putting a magnetic shield on the printed circuit covering the frequencies to be filtered out. These frequencies will not contribute to the cross-correlation and will not be present in the output.

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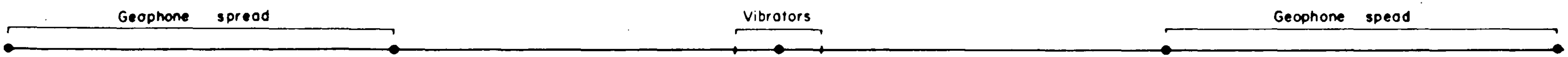

Fig. 11 In line spread

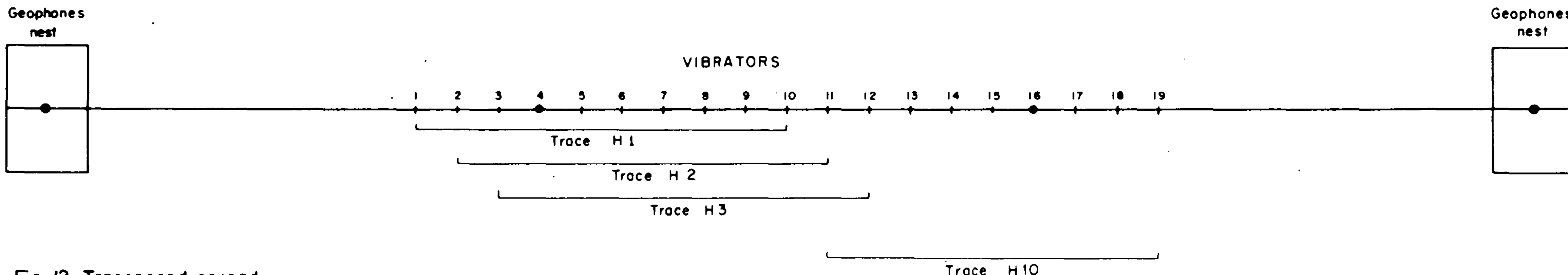

Fig. 12 Transposed spread

7. FIELD OPERATIONS

Two methods of field operations are possible - the in-line method and the transposed method.

The in-line method corresponds to the usual spread in conventional seismic operations. The line is laid with groups of geophones placed at predetermined locations, each location corresponding to a seismic trace. The vibrators are placed where the shot-point would be (see Figure 11).

In the transposed method, the geophones are grouped in two nests, one on each end of the line, and the vibrators operate along the line where the geophone stations would normally be. The advantages of this method are that large numbers of geophones can be used in each nest and extensive compositing of large numbers of independent ray paths may be done. Figure 12 shows an example of the transposed method similar to the one used by the party in the Carnarvon Basin during the author's visit. The geophones were nested in groups of 400 on each end of the line. Three vibrators were used simultaneously at each vibration position, and vibrations were made at each of the positions from 1 to 19. The recorded signals from positions 1 to 10 were composited into trace 1, the recorded signals from psoitions 2 to 11 were composited into trace 2, etc. up to trace 10.

Both methods allow the use of large geophone and/or vibrator patterns for cancellation of coherent noise and the use of compositing of a great number of independent ray-paths. The transposed method is extremely flexible in operation, and with it a great number of combinations or compositing arrangements may readily be used.

The "Vibroseis" has a tendency to cancel random noise. However, all other noise that originates from the source, such as surface waves, multiples, etc., will exist and be detected exactly as in the conventional method. For this reason similar techniques can be used to improve the quality of the desired signal, as shown by the following examples.

1. Noise spreads are recorded as a normal testing procedure to determine the nature of the noise and the best filters to adopt. Good noise spreads, having the same aspect as those obtained by conventional explosion methods, may be obtained with the "Vibroseis". This follows from the facts that all the ray paths and the nature of the coherent noise are the same for both methods. However, the results of a conventional explosion noise spread could not be used to determine the optimum "Vibroseis" pattern because the source is in one case on the surface and in the other case in the ground, and different noise is generated. Further, the frequency spectrum of the explosion is not the same as the frequency spectrum of the sweep signal.

2. Geophone and source patterns are designed from an analysis of the noise spread to eliminate the coherent noise.

3. Offset of the source from the geophones is made so that the surface noise will arrive much later than the signal and will not interfere with it.

4. Compositing or mixing, as explained above, is done to improve the signal-to-noise ratio.

5. Common depth point reflection methods are used, employing the same principle as used in conventional seismic work. The object is to obtain a common reflection point with several seismic paths and composite them into one trace.

6. Extended spreads are used in the same manner as for conventional seismic work and serve the same purposes, i.e. the detection of multiple reflections and the determination of seismic velocities.

There is no information on the "Vibroseis" records, from which the calculation of surface corrections can be made. If corrections are not available from another source (previous conventional survey for instance), special measurements must be made. The "Vibroseis" method can be adapted quite easily to make small-scale refraction recordings, from which the corrections can be computed. This, however, will cause some loss in operation time. The surface corrections, after having been computed, are applied to each trace during cross-correlation. Corrections should be applied to each sweep before compositing, but for obvious reasons this cannot be done in the field. Since all the data are erased after compositing in the recording truck, these corrections cannot be applied at a later stage. For this reason, all the sweeps that are to be composited in one trace should be recorded where surface conditions do not seriously change and where there are no serious differences in surface elevations.

8. COMMENTS

Little information is available at present to make a comparison with other methods. The economic aspect cannot be discussed yet; it is evident that certain factors, such as drilling, will influence greatly the comparative costs of this method and the conventional one. In the Carnarvon Basin, for instance, drilling was the determining factor in selecting the "Vibroseis". The technical performances of the "Vibroseis" under various conditions cannot be compared yet with other techniques. Opinions of geophysicists differ greatly as to the relative mertis of this method. The prospective surveys in the Sydney and Otway Basins will test its performance under various conditions, including very hard surfaces (volcanics and sandstones) and very loose surfaces (sand dunes). The results should show some of the possibilities of the method. It is felt that its potentialities have not yet been exploited because of lack of experience, lack of opportunities to carry out testing, and lack of development. The most important problems, i.e. cross-correlation in the field and dependable operation of the vibrators, have been solved. It was thought at first that penetration would be a major difficulty; energy returns are very weak, but heavy compositing seems to bring the signal to a sufficient level for any depth desired.

Some of the advantages and disadvantages of the method are listed below.

Disadvantages

1. "Vibroseis" reflection records do not yield data on weathering conditions; these must be obtained by a special effort or from other sources.

2. The vibrators cannot operate on certain surfaces, such as marshes, soft soils, steep slopes, etc.

3. In built-up areas care must be exercised with respect to pipes, sewers, mains, foundations, etc.

4. Correlator and playback centre must be in the field.

5. Signal is imparted to the ground on the surface.

6. Signal is very weak and heavy compositing is necessary.

Advantages

1. A "controlled" signal, which may be pre-selected to suit certain conditions, is used.

2. This method is more efficient because only the required frequencies are transmitted to the ground; no energy is wasted on the transmission of ineffective frequencies.

3. Random noise is filtered out.

4. No drilling and no shooting required.

5. Great flexibility of operation.

6. Surveys may be conducted in populated areas, where the background noise is high, or in places where dynamite or weight dropping cannot be used.

7. Large-scale compositing easily obtainable.

9. REFERENCES

CHENON, C.	-	Notes on Filter Theory. Bur. Min. Resour. Aust. Rec. (in preparation).
RICKER, N.	1940	The form and nature of seismic waves and the structure of seismograms. Geophysics V. 5 No. 4 p.348.

APPENDIX I

Output of the correlator head

by

C. Chenon

The magnetic tape on which the field signal is recorded moves across the correlating head with a constant velocity. This signal may be considered as a function of time k (t) with respect to any fixed point. The magnetic tape furnishes a magnetic field H (t), the intensity of which is directly proportional to k (t). Consider an element of the conductor representing the function f (τ), of length dS. The function H (t) moves across dS in the direction of the τ axis. Let H (τ -t) be the value of the function H (t) at a time τ. The value of the induced electromotive force de (t) will be

$$de(t) = \frac{\delta \overline{H}(\tau - t) d\overline{S}}{\delta t}$$

which may be written

$$de(t) = \frac{\delta H(\tau - t)}{\delta t} \, ds \cos \alpha$$

α being the angle between the direction of the field and normal to the conductor. But the direction of the field is the same as that of the axis so that

$$dS \cos \alpha = df(z)$$

and

$$de(t) = \frac{\delta H(\tau - t)}{\delta t} \, df(\tau)$$

The summation of all the electromotive forces de (t) for all the values of τ along the function f (τ), between the limits 0 and T will be :

$$e(t) = \int_0^T \frac{\delta H(\tau - t)}{\delta t} \, \frac{df(\tau)}{d\tau} \, d\tau$$

This expression shows that the induced electromotive force, measured at the poles of conductor f (t) is equal to the cross-correlation of the first differential of the field H (t) by the first differential of the function f (t); it may be written

$$e(t) = \int_0^T \frac{\delta k(\tau - t)}{\delta t} \, \frac{df(\tau)}{d\tau} \, d\tau$$

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APPENDIX II

Integration of the output of the correlator unit

by

C. Chenon

Consider the function

$$x(t) = \int_0^T f(\tau)\,k(\tau - t)\,d\tau$$

Differentiating with respect to t,

$$x'(t) = \int_0^T f(\tau)\,\frac{\partial k(\tau - t)}{\partial t}\,d\tau$$

But, as k $(\tau$ -t) is symmetrical with respect to t and τ ,

$$-\frac{\partial k(\tau - t)}{\partial t} = \frac{\partial k(\tau - t)}{\partial \tau}$$

so that

$$x'(t) = -\int_0^T f(\tau)\,\frac{\partial k(\tau - t)}{\partial \tau}\,d\tau$$

But $$\int_0^T \frac{\partial\,[f(\tau)k(\tau - t)]}{\partial \tau}\,d\tau = \int_0^T \frac{df(\tau)}{d\tau}\,k(\tau - t)\,d\tau + \int_0^T f(\tau)\,\frac{\partial k(\tau - t)}{\partial \tau}\,d\tau$$

The left hand side becomes

$$\Big[\,f(\tau)\,k(\tau - t)\,\Big]_0^T$$

But $f(\tau_0) = 0$ (beginning of printed circuit)
and $f(\tau_T) = 0$ (end of printed circuit)

Therefore

$$\Big[\,f(\tau)\,k(\tau - t)\,\Big]_0^T = 0$$

and

$$\int_0^T \frac{df(\tau)}{d\tau}\,k(\tau - t)\,d\tau = -\int_0^T f(\tau)\,\frac{\partial k(\tau - t)}{\partial \tau}\,d\tau$$

so that

$$x'(t) = \int_0^T \frac{df(\tau)}{d\tau}\,k(\tau - t)\,d\tau$$

Differentiating with respect to t,

$$x''(t) = \int_0^T \frac{df(\tau)}{d\tau}\,\frac{\partial k(\tau - t)}{\partial t}\,d\tau$$

This shows that the correlation between the first differential of f (t) and the first differential of k (t) is equal to the second derivative of x (t), when x (t) is the cross-correlation between f (t) and k (t).