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TWO-LAYER RESISTIVITY CURVES FOR THE WENNER AND SCHLUMBERGER ELECTRODE CONFIGURATIONS

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ILLUSTRATIONS

SUMMARY

It is shown that, except for a difference in position of the origin, two-layer resistivity curves for the Wenner configuration are the same as those for the Schlumberger configuration. Further, Hummel's relation is more nearly correct for the Schlumberger configuration than for the Wenner.

The ambiguity in the interpretation of resistivity depth probes on the bases of the Hummel and Maillet relations is shown.

The accuracy of depth estimates made with the two-layer curve method is compared with published three-layer curves.

1. INTRODUCTION

Electrical Sounding in geophysical exploration can be used with many different techniques. There are many different electrode configurations in use, and there are numerous techniques for interpreting the results.

One of the most widely used arrangements of electrodes is the Wenner configuration (Wenner, 1916). In the Wenner configuration, four electrodes are used: two current electrodes (C and C') and two potential electrodes (P and P') as shown in Figure 1. The spacing between each pair of electrodes is kept the same, as the whole system is expanded.

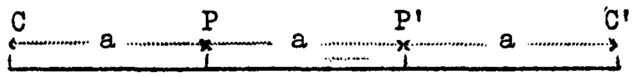
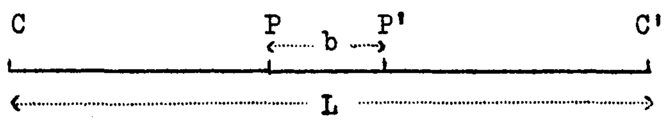


Figure 1. Wenner configuration of electrodes

Another arrangement of electrodes widely used is the Schlumberger configuration, described by Compagnie Generale de Geophysique (1955) and shown in Figure 2. In this configuration the spacing between the two potential electrodes is kept small compared with the spacing between the current electrodes, and the potential electrodes are moved only after the current electrodes have been moved several times. In practice the ratio of the spacing between potential and current electrodes is one fifth or less. As the current electrodes are moved apart, the potential or resistance values measured approach such low values that the instrument readings are no longer accurate. The potential electrodes must then be moved further apart and the reading repeated without moving the potential electrodes.



$$\frac{L}{5} > b$$

Figure 2. Schlumberger configuration of electrodes

To interpret the results of the electrical soundings is a difficult process, because the method uses a potential field and no unique solution is possible. The normal procedure is to make certain assumptions and then carry out an interpretation on the basis of the assumptions. If the assumption is made that the ground beneath the electrodes consists of a series of uniform horizontal layers, each with constant resistivity, it is possible to construct type curves for any number of layers of different values of resistivity and thickness. The greater the number of layers considered, the greater will be the number of possible curves, and for 3, 4, or 5 layers, the process of finding the best fitting curve is laborious and time consuming. Even when the best fit is found, the accuracy of the method is limited.

A rapid means of determining approximate values of resistivity and depth is to consider a multi-layer curve as a series of two-layer curves. Successive two-layer curves can be linked by means of Hummel's relation. This method involves some assumptions, which are commonly not justified, but in practice the results obtained compare fairly well with drilling information and data determined by seismic methods.

Sets of two-layer resistivity curves have been prepared, for use in conjunction with Hummel's relation, for both the Wenner and the Schlumberger configurations. When the two sets of curves are plotted on a double logarithmic log scale, they are found to be similar in shape but with different positions of origin. The reasons for the similarity between the two sets of curves have been investigated, and an estimate has been made of the error involved in assuming that the two sets have exactly the same shape. It is shown that the errors involved in using the same family of curves, but with different origins, for both the Schlumberger and the Wenner configurations, are small compared with other errors involved in the interpretation technique.

It can also be shown that the theoretical errors involved in the use of Hummel's principle are smaller with the Schlumberger configuration than with the Wenner configuration. As the number of field personnel required is fewer with the Schlumberger method than with the Wenner method, it is concluded that the Schlumberger method might be more efficient in practice.

2. GENERAL THEORY

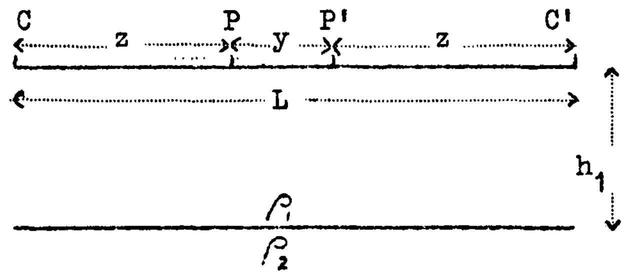


Figure 3. The two-layer case

In Figure 3, C is the current source and C' the current sink,

P and P' are potential electrodes,

ρ_1 = resistivity of first layer,

ρ_2 = resistivity of second layer,

h_1 = thickness of first layer.

By definition, $k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$

At any point in the line of electrodes at a distance x from C, the potential V is given by the following expression (Jakosky, 1950, p489):

$$V = \frac{I\rho_1}{2\pi}\left[\frac{1}{x} - \frac{1}{L-x} + 2\sum_{n=1}^{inf}\frac{k^n}{(x^2 + 4n^2h_1^2)^{\frac{1}{2}}} - 2\sum_{n=1}^{inf}\frac{k^n}{\left[(L-x)^2 + 4n^2h_1^2\right]^{\frac{1}{2}}}\right]$$

where I is the current flow from C to C'.

Wenner configuration

In the case of the Wenner configuration, $y = z = a$, and $L = 3a$. The potential difference E between P and P' can thus be expressed by :

$$E = \frac{I\rho_1}{2\pi}\left[\frac{1}{a} + 4\sum_{n=1}^{inf}\frac{k^n}{(a^2 + 4h^2h_1^2)^{\frac{1}{2}}} - 2\sum_{n=1}^{inf}\frac{k^n}{(a^2 + n^2h_1^2)^{\frac{1}{2}}}\right]$$

The apparent resistivity ρ_a is given (Jakosky, 1950, p490) by:

$$\rho_a = 2\pi a E/I$$

$$\therefore \rho_a/\rho_1 = 1 + 4\sum_{n=1}^{inf}\frac{k^n}{\left(1 + \frac{4n^2h_1^2}{a^2}\right)^{\frac{1}{2}}} - 2\sum_{n=1}^{inf}\frac{k^n}{\left(1 + \frac{n^2h_1^2}{a^2}\right)^{\frac{1}{2}}}$$

Schlumberger configuration

In the Schlumberger configuration, the spacing between the potential electrodes is kept small compared with the spacing between the current electrodes. The potential gradient between P and P' is assumed to be dV/dx.

In this case the apparent resistivity can be expressed by:

$$\rho_a = \frac{\pi L^2}{4I}\frac{dV}{dx}$$

By differentiating the function already given for V, the following expression is obtained when $x = L/2$ (Companie Generale de Geophysique, 1955):

$$\rho_a/\rho_1 = 1 + 2\sum_{n=1}^{inf}\frac{k^n}{\left(1 + \frac{16n^2h_1^2}{L^2}\right)^{\frac{3}{2}}}$$

3. COMPARISON OF THE TWO THEORETICAL EXPRESSIONS

For the limiting cases, the two functions tend to the same value:

a) Electrode spacing large compared to thickness h_1.

For the Wenner configuration, $h_1/a \rightarrow 0$, and

$$\frac{\rho_a}{\rho_1} \quad 1 + 4 \sum_{n=1}^{inf} k^n - 2 \sum_{n=1}^{inf} k^n$$

i.e.
$$\frac{\rho_a}{\rho_1} \quad 1 + 2 \sum_{n=1}^{inf} k^n$$

For the Schlumberger configuration, h_1/L 0, and

$$\frac{\rho_a}{\rho_1} \quad 1 + 2 \sum_{n=1}^{inf} k^n$$

which is the same as for the Wenner configuration.

b) Electrode spacing small compared to thickness h_1.

Both h_1/a and h_1/L tend to infinity, and both expressions for ρ_a/ρ_1 tend to unity.

For the range between the limits, the expression for the Wenner configuration was initially solved using Bessel functions, and the expression for the Schlumberger configuration was solved using a Ferranti Sirius Computer. The solutions of the two expressions were plotted on a double logarithmic scale as ρ_a/ρ_1 in terms of $L/3h_1$ (or a/h_1) for the Wenner configuration and $L/2h_1$ for the Schlumberger configuration for varying values of k. The shapes of the curves are similar for both configurations, but the sets of curves are displaced relative to each other by what appears to be a constant amount. A single set of curves has been drawn in Plate 1 to represent both configurations, but showing different origins. The actual displacement of the two sets of curves at selected points are shown in Table 1; the points were selected in the regions of the curves where $2h_1/L$ and h_1/a are large enough to be significant and where the curvature is greatest.

TABLE 1.

k	$2h_1/L$ (Schlumberger)	ρ_a/ρ_1 (Schlumberger)	value of $h_1/1$ (Wenner) to give same value of ρ_a/ρ_1	$\frac{h_1/a}{2h_1/L}$
-0.4	0.1	0.440	0.130	1.30
+0.4	0.1	2.126	0.132	1.32
-0.5	0.5	0.685	0.710	1.42
+0.5	0.5	1.409	0.670	1.34
-0.3	0.5	0.802	0.685	1.37
+0.3	0.5	1.229	0.678	1.36
				(Mean 1.36 ± 0.05)

This value for the ratio of h_1/a to $2h_1/L$ is the same as that obtained by a visual comparison of the two sets of curves over the range $k = -1$ to $k = +1$, and $2h_1/L$ and h_1/a from 0 to infinity.

The coordinates plotted are $L/3h_1$ in the case of the Wenner configuration and $L/2h_1$ in the Schlumberger configuration; if both were plotted with L/h_1 as coordinate, they would have a common origin.

The error involved in assuming that the two sets of curves are one set of curves with different origins is not significant.

4. MULTI-LAYER INTERPRETATION USING THE TWO-LAYER CURVE METHOD

In instances where there is a multiple number of horizontal layers having resistivities $\rho_1, \rho_2, \rho_3, \ldots$ and thicknesses h_1, h_2, h_3, ... respectively, a technique of resistivity interpretation is used, involving the repeated application of two-layer curves. The method was suggested by Hummel (1932), who showed that two layers having resistivities ρ_1 and ρ_2 and thicknesses h_1 and h_2 could be considered as one theoretical layer of thickness $(h_1 + h_2)$ and resistivity ρm, where

$$(h_1 + h_2)/\rho m = (h_1/\rho_1) + (h_2/\rho_2) \qquad \ldots.(1)$$

This relation is approximately valid if the assumption is made that there is no flow of current across either of the boundaries between the layers in the section beneath the potential electrodes (Plate 5a). This condition will be satisfied where ρ_2 is less than ρ_1 and less than ρ_3, but it will not be satisfied where ρ_2 is greater than ρ_1 and ρ_3.

In the condition where ρ_2 is greater than ρ_1 and ρ_3, another relation, given by Maillet (1947), will be approximately applicable:

$$(h_1 + h_2)\rho_m = h_1 \rho_1 + h_2 \rho_2 \qquad \ldots.(2)$$

This assumes that all the current crosses the boundaries in the area under consideration (Plate 5b), which is approximately correct if the spacing between the current electrodes is relatively large compared with the thickness of the top layer.

Plate 2 shows a plot of Hummel's relation, and Plate 3 a plot of Maillet's relation, each on a double logarithmic scale. It is seen that the two sets of curves are similar, each being the mirror image of the other about the line k=0.

The reason for the similarity in shape of the sets of curves can be shown as follows:

By definition, $k = (\rho_2 - \rho_1) / (\rho_2 + \rho_1) \qquad \ldots.(3)$

$$\therefore \quad \rho_2 = \rho_1 \frac{(1 + k)}{(1 - k)}$$

Substituting for ρ_2 in the Hummel relation (Equation 1),

$$\frac{h_1 + h_2}{\rho_m} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_1} \frac{(1 - k)}{(1 + k)}$$

$$\therefore \frac{\rho_m}{\rho_1} = \frac{h_1 + h_2}{h_1 + h_2 \frac{(1 - k)}{(1 + k)}} \qquad \ldots.(4)$$

Substituting for ρ_2 in the Maillet relation (Equation 2),

$$(h_1 + h_2)\rho_m = h_1\rho_1 + h_2 \rho_1 \frac{(1 + k)}{(1 - k)}$$

$$\therefore \frac{\rho_m}{\rho_1} = \frac{h_1 + h_2 \frac{(1 + k)}{(1 - k)}}{h_1 + h_2} \qquad \ldots.(5)$$

If we substitute - k for k in Equation 5, we get

$$\frac{\rho_m}{\rho_1} = \frac{h_1 + h_2 \frac{(1 - k)}{(1 + k)}}{h_1 + h_2} \qquad \ldots.(6)$$

This means that if both Equations 1 and 2 are plotted on a double logarithmic scale in terms of ρ_m/ρ_1 and $(h_1 + h_2)/h_1$, then the curve for a value of k equal to y from Equation 1 will be of the same shape as one obtained from Equation 2 with k equal to -y; but this will appear as a mirror image, since the value for ρ_m/ρ_1 in Equation 4 is the reciprocal of the value for ρ_m/ρ_1 in Equation 6.

Since the condition for satisfying the Hummel relation is that ρ_2 is less than ρ_1, i.e. k is negative, and the condition for satisfying the Maillet relation is that ρ_2 is greater than ρ_1, i.e. k is positive, both sets of curves can conveniently be incorporated in one diagram, as shown in Plate 4.

By use of this set of help curves (Plate 4) in conjunction with the two-layer type curves (Plate 1), it is possible to interpret a multi-layer case by successive reductions of the top two layers to single equivalent layers.

5. AMBIGUITY IN HUMMEL AND MAILLET RELATIONS

Equations 1 and 2 may be written in the forms:

$$(h_1 + nh_2)/\rho'_m = (h_1/\rho_1) + (nh_2/n\rho_2) \qquad \ldots.(1a)$$

and

$$(h_1 + nh_2)\,\rho'_m = h_1\rho_1 + (nh_2)(\rho_2/n) \qquad \ldots.(2a)$$

Hence, using Hummel's relation for a low-resistivity layer between two high-resistivity layers, a layer of a certain resistivity ρ_2, and thickness h_2 may be replaced by a layer of higher or lower resistivity and larger or smaller thickness respectively as long as the ratio h_2/ρ_2 is kept constant.

Using Maillet's relation for a high-resistivity layer between two lower-resistivity layers, a layer of a certain resistivity ρ_2 and thickness h_2 may be replaced by a layer of lower or higher resistivity and larger or smaller thickness, respectively.

This illustrates the basic ambiguities inherent in resistivity depth probe interpretations. Additional control in the form of borehole information or seismic data is required to make depths and resistivity estimates accurate or reliable. Nevertheless, valuable qualitative information may be obtained.

6. ACCURACY OF DEPTH ESTIMATES WITH THE TWO-LAYER CURVE METHOD OF INTERPRETATION

Some published three-layer curves (Compagnie Generale de Geophysique, 1955) have been used as examples for applying the two-layer curve technique of interpretation, using both the Maillet and Hummel relations. The three-layer curves are shown in Plates 6 to 9 and the results of the two-layer curve interpretations are shown in Table 2.

In the interpretations, the ratio of the resistivities of the three layers and the thickness of the first layer were taken from the published three-layer curve information; hence the interpretation gives an idea of the accuracy of determining the thickness of the middle layer.

TABLE 2

Three-layer curve Example No.	Plate No.	h_2/h_1	Ratio $\rho_1:\rho_2:\rho_3$	Discrepancy in depth with Hummel curves (%)	Discrepancy in depth with Maillet curves (%)
1	6	9	$1 : \frac{1}{4} : 1$	+ 5	+ 15
2	6	1	$1 : \frac{1}{4} : 1$	-22	+ 55
3	6	24	$1 : \frac{1}{19} : 1$	-17	+ 10
4	6	2	$1 : \frac{1}{19} : 1$	+ 2	+160
5	6	5	1 : 4 : 1	+50	+ 15
6	6	.5	1 : 4 : 1	+100	+ 60
7	6	5	1 : 19 : 1	+150	+ 80
8	6	1	1 : 19 : 1	+200	+120
9	7	9	$1 : \frac{1}{4} : \frac{1}{16}$	-12	- 30
10	7	1	$1 : \frac{1}{4} : \frac{1}{16}$	-30	- 32
11	7	24	$1 : \frac{1}{19} : \frac{1}{361}$	-20	- 33
12	7	1	$1 : \frac{1}{19} : \frac{1}{361}$	-22	- 13
13	8	24	1 : 4 : 16	-10	+ 5
14	8	1	1 : 4 : 16	+120	+140
15	8	9	1 : 19 : 361	-27	+160
16	8	1	1 : 19 : 361	+550	+850
17	9	3	1 : 9 : inf	- 2	+150
18	9	1	1 : 39 : 0	+360	+266
19	9	9	1 : 4 : 0	0	+ 20
20	9	3	$1 : \frac{1}{19} :$ inf	-10	+100

It can be seen from Table 2 that there are some large discrepancies in the depths as determined by the two-layer curve method. An analysis of the results shows that with the two-layer curve method:

(a) The ratio h_2/h_1 is important in that the lower its value for a given ratio of resistivities, the higher will be the inaccuracy in the depth determination. This is more evident when the thicknesses are plotted on a logarithmic scale.

(b) Where ρ_2 is much greater than ρ_1, much larger thicknesses are obtained for the middle layer (see Examples 7 and 8). Thus, in the case where ρ_2 is greater than ρ_1 and ρ_3, and where the ratio of ρ_2/ρ_1 is much greater than 4, it is impossible to interpret the thickness of the middle layer with reasonable accuracy.

(c) Where ρ_2 has a transitional value between ρ_1 and ρ_3, unless the middle layer is thick enough (i.e. unless the ratio h_2/h_1 is above a certain value), it may not be detectable.

This can be seen in Examples 9 to 12, where $\rho_1 > \rho_2 > \rho_3$. Examples 10 and 12, where the middle layers are thin, may equally well be interpreted as two-layer cases and the discrepancies quoted have no real significance. Examples 9 and 11 show that when the ratio h_2/h_1 is greater than about 9, the middle layer can easily be detected from the curves, and the discrepancies in its thickness are reasonable.

A similar situation arises in Examples 13 to 16, where $\rho_1 < \rho_2 < \rho_3$. In Example 13, where h_2/h_1 is large, the middle layer is easily detected. In Example 15 (where $h_2/h_1 = 9$ appears to be a limiting value) there is only a slight change of slope on the three-layer curve, so that the middle layer can only just be detected; the discrepancies quoted for the depth to the third layer, however, give a reasonable estimate of the accuracy of its determination. However, for Examples 14 and 16, where the middle layer is thin (i.e. $h_2/h_1 << 9$), the middle layer cannot be detected and the discrepancies quoted for the depth to the bottom of the middle layer are not really significant. It is possible to interpret the bottom of the middle layers at any depth below the given three-layer solution for curves such as these.

(d) Within the context of the above reservations, it can be seen that, where ρ_2 is greater than ρ_1 and ρ_3, Maillet's relation gives the more accurate result (Examples 5 to 8). In cases where ρ_2 is less than ρ_1 and ρ_3, more accurate depth estimates are given by Hummel's relation (Examples 1 to 4).

7. CONCLUSIONS

There are good theoretical grounds for concluding that the use of the Schlumberger configuration of electrodes for the production of electrical sounding field curves will give more accurate results than the Wenner configuration.

It appears that when two-layer curves for the two configurations are plotted on a logarithmic scale, they are for all practical purposes identical, except for a difference in origin.

The use of Hummel and Maillet relations is reasonable under certain conditions. However, the ratio of the thicknesses of the first two layers and the distribution of the resistivities are shown to have significant effects on the resulting interpretations, when compared with published three-layer curves.

8. REFERENCES

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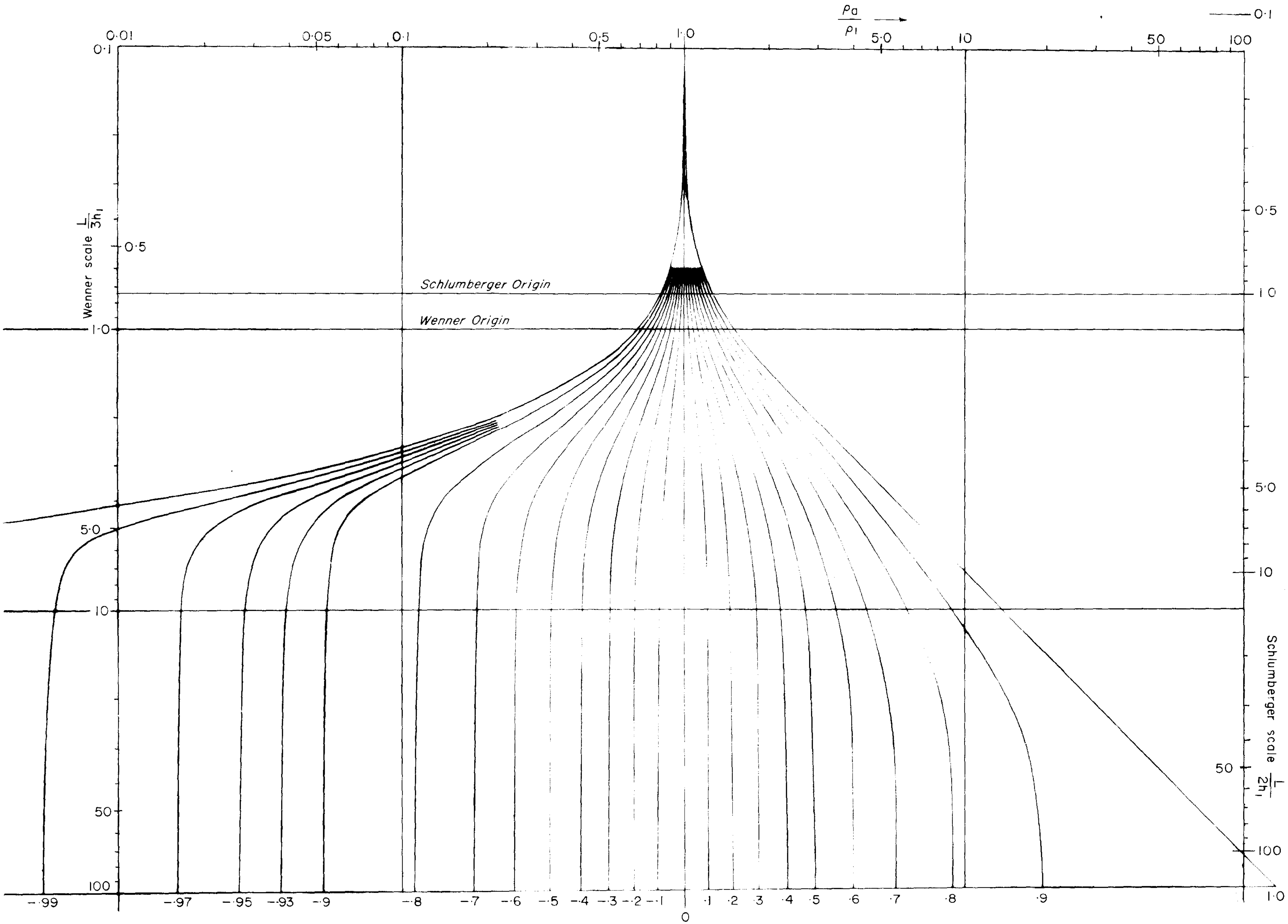

Two-layer Type Curves for resistivity depth probe interpretation Value of k from −1·0 to +1·0

PLATE I

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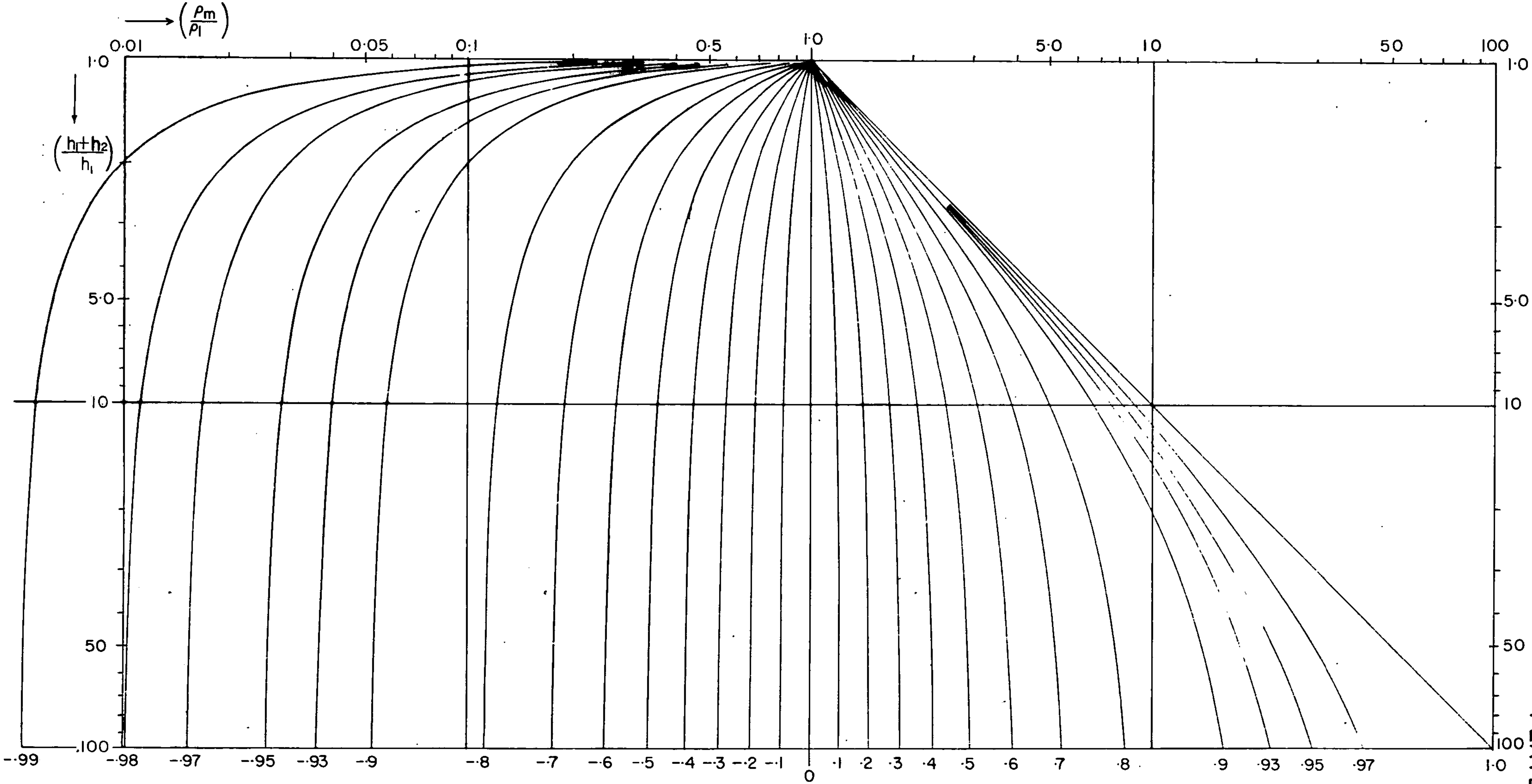
PLATE 2



Help curves for resistivity depth probe interpretation based on the Hummel relation $\frac{h_1+h_2}{\rho m} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2}$ for varying $k = \left(\frac{\rho_2-\rho_1}{\rho_2+\rho_1}\right)$

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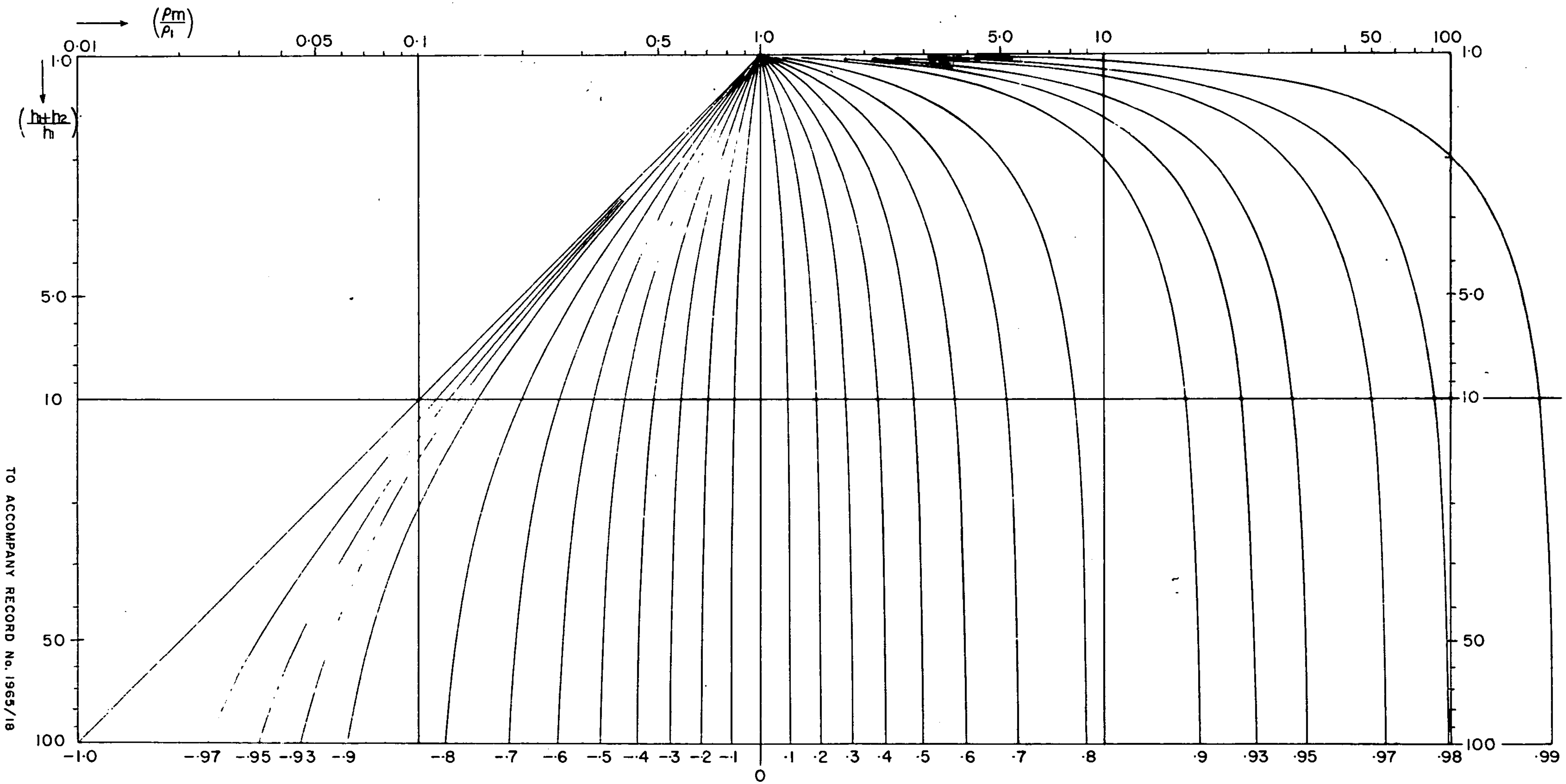

Help curves for resistivity depth probe interpretation based on the Maillet relation $(h_1+h_2)\rho m = h_1\rho_1 + h_2\rho_2$ for varying $k = \left(\frac{\rho_2-\rho_1}{\rho_2+\rho_1}\right)$

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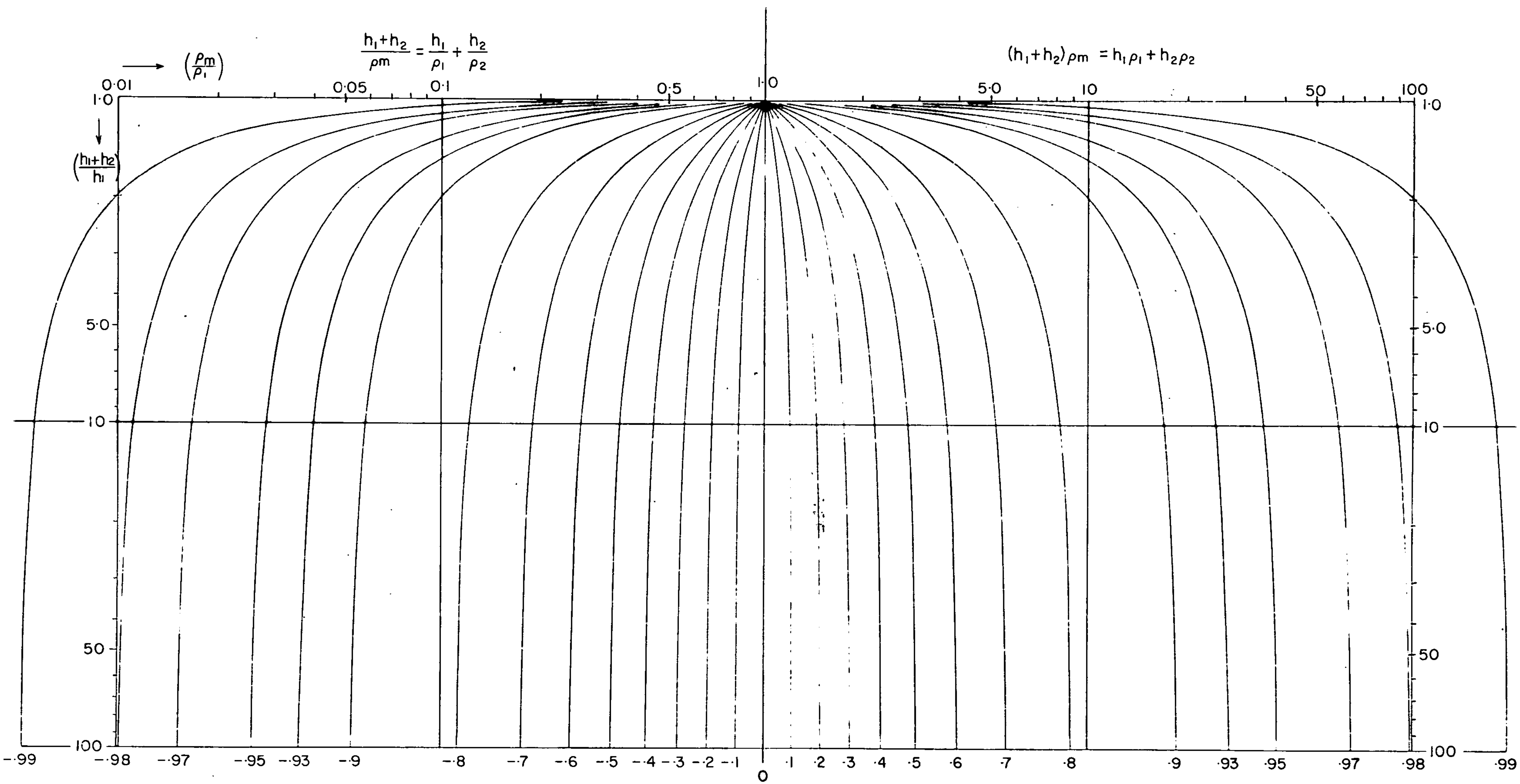
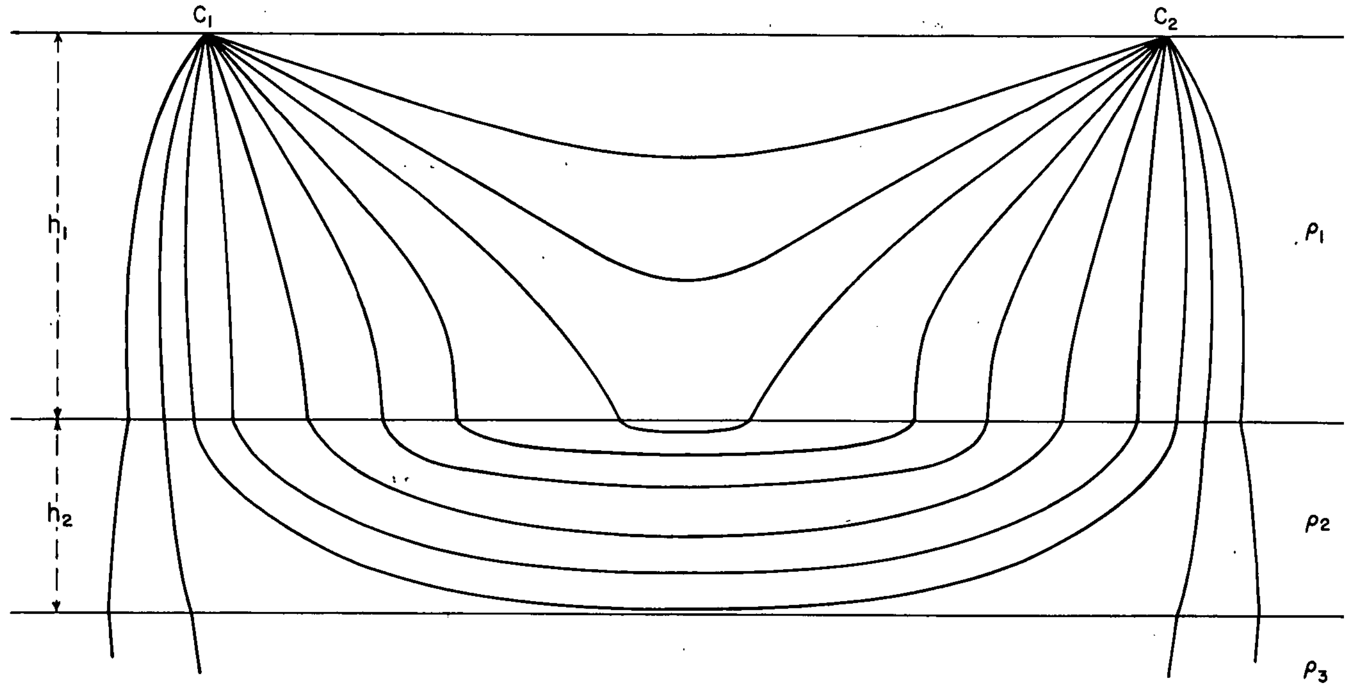


Help curves for resistivity depth probe interpretation for a combination of Hummel and Maillet relations for varying $k = \left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right)$

PLATE 5

$\rho_1 > \rho_2$
$\rho_3 > \rho_2$

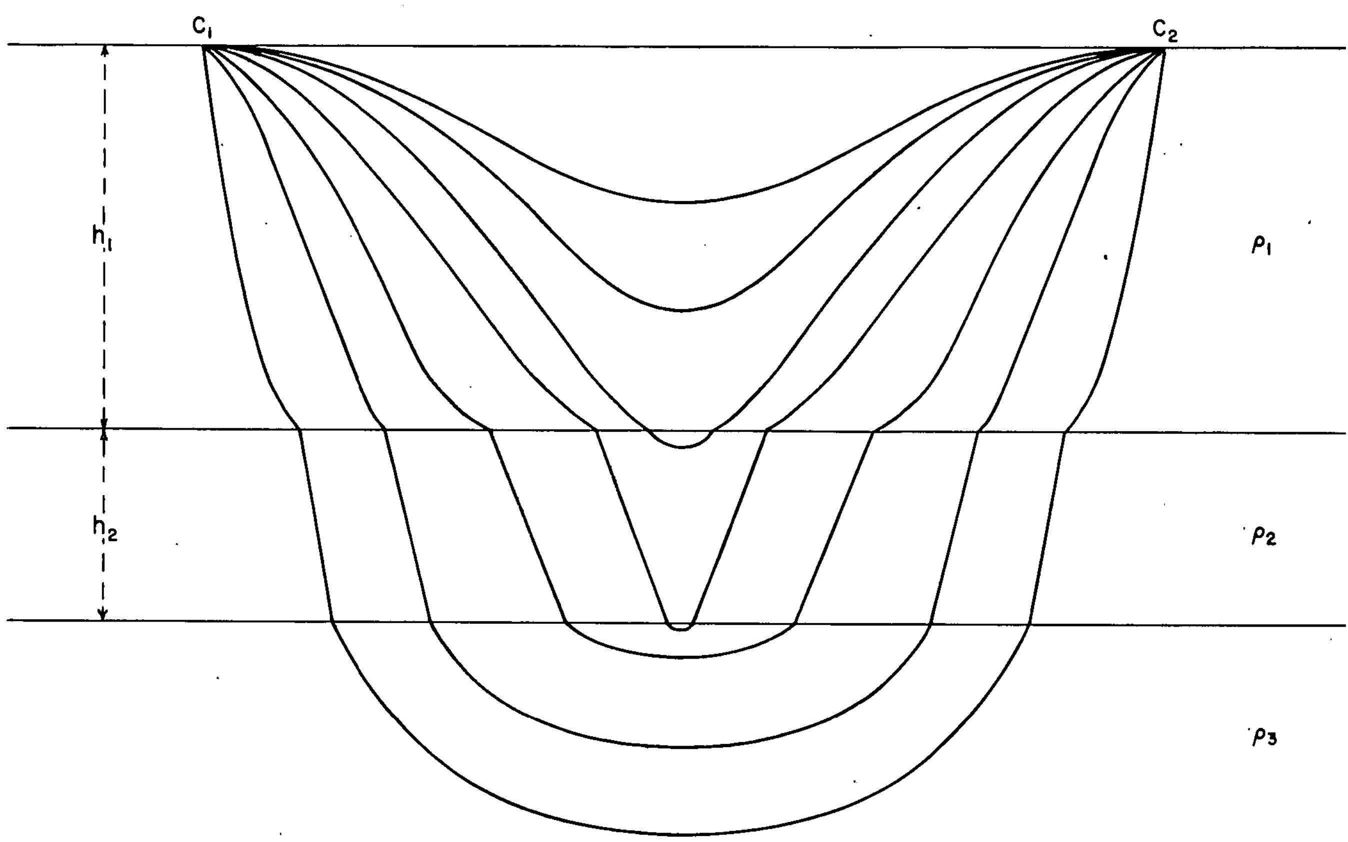
Case where $\frac{h_1+h_2}{\rho_{av}} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2}$ is valid



a) Lines of flow where Hummel's relation is applicable

$\rho_1 < \rho_2$
$\rho_3 < \rho_2$

Case where $(h_1+h_2)\rho_{av} = h_1\rho_1 + h_2\rho_2$ is valid



b) Lines of flow where Maillet's relation is applicable

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RESISTIVITY INTERPRETATIONS

EXAMPLES 1 TO 8

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RESISTIVITY INTERPRETATIONS

EXAMPLES 9 TO 12

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RESISTIVITY INTERPRETATIONS

EXAMPLES 13 TO 16

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RESISTIVITY INTERPRETATIONS

EXAMPLES 17 TO 20

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