COMMONWEALTH OF AUSTRALIA

DEPARTMENT OF NATIONAL DEVELOPMENT

BUREAU OF MINERAL RESOURCES, GEOLOGY AND GEOPHYSICS

RECORD No. 1965/62

THE CONSTRUCTION OF A CIRCULAR SLIDE RULE FOR SEISMIC DIP COMPUTATIONS

by

K.F. FOWLER

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ILLUSTRATIONS

SUMMARY

A slide rule has been designed generally to facilitate the computations for plotting 'migrated' seismic reflection sections, and particularly for use in calibrating the Sinclair Dip Plotter Type CKB. It is based on the theory of linear increase of seismic velocity with depth.

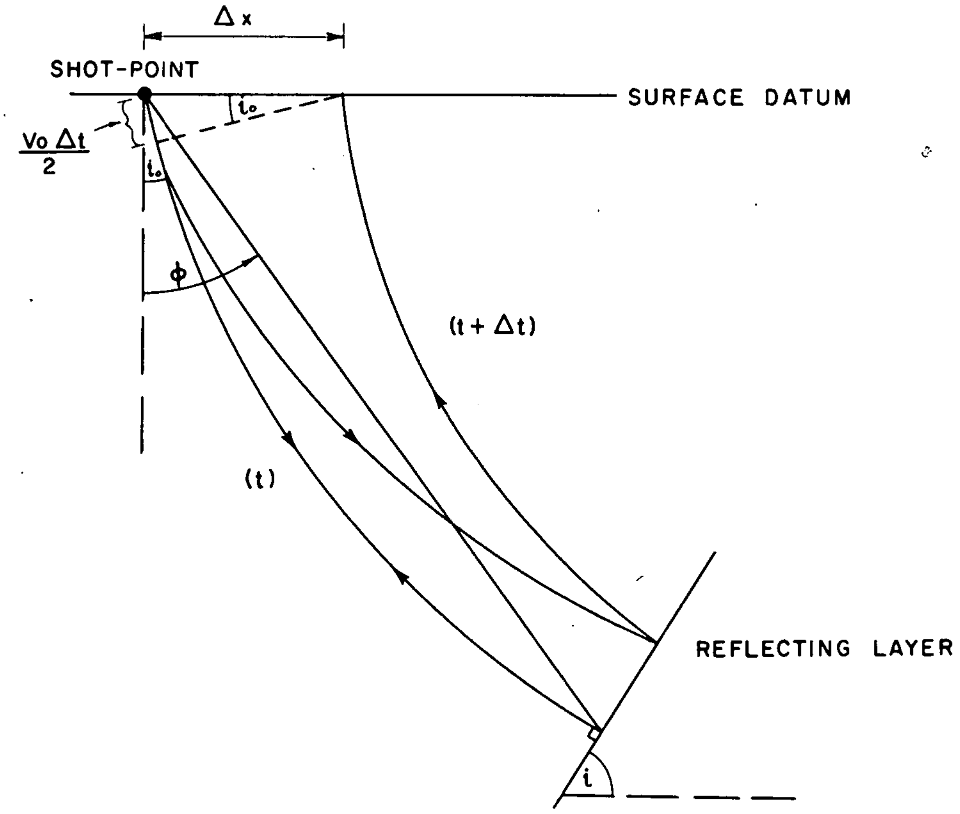


FIGURE 1: Vertical section showing reflected ray paths for a dipping reflector

1. INTRODUCTION

The slide rule described in this Record was initially designed to facilitate the computations involved in the hand-plotting of 'migrated' seismic reflection sections. Shortly after this design was completed, however, the Bureau of Mineral Resources bought a Sinclair Dip Plotter Type CKB, a semi-automatic device for the plotting of 'migrated' dip sections, which requires some tedious computations in the calibration of the instrument. The slide rule was subsequently modified to include an additional parameter, the polar angle of a point of reflection. This increased the versatility of the slide rule and made it of particular use in calibrating the Sinclair Dip Plotter .

2. THEORY

The following is a list of symbols used:

V_o	Velocity at surface datum
v	Velocity at depth below datum
a	Rate of increase of velocity with depth (acceleration constant)
t	Two-way reflection time at shot-point
i_o	Angle of emergence of ray at surface
i	Angle of dip (in same plane as i_o) of reflecting horizon
ϕ	Polar angle of point of reflection
Δx	Geophone spread length
Δt	Two-way time difference over spread length x

A linear velocity-depth function is assumed:

$$v = V_o + ad,$$

from which the following relations can be derived:

$$\tan(i/2) = \tan(i_o/2)e^{at/2} \qquad \text{...........(1)}$$

$$\sin i_o = V_o \Delta t / 2 \Delta x \qquad \text{...........(2)}$$

$$\phi = (i_o + i)/2 \qquad \text{...........(3)}$$

For small values of $i_o/2$ and $i/2$, Equation 3 can be written as

$$\phi = \tan(i_o/2) + \tan(i/2) \qquad \text{...........(4)}$$

From Equation 1, this becomes

$$\phi = \tan(i_o/2)(1 + e^{at/2}) \qquad \text{.............(5)}$$

Equations 1 and 5 can be written in logarithmic form:

$$K \log \tan(i/2) = K \log \tan(i_o/2) + K(at/2) \log e \qquad \text{.............(6)}$$

and

$$K \log \phi = K \log \tan(i_o/2) + K \log(1 + e^{at/2}) \qquad \text{.............(7)}$$

where K is a constant. It is convenient to take K=2. Equations 6 and 7 can then be written in the forms

$$2 \log \tan(i/2) = 2 \log \tan\left(\tfrac{1}{2} \arcsin \frac{V_o \Delta t}{2\Delta x}\right) + at \log e \qquad \text{.......(8)}$$

$$2 \log \phi = 2 \log \tan\left(\tfrac{1}{2} \arcsin \frac{V_o \Delta t}{2\Delta x}\right) + 2 \log(1 + e^{at/2}) \qquad \text{.......(9)}$$

Equations 8 and 9 form the basis of the slide rule, which has been constructed to perform mechanically the additions shown in these two equations. It will be noted that both equations contain a common term because of the approximations made in Equation 4. This simplifies the construction of the slide rule and also its use, as it requires only one setting of the V_o and Δt values to give values for i_o, i, and ϕ. It will also be noted that both i_o and i are expressed in the same form, i.e. the tangential form. This means that only one angular scale is required for both i_o and i. ϕ requires a separate scale as it is expressed in a different form.

The slide rule is calibrated to give directly values of i_o, i, and ϕ for known values of a, V_o, Δt, Δx, and t. For the purpose of calibrating the Sinclair Dip Plotter, values of ϕ can be assumed and, with a knowledge of a V_o, t, and Δx, corresponding values of Δt, i_o, and i can be determined.

The slide rule has been calibrated for Δx = 1320 ft, but can be used for other values of spread length by using an apparent value of V_o (instead of the true value), given by

$$V_o \text{ (apparent)} = V_o \cdot \frac{1320}{\Delta x}$$

3. APPROXIMATIONS AND ERRORS

The only approximation used in calibrating the slide rule is the substitution of $\tan(i_o/2)$ and $\tan(i/2)$ for $i_o/2$ and $i/2$ respectively, as shown in Equation 4. This results in the following errors in ϕ:

ϕ	Error
37.5°	+4.7%
30°	+3.0%
20°	+1.3%
10°	+0.3%

No approximations are used in determining i_o and i, hence the accuracy of these values will be limited only by the accuracy of reading the slide rule.

A true value of ϕ can be found from i_o and i from Equation 3.

Comparison with the Sinclair Dip Plotter

In the theory of the Sinclair Dip Plotter, the approximation

$$i/2 = (i_o/2)\ e^{at/2} \qquad \text{..........(10)}$$

is made for Equation 1, but a second approximation is made by substituting $(\sin i_o)/2$ for $i_o/2$, so that ϕ is given by

$$\phi = \frac{\sin i_o}{2} \quad (1 + e^{at/2}) \qquad \text{..........(11)}$$

which results in the following errors in ϕ:

ϕ	Error
37.5^o	-1.2%
30^o	-0.7%
20^o	-0.3%
10^o	-0.1%

These errors are less than those for the slide rule, and Equation 11 would form a more accurate basis for the construction of a slide rule than Equation 5. If it were found in practice that the slide rule described in this Record was not sufficiently accurate for calibrating the Sinclair Dip Plotter, a more accurate one could be made based on the following equations:

$$\log (i/2) = \log \frac{\sin i_o}{2} \quad + \quad at/2 \qquad \text{..........(12)}$$

$$\log \phi \quad = \log \frac{\sin i_o}{2} \quad + \ \log (1 + e^{at/2}) \qquad \text{.........(13)}$$

Equation 12 is an approximation of Equation 10, but gives values of i_o and i that are very close to the correct values, the only disadvantage being that separate scales would be required for i_o and i.

4. CONSTRUCTION

The Circular Slide Rule is constructed using polar coordinate graph paper, calibrated in degrees. For this reason it is found convenient to convert all circular measurements from radians (as given in Equations 8 and 9) to degrees and decimal fractions of degrees. The slide rule can thus be made any size required and the radial scales can be positioned for maximum convenience.

The slide rule consists of three parts: the fixed base, the moveable disc, and the index arm, which are described below.

The fixed base

This consists of a full 360° of 12-inch diameter polar coordinate graph paper with one radial line taken as the origin (Plate 1). This line is marked off linearly in values of V_o from 5000 ft/s to 12,000 ft/s in intervals of 1000 ft/s, with 200 ft/s subdivisions. It is convenient to repeat this V_o scale at 90° intervals.

Table 1 gives the relations between V_o, $\triangle t$, and the plotting angles. 'Constant $\triangle t$' curves can be plotted and drawn on the fixed base from this table by measuring the plotting angle degrees in an anticlockwise direction from the line of origin.

Table 2 gives the relation between the angle i and the plotting angle. From these values an i scale can be drawn round the outer edge of the graph paper.

Table 3 gives the relation between the angle ϕ and the plotting angle. From these values a ϕ scale can be drawn round the outer edge of the graph paper but separate from the i scale, as the two scales overlap.

On the fixed base, all plotting angles are measured anticlockwise from the line of origin.

The moveable disc

This consists of 225° of 10-inch diameter polar coordinate graph paper mounted on a transparent circular disc of perspex, which enables a portion of the fixed base to be seen through the remaining 135° (Plate 2). On this disc are drawn two separate sets of 'constant a' curves, each set bearing identical t scales. The disc is mounted so that it can rotate coaxially above the fixed base.

The leading edge of this graph paper is taken as the origin for all angular measurements on this disc, which are measured clockwise from this origin.

Radial lines are drawn at 10° and 130°, and are marked off linearly in values of t from 0 to 4 seconds at intervals of 0.2 second, with 0.05-second subdivisions.

The two sets of 'constant a' curves can be plotted and drawn from the values given in Tables 4 and 5, which show the relations between a, t, and the plotting angle.

The index arm

This is a piece of transparent perspex bearing a radial hair-line and mounted to rotate coaxially with the moveable disc and fixed base.

5. USE OF SLIDE RULE

General

Usually, V_o, a, Δt, Δx, and t are known quantities and it is required to find i_o, i, and ϕ. The following operations will give these quantities directly:

(1) The leading edge of the moveable disc is placed on the intersection of the appropriate Δt and V_o lines, and i_o can be read off on the i scale, not along an extension of the leading edge, but by placing the hair-line of the index arm along the first t scale. Where this intersects the i scale is the required value of i_o.

(2) Without moving the disc, place the hair-line of the index arm on the intersection of the appropriate a and t lines in the first set of 'constant a' curves. This gives the value of i directly.

(3) Without moving the disc place the hair-line of the index arm on the intersection of the appropriate a and t lines in the second set of 'constant a' curves. This gives the value of ϕ directly.

Calibration of the Sinclair Dip Plotter

Two sets of calibrations are necessary on the Sinclair Dip Plotter in which the slide rule can be of use:

(1) It is necessary to calibrate the instrument for the relation between Δt, t, and ϕ. To do this, three values of t are taken, one value near the beginning of the records, one near the end, and one in the middle. It is required to find the value of Δt at each of these times t for four values of ϕ (10°, 20°, 30°, and 37.5°). The hair-line of the index arm is placed over the value of ϕ on the ϕ scale and the moveable disc rotated until the intersection of the known t and a values on the ϕ set of curves lies on the hair-line. The required value of Δt will then be found on the fixed base at the intersection of the leading edge of the moveable disc and the known value of V_o.

(2) In order to draw in the reflection at the correct dip angle, the plotting cursor has to be rotated through an angle $(i-\phi)$. This is done by the use of a correction graph (an example is shown in Plate 3), which relates time (or depth), ϕ, and $(i-\phi)$ for a given velocity-depth function. The slide rule is used in the preparation of these correction graphs by setting the hair-line of the index arm on a particular value of ϕ, moving the moveable disc until the intersection of the known a value and a particular value of t on the ϕ set of curves appears on the hair-line, and then moving the index arm so that the hair-line is over the intersection of the same values of t and a on the i set of curves; i can then be read off under the hair-line on the i scale.

TABLE 1

V_o / Δt curves

Plotting angle in degrees

V_o in ft/s	Δt in milliseconds: 1	2	3	4	5	6	7	8
5000	350	312.2	294.5	277.6	265.2	257.4	249.8	243.1
6000	338	303.1	285.4	270.2	258.0	248.2	241.6	233.8
7000	330	295.4	277.5	260.9	250.5	241.2	232.8	226.9
8000	323	288.7	270.0	254.5	243.5	233.8	226.2	219.2
9000	317.2	282.9	263.0	248.2	237.0	228.0	220.4	213.8
10000	313	277.6	257.4	243.1	232.0	222.7	215.1	208.6
12000	308	270.2	248.2	233.8	222.7	213.8	206.0	199.0

Plotting angle in degrees

V_o in ft/s	Δt in milliseconds: 10	12	14	16	18	20	40	50
5000	232.0	222.7	215.1	208.6	202.6	197.2	162.9	151.7
6000	222.7	213.8	206.0	199.0	193.8	188.4	153.9	142.5
7000	215.1	205.8	198.2	191.7	185.9	180.7	145.9	134.8
8000	208.6	199.0	191.7			174.1	139.3	128.2
9000	202.6	193.8	185.9			168.2	133.4	
10000	197.2	188.0	180.7	174.1	168.2	162.9	128.2	116.8
12000	188.4	179.1	171.7	164.9	158.9	153.9	118.9	107.6

Plotting angle in degrees

V_o in ft/s	Δt in milliseconds: 60	80	120	160	200	240	280	320
5000	142.5	128.2	107.6	92.8	80.9	70.9	62.1	54.0
6000	133.4	118.9	98.2	83.1	70.9	60.4	50.9	41.8
7000	125.6	111.2	90.1	74.8	62.1	50.9	40.3	
8000	118.9	103.7	83.1	67.3	54.0	41.8		
9000		98.2	76.8	60.4	46.4			
10000	107.6	92.8	70.9	54.0	38.8			
12000	98.2	83.1	60.4	41.8				

TABLE 1 (cont.)

V_o in ft/s	Δt in milliseconds: 22	24	26	28	30	32	34	36
5000	192.4	188.0	184.2	180.7	177.0	174.1	170.8	168.2
7000	176.0	171.7	167.4	163.8	160.4	157.2	154.2	151.4
10000	158.2	153.9	149.6	145.9	142.5	139.3	136.2	133.4
12000	149.0	144.5	140.5	136.9	133.4	130.2	127.1	124.1
	Plotting angle in degrees							

V_o in ft/s	Δt in milliseconds: 38	42	44	46	48	52	54	56
5000	165.3	160.4	158.2	155.8	153.9	149.6	147.9	145.9
7000	148.7	143.5	141.2	139.1	137.0	132.8	131.0	129.1
10000	130.7	125.6	123.4	121.1	118.9	114.9	112.9	111.2
12000	121.6	116.5	114.1	111.8	109.7	105.6	103.7	101.8
	Plotting angle in degrees							

V_o in ft/s	Δt in milliseconds: 58	62	64	66	68	70	72	74
5000	144.3	141.0	139.3	137.9	136.2	134.8	133.4	132.0
7000	127.4	124.0	122.4	120.9	119.3	117.8	116.5	115.1
10000	109.3	106.0	104.3	102.7	101.1	99.7	98.8	96.8
12000	99.9	96.5	94.9	93.2	91.7	90.1	88.7	87.2
	Plotting angle in degrees							

TABLE 1 (cont.)

V_o in ft/s	Δt in milliseconds: 76	78	84	88	92	96	100	104
5000	130.7	129.4	125.6	123.4	121.1	118.9	116.8	114.9
7000	113.7	112.4	108.6	106.2	103.9	101.8	99.7	97.6
10000	95.4	94.0	90.1	87.7	85.4	83.1	80.9	78.8
12000	85.9	84.5	80.5	78.0	75.5	73.2	70.9	68.8

Plotting angle in degrees

V_o in ft/s	Δt in milliseconds: 108	112	116	124	128	132	136	140
5000	112.9	111.2	109.3	106.0	104.3	102.7	101.1	99.7
7000	95.7	93.8	92.0	88.5	86.8	85.1	83.5	82.0
10000	76.8	74.8	72.8	69.1	67.3	65.6	63.8	62.1
12000	66.6	64.5	62.5	58.5	56.6	54.7	52.8	50.9

Plotting angle in degrees

V_o in ft/s	Δt in milliseconds: 144	148	152	156	170	180	190	210
5000	98.8	96.8	95.4	94.0	89.6	86.5	83.7	78.3
7000	80.5	79.0	77.6	76.2	71.4	68.2	65.1	59.2
10000	60.4	58.8	57.2	55.6	50.2	46.4	42.2	
12000	49.1	47.3	45.5	43.7				

Plotting angle in degrees

TABLE 1 (cont.)

V_o in ft/s	Δt in milliseconds: 220	230	250	260	270	290	300	310
5000	75.8	73.3	68.7	66.4	64.2	60.0	58.0	56.0
7000	56.4	53.7	48.2	45.7	43.0			
	Plotting angle in degrees							

V_o in ft/s	Δt in milliseconds: 330	340	350	360	370	380	390	400
5000	52.1	50.2	48.2	46.4	44.5	42.2	40.7	38.8
	Plotting angle in degrees							

Note. The plotting angles are to be measured anticlockwise from the line of origin.

TABLE 2

i scale

i	Plotting angle	i	Plotting angle	i	Plotting angle
0°30'	260.4	13°	98.1	31°	53.8
1°	225.9	13°30'	96.2	32°	52.2
1°30'	205.8	14°	94.3	33°	50.6
2°	191.4	14°30'	92.6	34°	49.0
2°30'	180.3	15°	90.9	35°	47.4
3°	171.3	15°30'	89.2	36°	45.9
3°30'	163.6	16°	87.6	37°	44.5°
4°	157.0	16°30'	86.1	38°	43.1
4°30'	151.1	17°	84.6	39°	41.7
5°	145.9	17°30'	83.1	40°	40.3
5°30'	141.1	18°	81.7	42°	37.7
6°	136.7	18°30'	80.3	44°	35.1
6°30'	132.8	19°	79.0	46°	32.6
7°	129.1	19°30'	77.6	48°	30.3
7°30'	125.6	20°	76.3	50°	28.0
8°	122.4	21°	73.9	55°	22.5
8°30'	119.4	22°	71.5	60°	17.3
9°	116.5	23°	69.2	65°	12.5
9°30'	113.8	24°	67.1	70°	7.8
10°	111.2	25°	64.9	75°	3.2
10°30'	108.8	26°	62.9	80°	-1.3
11°	106.5	27°	61.0	85°	-5.7
11°30'	104.3	28°	59.1	90°	-10.0
12°	102.1	29°	57.3		
12°30'	100.1	30°	55.5		

Note. The plotting angles are given in degrees and are to be measured anticlockwise from the line of origin. A negative anticlockwise rotation is equivalent to a clockwise one.

TABLE 3

φ scale

φ	Plotting angle	φ	Plotting angle	φ	Plotting angle
0°30'	135.9	10°30'	-15.5	21°	-50.0
1°	101.4	11°	-17.9	22°	-52.4
1°30'	81.3	11°30'	-20.1	23°	-54.6
2°	68.0	12°	-22.2	24°	-56.7
2°30'	55.8	12°30'	-24.2	25°	-58.7
3°	46.8	13°	-26.2	26°	-60.7
3°30'	39.1	13°30'	-28.0	27°	-62.5
4°	32.5	14°	-29.9	28°	-64.4
4°30'	26.6	14°30'	-31.6	29°	-66.1
5°	21.4	15°	-33.3	30°	-67.8
5°30'	16.6	15°30'	-34.9	31°	-69.4
6°	12.3	16°	-36.5	32°	-71.0
6°30'	8.4	16°30'	-38.1	33°	-72.6
7°	4.6	17°	-39.6	34°	-74.0
7°30'	1.2	17°30'	-41.0	35°	-75.5
8°	-2.0	18°	-42.4	36°	-76.9
8°30'	-5.1	18°30'	-43.7	37°	-78.2
9°	-7.9	19°	-45.1	40°	-82.1
9°30'	-10.6	19°30'	-46.4		
10°	-13.1	20°	-47.6		

Note. The plotting angles are given in degrees and are to be measured anticlockwise from the line of origin. A negative anticlockwise rotation is equivalent to a clockwise one.

TABLE 4

a/t curves for i

Acceleration constant a in ft/s²	Time t in seconds: 0	1	2	3	4
.05	10.0	11.2	12.5	13.7	15.0
.10	10.0	12.5	15.0	17.5	20.0
.15	10.0	13.7	17.5	21.2	24.9
.20	10.0	15.0	20.0	24.9	29.9
.25	10.0	16.2	22.4	28.7	34.9
.30	10.0	17.5	24.9	32.4	39.9
.35	10.0	18.7	27.4	36.1	44.8
.40	10.0	20.0	29.9	39.9	49.8
.45	10.0	21.2	32.4	43.6	54.8
.50	10.0	22.4	34.9	47.3	59.8
.55	10.0	23.7	37.4	51.1	64.7
.60	10.0	24.9	39.9	54.8	69.7
.65	10.0	26.2	42.3	58.5	74.7
.70	10.0	27.4	44.8	62.3	79.7
.75	10.0	28.7	47.3	66.0	84.7
.80	10.0	29.9	49.8	69.7	89.6
.85	10.0	31.2	52.3	73.5	94.6
.90	10.0	32.4	54.8	77.2	99.6
.95	10.0	33.6	57.3	80.9	104.6
1.00	10.0	34.9	59.8	84.7	109.5

Plotting angle in degrees

Note. The plotting angles are to be measured clockwise from the leading edge.

TABLE 5

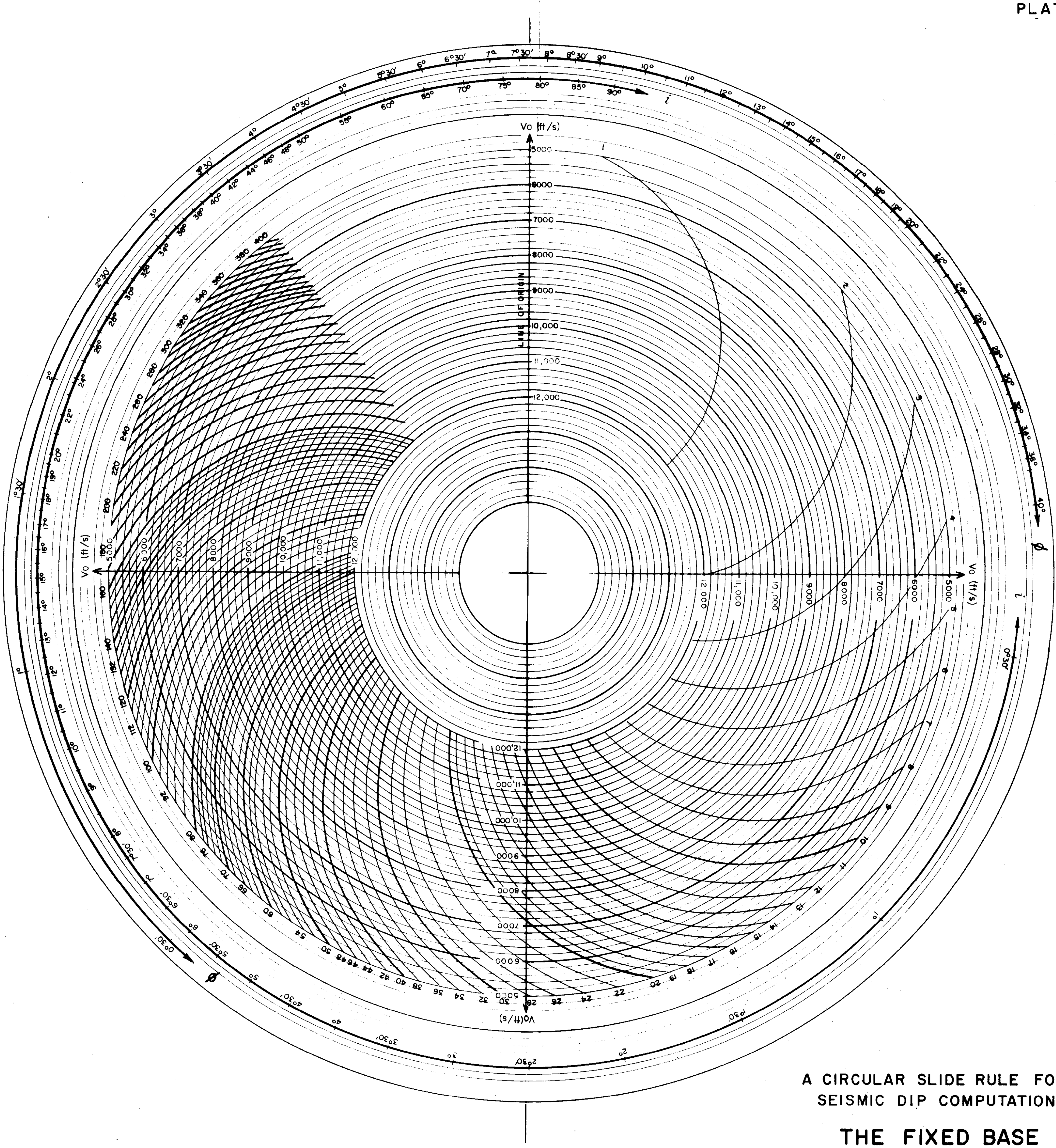
a/t curves for ∅

Acceleration constant a in ft/s²	Time t in seconds: 0	1	2	3	4
.05	134.5	135.1	135.8	136.4	137.0
.10	134.5	135.8	137.0	138.4	139.7
.15	134.5	136.4	138.4	140.4	142.5
.20	134.5	137.0	139.7	142.5	145.4
.25	134.5	137.7	141.1	144.7	148.5
.30	134.5	138.4	142.5	146.9	151.6
.35	134.5	139.0	144.0	149.2	154.9
.40	134.5	139.7	145.4	151.6	158.3
.45	134.5	140.4	146.9	154.1	161.8
.50	134.5	141.1	148.5	156.6	165.4
.55	134.5	141.8	150.0	159.1	169.1
.60	134.5	142.5	151.6	161.8	172.8
.65	134.5	143.3	153.3	164.5	176.7
.70	134.5	144.0	154.9	167.2	180.6
.75	134.5	144.7	156.6	170.0	184.7
.80	134.5	145.4	158.3	172.8	188.8
.85	134.5	146.2	160.0	175.7	192.9
.90	134.5	146.9	161.8	178.7	197.2
.95	134.5	147.7	163.6	181.7	201.5
1.00	134.5	148.5	165.4	184.7	205.8

Plotting angle in degrees

Note. The plotting angles are to be measured clockwise from the leading edge.

PLATE I



A CIRCULAR SLIDE RULE FOR SEISMIC DIP COMPUTATION

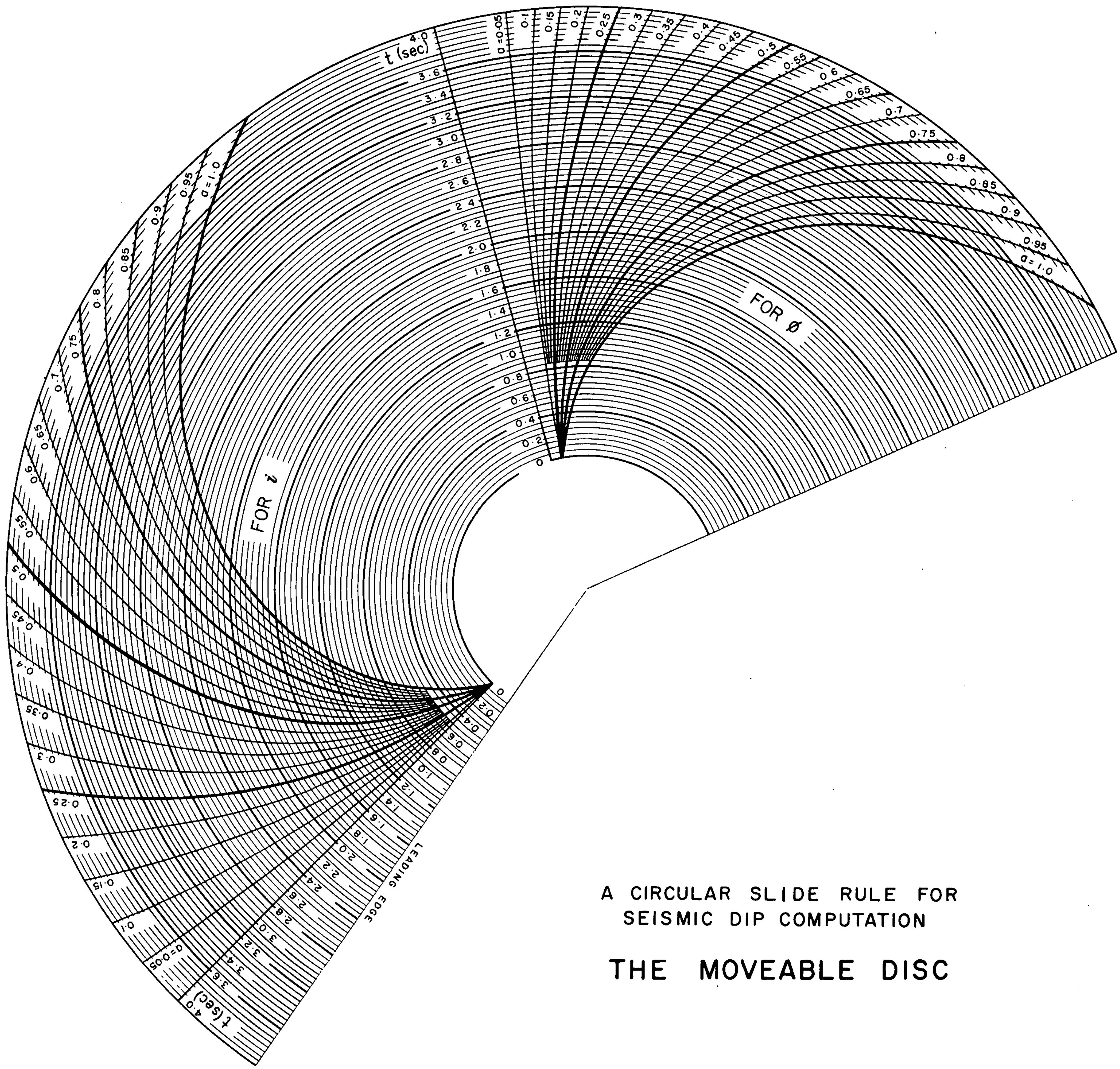
THE FIXED BASE

G 85/3-41

(BASED ON G 85/3-1 and 2)

Geophysical Branch, Bureau of Mineral Resources, Geology and Geophysics

TO ACCOMPANY RECORD No 1965/62



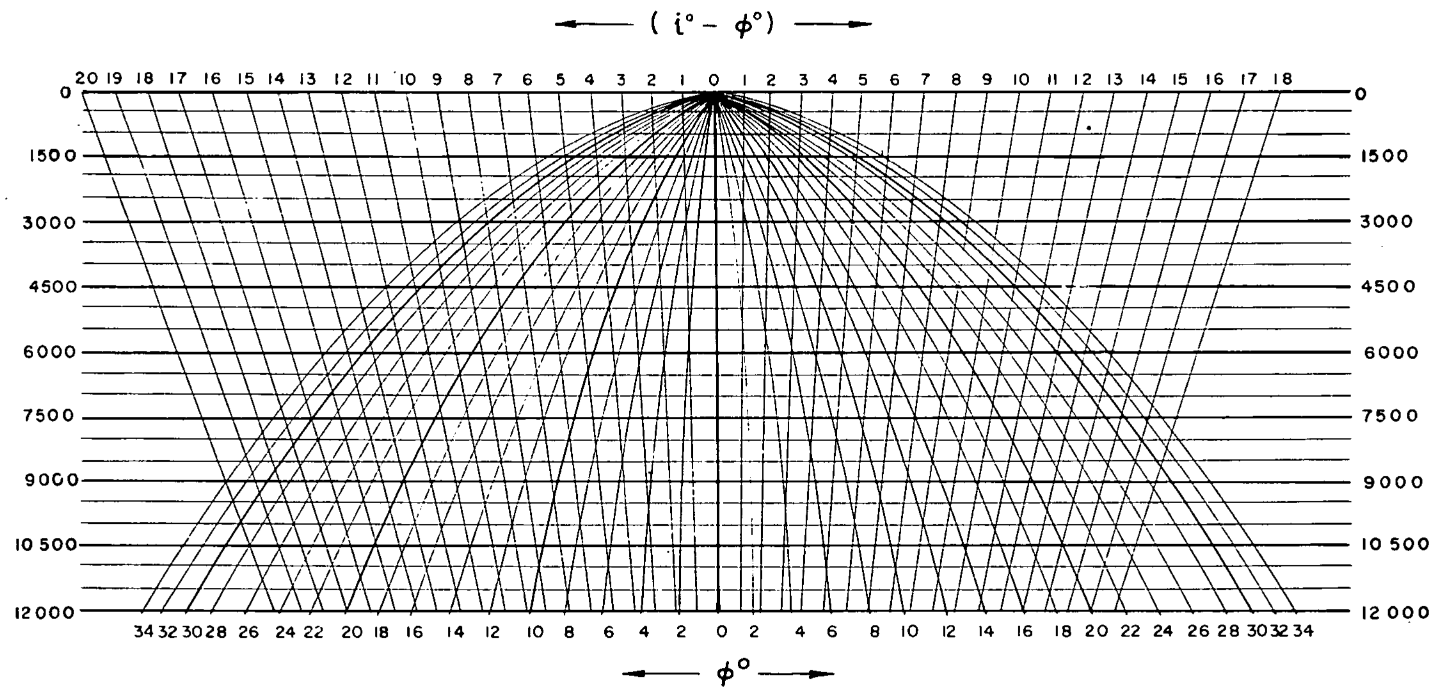
A CIRCULAR SLIDE RULE FOR SEISMIC DIP COMPUTATION

THE MOVEABLE DISC

Geophysical Branch, Bureau of Mineral Resources, Geology and Geophysics

G 85/3—3

TO ACCOMPANY RECORD No 1965/62



EXAMPLE OF A CORRECTION GRAPH FOR THE SINCLAIR DIP PLOTTER

V = 8000 + 0·6 Z

PLATE 3

Geophysical Branch, Bureau of Mineral Resources, Geology and Geophysics G 85—149

TO ACCOMPANY RECORD No. 1965/62