COMMONWEALTH OF AUSTRALIA

DEPARTMENT OF NATIONAL DEVELOPMENT

BUREAU OF MINERAL RESOURCES, GEOLOGY AND GEOPHYSICS

RECORD No. 1966/23

A VERTICAL SECTION INTEGRATOR FOR THE COMPUTATION OF GRAVITY ANOMALIES

by

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ILLUSTRATIONS

This is the effect of a section of the body bounded by radii r_1 and r_2 and by angles ϕ_1 and ϕ_2 and stretching to infinity in one horizontal direction. The effect of the whole body will be the sum of the effects due to a number of such sections. It is this summation that the instrument has to make.

3. BASIC DESIGN

To evaluate the required integral, it is necessary to apply to a recording mechanism, such as a planimeter wheel, a compound motion, of which one part is proportional to a radial displacement and the other part proportional to the cosine of an angular displacement from a fixed datum. The effect of the radial displacement is easily introduced. In order to introduce the effect of angular displacement, advantage is taken of a property of cycloidal curves.

If a circle rolls without slipping in internal contact with the circumference of a larger circle, the locus of a point fixed with reference to the smaller circle is a hypocycloid. If the point is on the circumference of the smaller circle, and the radius of the smaller circle is half the radius of the larger circle, the hypocycloid degenerates into a straight line through the centre of the larger circle. The displacement of the point from the centre of the larger circle is proportional to the cosine of the angle between the line of centres of the circles and the path of the point. The relations are shown diagrammatically in Plate 1. This property provides a means of applying the required motion.

The basic design is shown schematically in Plate 2. As the mechanism of rolling circles is difficult to realise in practice, the circles are coupled by gears so that the speed of rotation of the smaller circle is twice that of the larger one. In Plate 2, circle 3 is the rolling circle rolling within circle 4. We suppose that circle 3 is rigidly connected to the large toothed disc 1, and circle 4 to the toothed disc 2, and that discs 1 and 2 are driven by rigidly connected concentric pinions, the numbers of teeth being so chosen that rotation of disc 1 through any angle causes rotation of disc 2 through twice the angle. The centre of the planimeter wheel is rigidly connected to the point A on disc 4. The zero position is when point A is in contact with the circumference of circle 3. If disc 1 is now rotated through any angle (say 30°) point A will move to position A1 and the distance of the planimeter wheel from O will be $1.8 \cos 30^\circ$ inches. (The sizes shown are those used in the final instrument).

The planimeter wheel bears on another disc concentric with disc 1 and circle 3 and a mechanism is introduced for converting radial motions of the tracing point into rotations of this disc. The tracing point is constrained to move along a line through O, rigidly connected to disc 1.

If the tracing point is now carried round the desired section, the planimeter reading is proportional to the required integral. An advantage of the design is that the axis of the planimeter wheel is always radial, so that errors due to slipping are minimised.

4. PRACTICAL REALISATION

The practical realisation is shown schematically in Plate 3. The tracing point is mounted on a carriage which moves along an arm passing through the axis of the instrument and which is rigidly fixed to the case. The planimeter wheel is mounted on a carriage that moves along a radial axis parallel to the arm carrying the tracing point. This carriage terminates in a bush which is a slide fit in an eccentric hole in the top plate (disc 2 in Plate 2). Gearing is introduced so that, when the case of the instrument is rotated through a given angle, the top plate turns through twice the angle. The theoretical relations are then satisfied. The effective diameter of the rolling circle is equal to the maximum distance of the planimeter wheel from the axis. This position is the zero line of the instrument, and in use, the level surface of drawings must be positioned accurately at right angles to it. The planimeter wheel bears on a disc, and radial motions of the tracing point are converted to rotations of the disc by means of a steel band. In order to leave the space under the instrument as free as possible for manipulating drawings, the instrument is mounted on an off-centre bracket. A stop is provided on the case (not shown in Plate 3) so that, when the carriage is against the stop, the tracing point is accurately under the central axis.

In an actual determination of the gravity effect of a two-dimensional body on a point P, this point must be placed directly beneath the centre of the instrument. The stylus is then placed on a chosen point on the circumference of the section, the planimeter wheel is then turned to zero and the stylus is moved by tracing the contour in a clock-wise direction. The planimeter is then read, a complete revolution of the wheel being one unit.

With an effective diameter of the rolling circle of 1.8 inches, the formula for converting planimeter readings to gravity anomalies is

$$g = 1.10 \times 10^{-2} \sigma R f \text{ milligal},$$

where σ = density difference,
R = number of revolutions of the planimeter wheel, and
f = scale of the drawing in feet per inch.

A photograph of the instrument is shown in Plate 4. The tracing point is moved radially by means of a hand wheel driving a pinion, which engages in a rack cut on a solid steel rod. The carriage also moves along a slotted steel tube, which contains the band controlling the rotation of the planimeter wheel. A lock is provided, to lock the instrument in the zero position.

5. ACCURACY OF THE INSTRUMENT

As a check on the performance of the instrument, the anomaly due to a horizontal cylinder of radius 150 feet, with its axis at a depth of 300 feet, was measured, using a drawing made at a scale of 1 inch = 100 feet. The density difference was taken as 1. The anomaly was obtained at points along a traverse at right angles to the axisof the cylinder, the zero point being vertically above the axis.

The results were as follows:

Distance from zero (feet)		g from instrument (mgal)	g by calculation (mgal)	Error of instrument (mgal)
0	(using	0.963	0.957	+0.006
100	g = 1·13R)	0.852	0.861	-0.009
200		0.666	0.663	+0.003
300		0.485	0.479	+0.006
400		0.341	0.345	-0.004
500		0.248	0.253	-0.005
600		0.188	0.192	-0.004
700		0.149	0.149	0.000
800		0.115	0.118	-0.003
900		0.094	0.096	-0.002
1000		0.080	0.079	+0.001

The errors show no systematic trend. The maximum error is 0.009 milligal. The test is a severe one, because the body is much smaller, and the density difference much larger, than are generally encountered in practice. It is concluded that the accuracy of the instrument is sufficient for all practical requirements.

6. ADDITIONAL APPLICATIONS OF THE INTEGRATOR

Non-infinite bodies

The integrator can be used to determine the gravity effect at a point P on the surface due to a finite body, providing the cross-section of the body is small compared with its depth below P and providing the length of the body is known. If the body subtends angles β_1 and β_2 from the vertical at P, the gravity effect due to the body is

$$g' = \tfrac{1}{2}g\,(\sin\beta_1 + \sin\beta_2)$$

where g is the gravity effect of an infinite body of similar cross-section, as measured by the integrator.

Terrain corrections

Probably the most important application of the instrument is its use in determining terrain corrections.

Plate 5 (a) shows the situation where the body whose gravity effect is to be determined is at the surface and the shape of the section of the body subtended by an angle of 2β from point P.

Since P, with respect to which the terrain correction is being evaluated, always lies on the mass, the cross-section to be traced must vanish at that point, as shown in Plate 5 (b and c). The gravity effect of such a section will be

$$g' = g\sin\beta$$

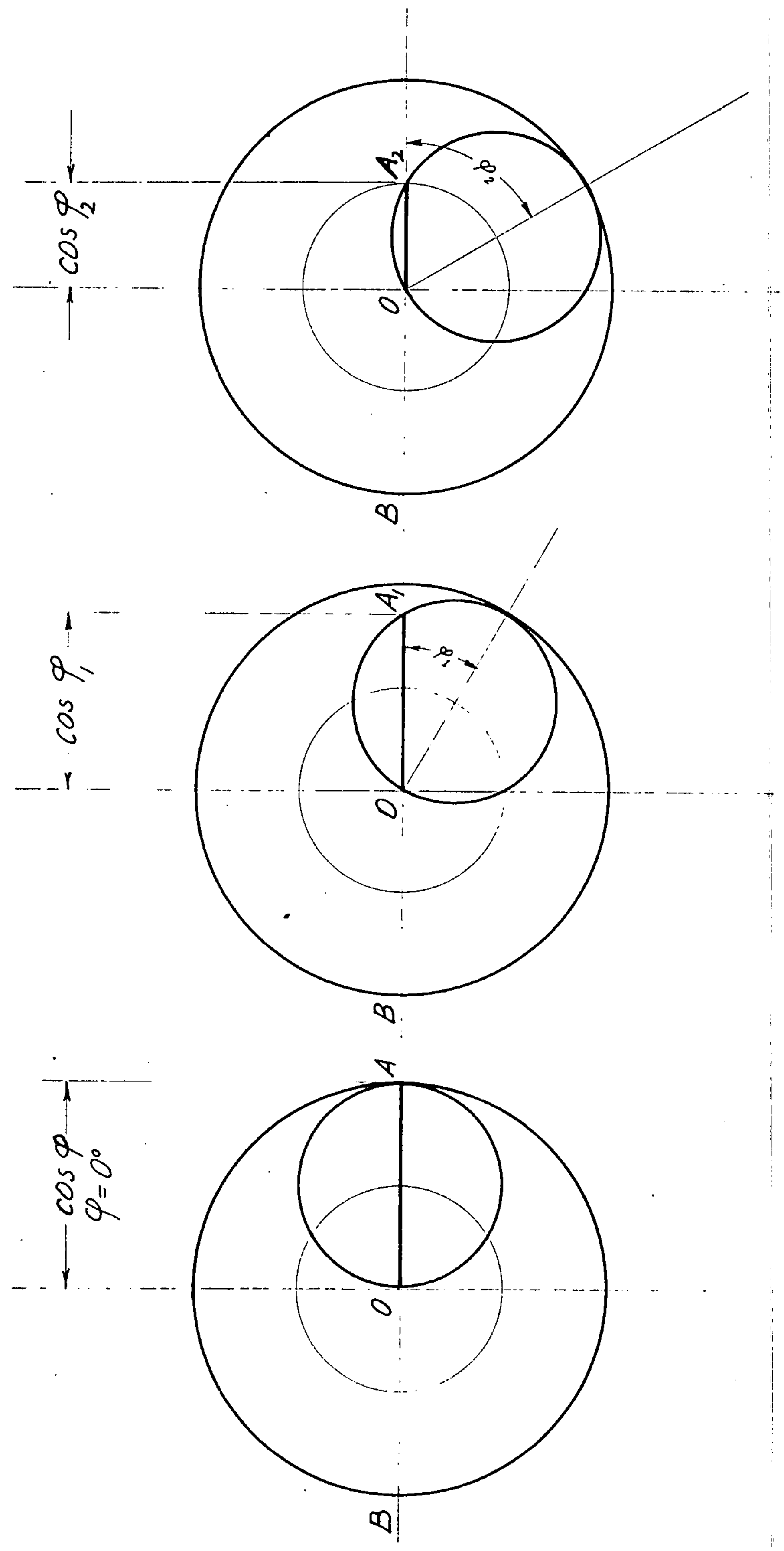
In the example shown in Plate 5 (c), DPC is the terrain profile along line AB, and if the integrator stylus is traced from P to A, D, P, C, B, and P, the planimeter reading R_1 will be proportional to the gravity effect of the section PLMM'L'PNOO'N'. To get the total terrain effect at P, similar operations have to be performed for sections EF and GH.

The total planimeter reading will be

$$R = R_1\sin\beta + R_2\sin\alpha + R_3\sin\gamma$$

The terrain effect will then be given by substituting this value of R in equation (1).

The actual terrain in the area around P will determine how best to divide the area into segments and how many segments to employ. Care should be exercised to trace the boundary of any cross-section of additional mass in a clockwise direction and the cross-section of a void in an anticlockwise direction.



EXAMPLE OF CYCLOIDAL CURVES

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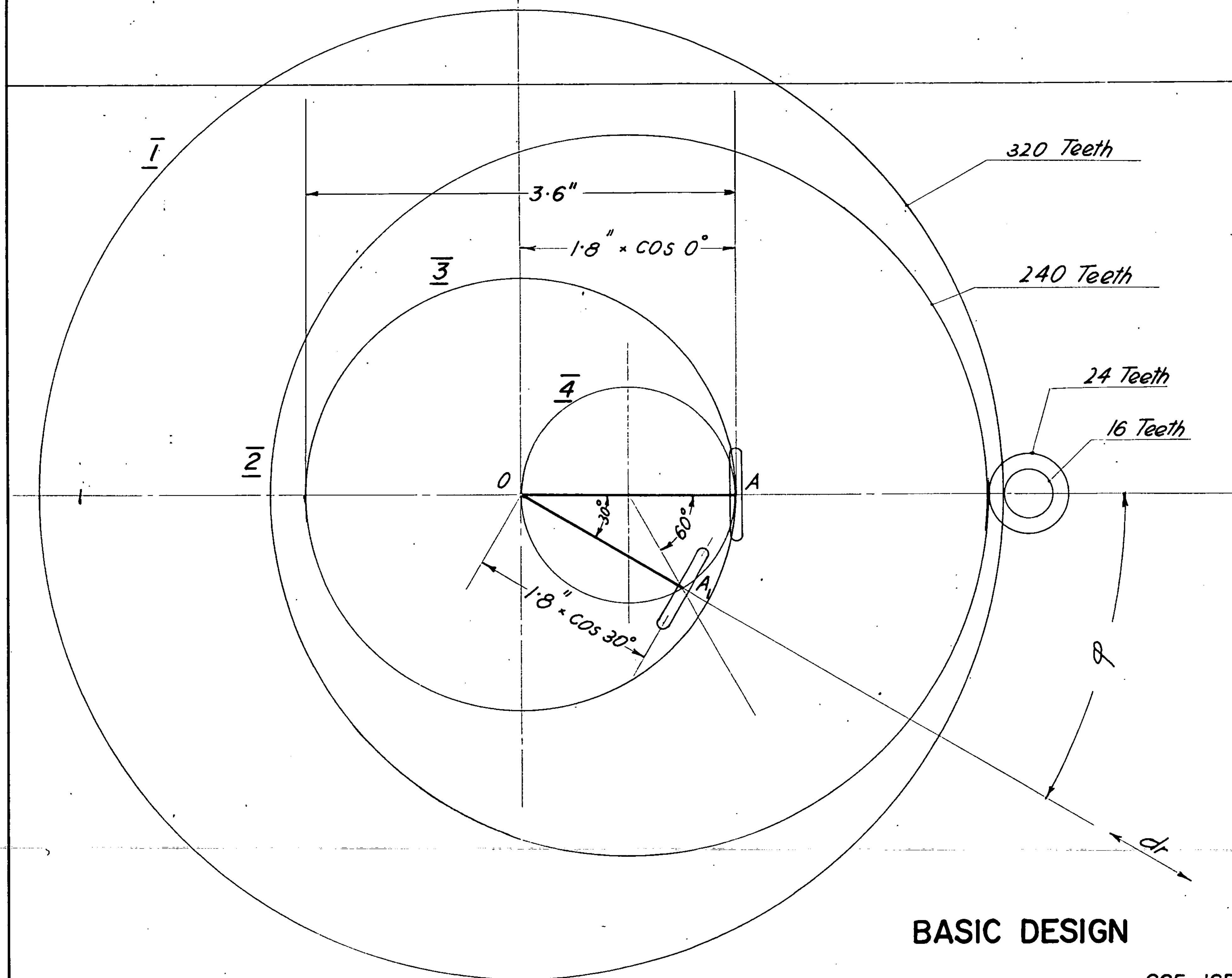
PLATE 2



BASIC DESIGN

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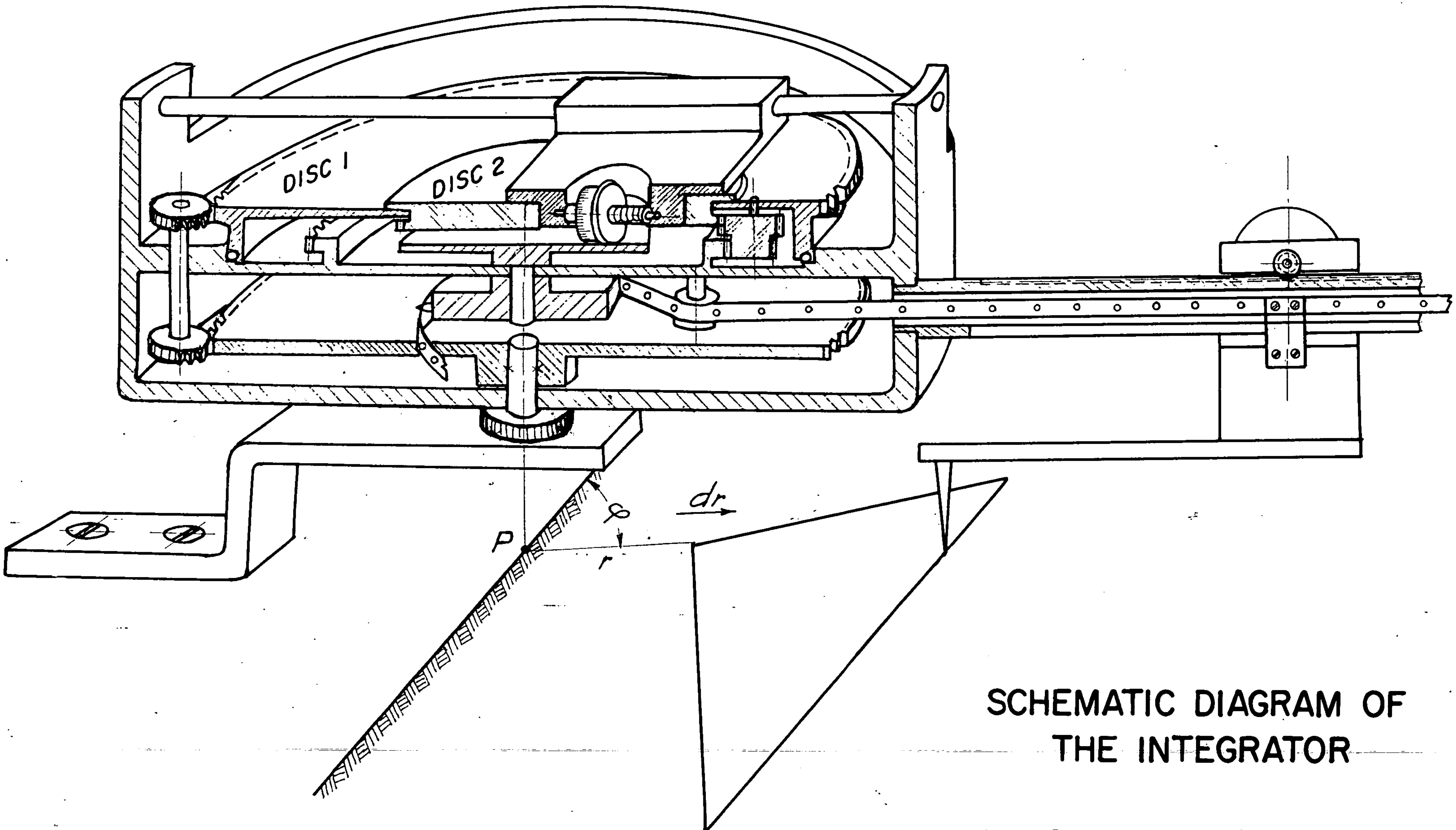

SCHEMATIC DIAGRAM OF THE INTEGRATOR

PLATE 3

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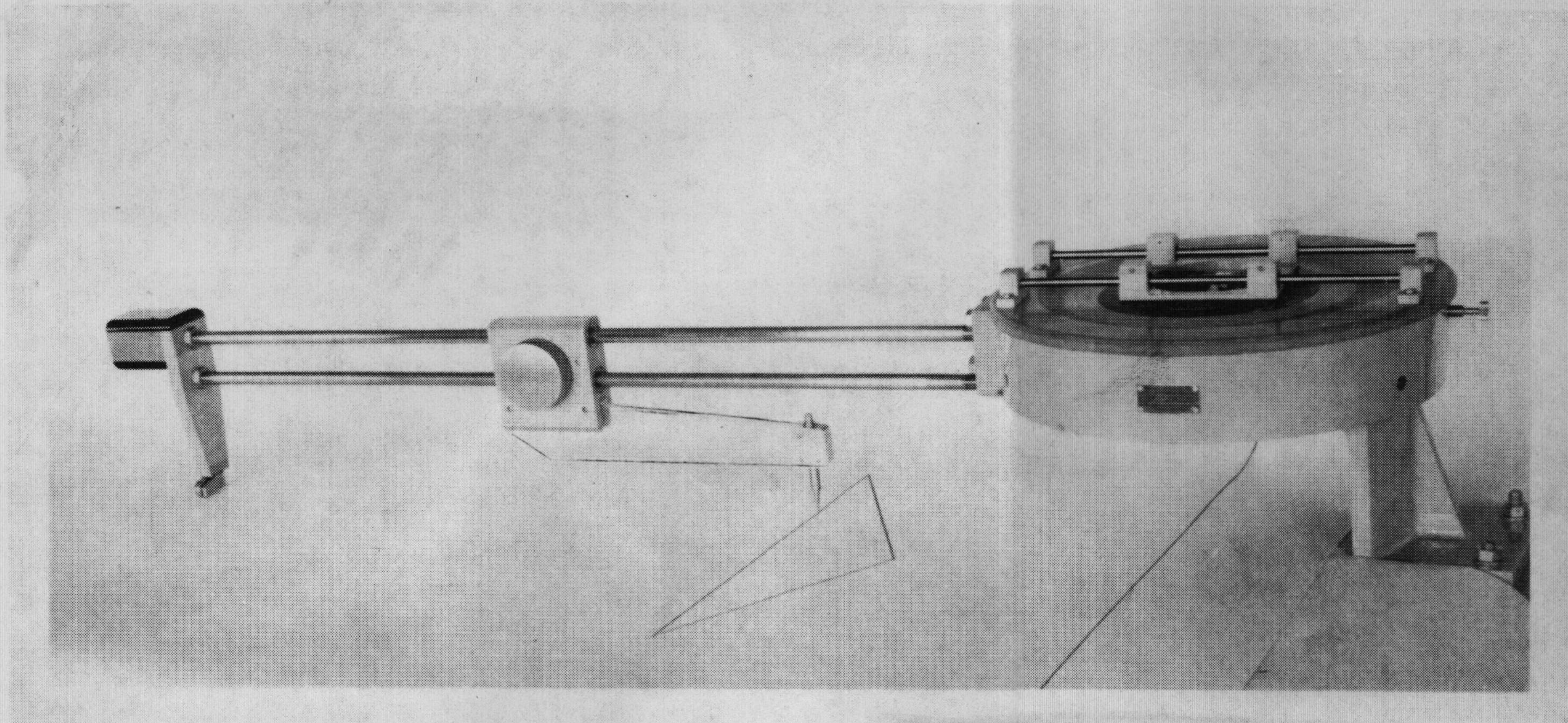

THE VERTICAL SECTION INTEGRATOR

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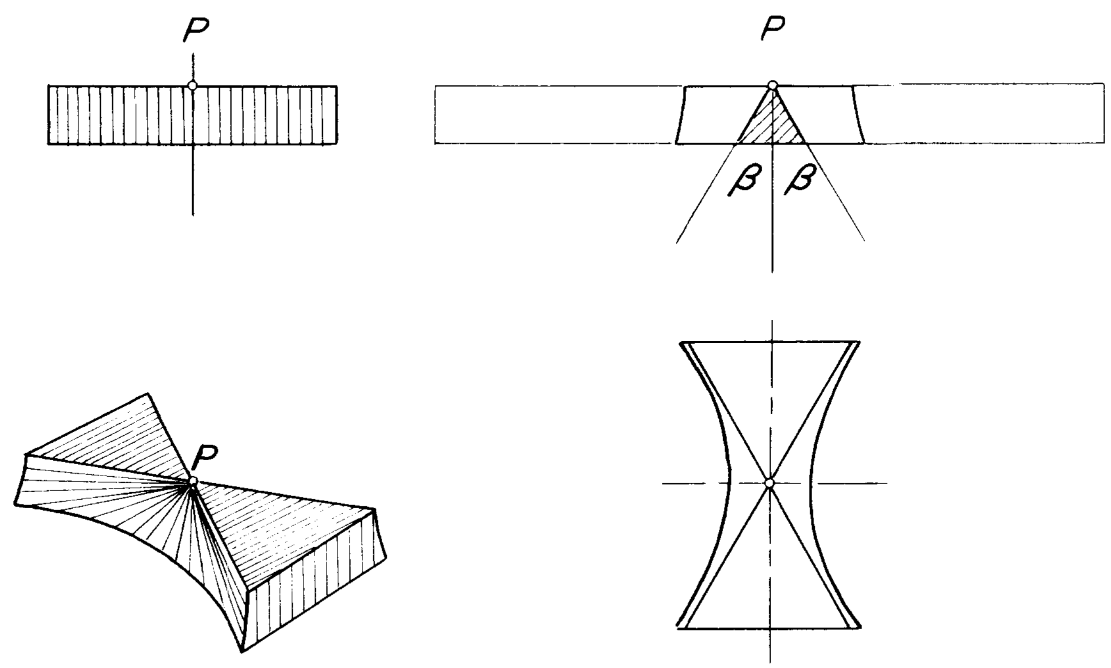
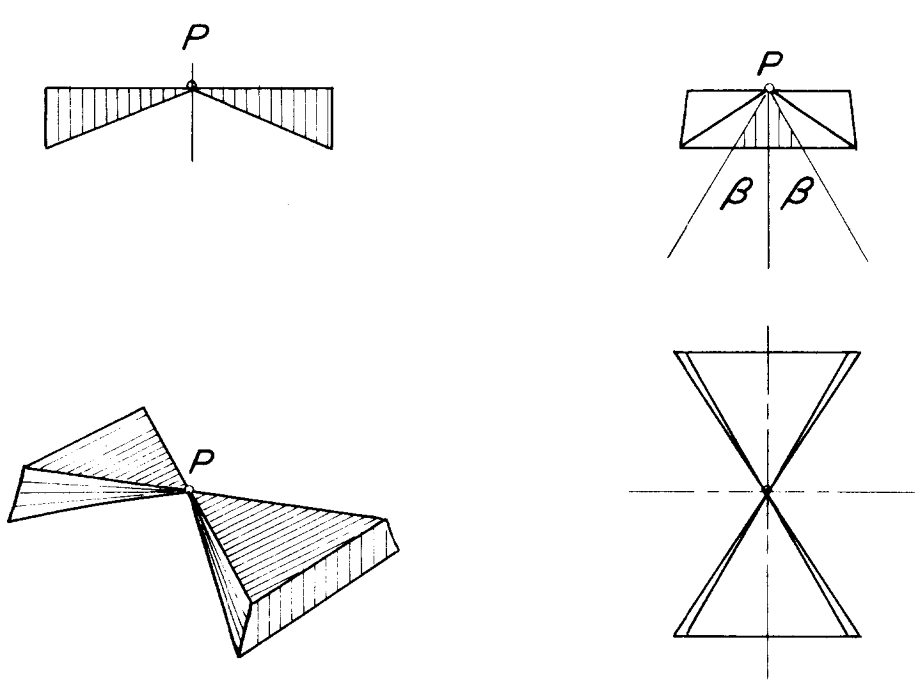
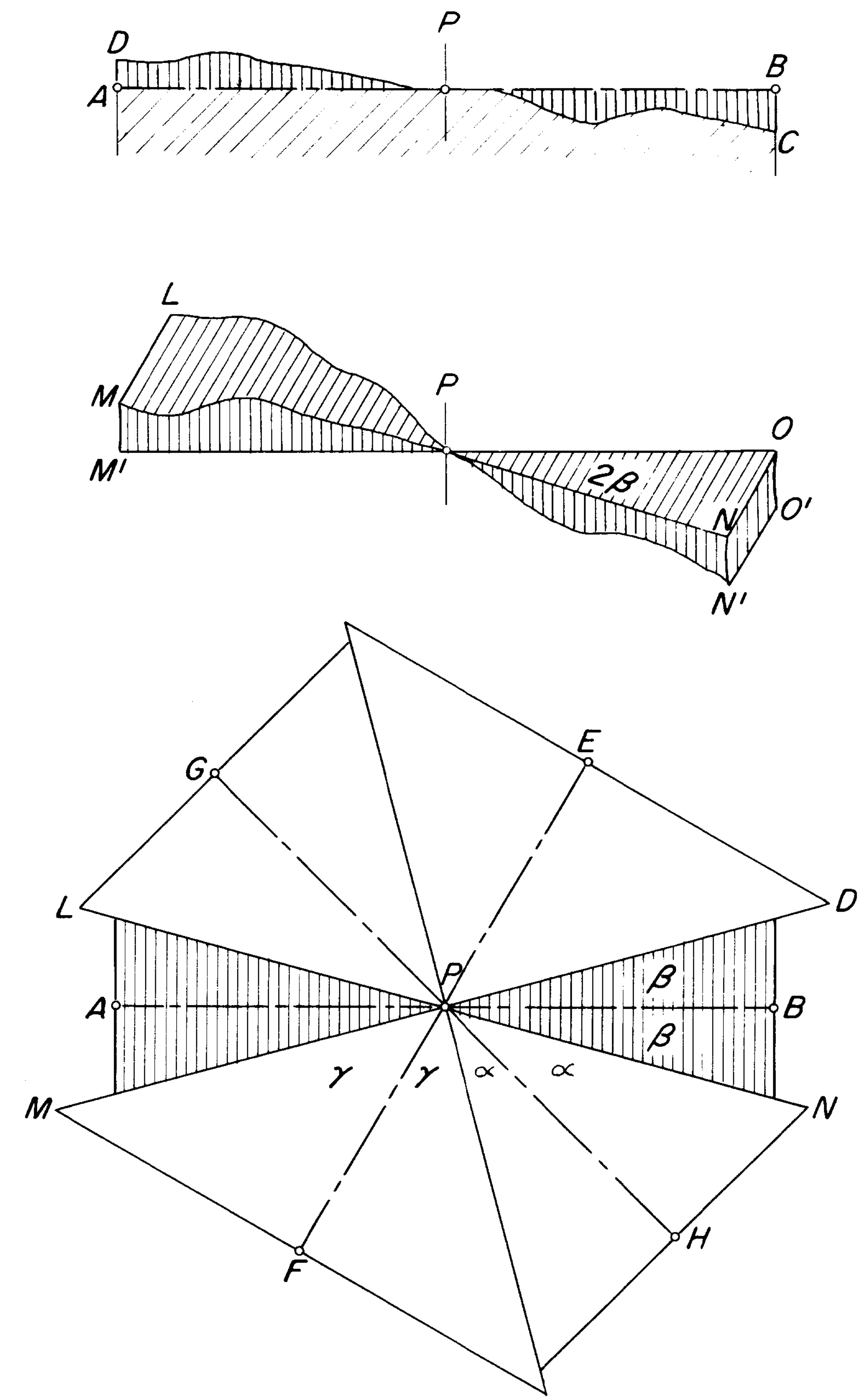
PLATE 4



(a)



(b)



(c)

APPLICATION TO TERRAIN CORRECTIONS