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RECORD No. 1966/109

PENDULUM GRAVITY TIES BETWEEN TOKYO AND MELBOURNE, 1962-1964

by

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SUMMARY

The results of a doubly read gravity tie between Melbourne (Australia) and Tokyo (Japan) are discussed. The measurements were made using the Bureau of Mineral Resources' GSI quartz pendulum apparatus.

The difference between the adjusted results of this tie and the figures obtained during earlier ties is about 2.5 milligals. Possible reasons for this difference are discussed and it would seem from work subsequently carried out in Australia that there are serious instrumental faults inherent in the equipment.

1. INTRODUCTION

This Record follows on from one written earlier by the author (Langron, 1963) describing a visit to Japan in 1962, during which the first of the Tokyo measurements used in this Record are discussed. Descriptions of the apparatus used are given in the earlier Record and will not be repeated here.

The author brought the pendulums and crystal clock back to Melbourne by air as hand luggage; the remainder of the apparatus followed by sea freight. Measurements in Melbourne were obtained during the period January to March 1963.

The swinging chamber was then sea-freighted back to Tokyo and the pendulums sent by safe hand of a Qantas aircraft captain, so that with these components and by use of their own associated electronics, members of the gravity division of the Geographical Survey Institute (GSI) carried out a repeat set of observations at Tokyo during July 1963.

The pendulums and swinging chamber were then returned to Melbourne in similar fashion, and the repeat set of measurements was obtained in Melbourne during January 1964.

These four sets of measurements constitute the data for the gravity tie, Melbourne to Tokyo, discussed in this Record.

The author wishes to acknowledge the cooperation of the Director and Staff (in particular Y. Fuzii) of the GSI in obtaining the data at Tokyo, and W. Burch, T. Lai, and J. Shirley for assistance during the measurements at Melbourne.

2. INSTRUMENTAL ASPECTS

The report cited earlier also gives the results of a short field test between international gravity stations at Tokyo and Hakone (Japan). The measurements used in the calculation of this gravity interval were carried out during September and October 1962. The significant results of this field test were :

(a) The value for the gravity interval as measured by the pendulum apparatus compared favourably with that obtained from numerous gravity meter connections.

(b) The equipment could be considered to have performed satisfactorily during the period of the tests.

(c) Readings at Hakone indicated that pendulum No. 2 had a tendency to 'drift', although the sets of readings at Tokyo did not give any clear indication of any such tendency. There was no evidence of 'tares' in the pendulums during the course of the measurements.

Work with the apparatus since that time has, however, brought to light several instrumental defects, some of which are considered to be very serious. For example the drift of pendulum No. 2 is on some occasions quite erratic and 'tares' of up to 2 to 3 milligals appear in the results. More detailed investigations of these and allied matters are discussed in reports dealing with pendulum measurements along the chain of stations along the east coast of Australia (Shirley, in preparation-a), and laboratory tests and modifications carried out on the pendulum equipment (Shirley, in preparation-b).

Only those points pertinent to the present report will be considered here. Briefly these fall into two categories, viz.

(a) damage to the equipment in transit, and

(b) faults inherent in the equipment.

With reference to (a), considerable damage was done to the parts of the equipment that were shipped by sea freight. The only item that did not appear to have been damaged was the swinging chamber. Before the first set of measurements at Melbourne could be undertaken several units of the equipment had to be rebuilt, the principal faults being weak chassis and poor soldering joints.

Inspection with a 10X magnification eye-glass at intervals during the course of the measurements revealed no damage to the knife edges of the pendulums or to the agate plate of the swinging chamber, although during the second trip to Melbourne the package containing the pendulums was seen to be fairly roughly handled at Melbourne airport. Pendulum No. 2 contains several flaws and minor cracks and generally is poorer physically than pendulum No. 3.

In connection with (b), reference should be made to the graph of the pendulum periods (Plate 3). It is clear that pendulum No. 2 is prone to instability. In general, pendulum No. 3 is stable and one would feel satisfied with the set of results obtained with it. However, one of the peculiarities that has come to light during further work is that even though the individual sets of measurements appear to be consistent within themselves (and this is the basis upon which a set of readings had been judged to be satisfactory before dismantling the apparatus at a particular site), the 'between sets' errors may be considerable. For example, a peculiarity on occasions is that a change in 'level' of the graphed periods has been effected simply by removing, recleaning, and replacing the pendulums, even though for the initial swing the pendulums were thoroughly washed and inspected before placing them in position. Some warping of the agate plate or damage to the knife edge could be suspected, but the plunger arrangement in the swinging chamber ensures that for each swing the knife edge is taken to the same position on the agate plate. This and similar matters are discussed at length by Shirley (in preparation-b).

For the measurements in Japan the crystal was checked against time signals from the Japanese time service JJY. In Melbourne it was impossible to record time signals from any of the overseas time services even though special filters and aerials were used and though most of the measurements were made at night when radio reception improved. For the measurements in Melbourne no suitable local time service was available whereby permanent records for time check purposes could be obtained; the checks were made using a cathode ray oscillograph (CRO) to compare the 1-kc/s output from the divider unit with a 1-kc/s signal transmitted to the National Gravity Base Station (N.G.B.S.) per telephone line by the Postmaster General's Department (P.M.G.) Melbourne. The P.M.G. service was stated to be stable to better than 1 part in 10^9 on transmission. It was assumed that if the Lissajous figure obtained on the CRO did not make more than one complete 'flop' in less than about 26 hours, then the crystal setting was correct and the required accuracy (5 parts in 10^7) was obtained.

Since these measurements were made, time services provided by the Weapons Research Establishment, Salisbury, South Australia, and, later, a time signal broadcast provided by the P.M.G. have been used for pendulum measurements in Australia.

TABLE 1

Summary of observational data

Station	Pendulum No. 3-1				Pendulum No. 2-1			
	Number of pairs of observations	Period (seconds)	Standard Deviation (10^{-8}second)	Mean error (10^{-8}second)	Number of pairs of observations	Period (seconds)	Standard Deviation (10^{-8}second)	Mean error (10^{-8}second)
Tokyo A1	7	1.00556477	23	9	7	1.00555925	56	22
Tokyo A2	7	56419	26	10	7	55852	60	23
Melbourne A	12	45965	40	12	12	45384	40	12
Tokyo B	16	56405	80	20	12	55912	127	37
Melbourne B	17	45918	27	7	17	45620	194	49

Note : All period values have been corrected to a temperature of 34°C.

Mean error = Standard deviation/$\sqrt{}$(total number of pairs of readings).

The crystal clock behaved satisfactorily during the course of the measurements and there is no reason to suspect that significant errors due to timing are present in the records. Intermittent trouble was experienced with the recorder unit, but the divider unit was by far the most troublesome portion of the equipment, and its performance had to be checked frequently. Some trouble was found in maintaining a high vacuum, generally in connection with joins where clips were used to fasten rubber hoses to metal pipes. It is proposed to shorten all hoses and to replace some of the hose with metal pipes; it is also proposed to replace the present method of vacuum measurement, which is inconvenient and dangerous (a Geissler tube and a high tension supply is used), by a conventional vacuum gauge.

N.G.B.S. Melbourne is not an ideal site for a pendulum station. Radio reception of overseas time services is poor, mainly because of heavy electrical equipment in the surrounding factories and the passage of tram-cars along Gordon Street (see Plate 2). For similar reasons, ground vibration at times is also very heavy. The GSI base, although situated in the basement of a laboratory building in which electrical equipment is in use, does not appear to suffer from these shortcomings. It is very close to the Japanese International time service JJY, and the signal strength is much greater than that of any radio time service that can be received at Footscray.

3. REDUCTION OF DATA

Details of the periods obtained for individual measurements at Melbourne and Tokyo are included in Appendix A. It is to be noted that in the calculation of the standard deviation, the two values of period for each record have been averaged before proceeding with the reduction of the data. Inoue and Seto (1961) have treated the two values of period obtained from each record as being independent, whereas in fact this is not so. This makes a small but significant difference in the final values obtained using the two treatments. Their convention for mean error (M.E.) has been modified in a similar way.

For convenience, the results to this stage are summarised in Table 1. The table shows that the results with pendulum No. 3-1 are in general of better quality than those of pendulum No. 2-1. There are other features which are not brought out by this summary, but the plot of the periods in Plate 3 suggests that some of the values would appear to be incorrect and could therefore be neglected. For example, in the set of periods for pendulum 2-1 at Melbourne B, it would be tempting to consider only the first nine pairs as forming a fairly consistent set of measurements. As the measurements proceeded with this particular set-up, this pendulum tended to become more unstable.

Also it is questionable whether any of the measurements for Tokyo B should be included in the calculation. However, the values shown on the plot have all been obtained under acceptable physical conditions (of temperature, pressure, level, vibration, etc) and no abnormal features were noted during the course of the measurements. All values meeting these requirements are shown and have been included in the calculation.

The data have been reduced using a least-squares adjustment process similar to that presented by Inoue and Seto (op cit). The calculation is shown in Appendix B. With this method, the following value for the gravity interval between Melbourne and Tokyo ($g_M - g_T$) is obtained :

$$g_M - g_T = 205.3 \pm 0.9 \text{ milligals}$$

This can be compared with the results obtained by other workers as shown in Table 2.

Because the value obtained during the present work differed so markedly from the values shown in Table 2, it was decided to investigate the results from each pendulum independently. This course seemed an obvious one because of the generally superior results of pendulum No. 3-1 and the suspicion that pendulum No. 2-1 might have been giving spurious results during the repeat readings at Tokyo and Melbourne. It might be argued that the results from the pendulums cannot be treated independently, but it is felt that because the vibration of the centre pendulum is always kepth within the prescribed limits the above analysis is warranted. Details of this analysis are included in Appendix C. The resultant values for the interval ($g_M - g_T$) using this treatment cannot be accepted as a significant improvement on the value obtained by the least-squares adjustment above.

Some analysis has also been carried out on results from particular swings in an attempt to improve the estimated value of the interval. This has been done only in cases where all physical requirements for the measurement appear to be satisfactory and yet the value obtained for the period is significantly different from its expected value based on other values for the particular set-up and on an assumed value for the gravity interval based on Table 2. This investigation did not lead to any useful conclusions.

TABLE 2

Summary of gravity ties between Melbourne and Tokyo

Source	Interval $g_M g_T$ (mgals)	Instrument
Inoue & Seto (1961)	202.1 $\pm$ 0.2	GSI pendulum
*USAMSFE (pers. comm.)	202.2	La Coste G23 gravity meter.
*Woollard & Rose (1963)	201.7	La Coste G1 gravity meter.
*Woollard & Rose (1963)	202.6	Worden 10F gravity meter
*Woollard & Rose (1963)	202.4	Worden 147 gravity meter.
**Williams, Goodspeed, & Flavelle (1961)	202.1	Worden 140 gravity meter.
**Williams, Goodspeed, & Flavelle (1961)	202.0	Worden 169 gravity meter.

* This value includes the BMR value for the interval between BMR pendulum stations at Melbourne (N.G.B.S., P.S.1) and Sydney (P.S.5).

** This value includes the BMR value for the interval between BMR pendulum stations at Melbourne (N.G.B.S., P.S.1) and Sydney (P.S.5). The values for the intervals P.S.5 to Hakone as given by Williams et al have been adjusted using revised instrument calibration factors. The tie to Tokyo (GSI) has been calculated using data published by Woollard and Rose (1963) and information supplied by the University of Hawii (personal communication).

4. DISCUSSION OF RESULTS

The results of the present work are significantly different from those of the other surveys listed in Table 2. Several types of analysis which are considered to be reasonable have been applied to the present data without greatly changing the result.

To provide additional data for check purposes, the equipment was used over part of the Western Pacific Calibration Line, between Melbourne and Cairns. The result of this work is discussed by Shirley (in preparation-a) and clearly indicates that there are serious instrumental faults affecting the measurements.

As all physical requirements (temperature, pressure, vibration, timing, etc.) have been met in the measurements and as there did not appear to be any systematic error in the results it was decided to submit the knife edges and agate plate for optical examination. The tests were carried out by the Defence Standards Laboratories, Melbourne, and indicated quite clearly that there was some crumbling of the knife edges and considerable warping of the agate plate (up to 25 interference fringes using mercury green light). The knife edges of pendulums Nos. 2 and 3 both exhibit considerable asymmetry. The knife edge of pendulum No. 1 contains a distinct plateau with flattening producing a deficiency of approximately .0001 inch in the knife edge. It is felt that these defects must be held responsible for the poor results.

Accordingly, the matter of repairs has been taken up with Sokkisha Ltd, the manufacturers of the equipment. Details of the tests carried out on the equipment in Australia are given by Shirley (in preparation-b).

One puzzling feature about this work is that whereas the original test run over a known gravity interval between Tokyo and Hakone produced satisfactory results, the results since that time have been unsatisfactory. For the measurement of the first interval (Tokyo A2 - Melbourne A) it can be reasonably assumed that as the pendulums were carried personally by the author, they arrived in Melbourne in good condition. The plotted periods for Melbourne A (Plate 3) show a comparatively small scatter, whereas in fact there is a discrepancy of about $2\frac{1}{2}$ milligals between the gravity interval calculated from these data and the 'accepted' interval from Table 2. It is possible that the swinging chamber, which was transported by sea, suffered damage en route, although any such damage was not apparent on arrival.

It is much more likely that the pendulums (and the swinging chamber or both) could have suffered damage in subsequent trips between Melbourne and Tokyo.

On the basis of the present data the best adjusted value for the gravity interval between Melbourne and Tokyo is considered to be :

$$g_M - g_T = +205.3 \pm 0.9 \text{ milligals.}$$

However, for the reasons given, this value should not be regarded as a very reliable estimate for the true gravity interval.

5. REFERENCES

INOUE, E. and SETO, T. 1961 Pendulum determinations of the gravity difference between Tokyo and Melbourne. Bull. Geograph. Surv. Inst. Japan 6(4), 201-211.

LANGRON, W. J. 1963 Visit to Japan, 1962. Bur. Min. Resour. Aust. Rec. 1963/125.

SHIRLEY, J. E.	a	Pendulum gravity measurements in Australia, 1964. <u>Bur. Min. Resour. Aust. Rec</u>. (in preparation).
	b	Modifications and tests of the GSI Pendulum apparatus. <u>Bur. Min. Resour. Aust. Rec</u>. (in preparation).
TOPPING, J.	1962	ERRORS OF OBSERVATION AND THEIR TREATMENT. The Institute of Physics and the Physical Society, Monographs for Students.
WILLIAMS, L. W., GOODSPEED, M. J., and FLAVELLE, A. J.	1961	International gravity meter ties, 1959. <u>Bur. Min. Resour. Aust. Rec</u>. 1961/24.
WOOLLARD, G. P. and ROSE, J. C.	1963	INTERNATIONAL GRAVITY MEASUREMENTS. University of Wisconsin, Geophysical and Polar Polar Research Center.

APPENDIX A

Details of measurements at each station

In the following tables,

- T is the period in seconds.
- T_i is the average residual period in 10^{-8} second for each pair of readings.
- $\bar{T}$ is the mean residual period in 10^{-8} second.

Tokyo A1, Pendulum No. 2-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
T1A	13.9.62	1.00555846			
		878	862	- 63	3969
T2A	13.9.62	888			
		906	897	- 28	784
T3A	13.9.62	857			
		843	850	- 75	5625
T4A	13.9.62	940			
		943	942	+ 17	289
T5A	15.9.62	970			
		940	955	+ 30	900
T6A	15.9.62	978			
		985	982	+ 57	3249
T7A	17.9.62	977			
		996	987	+ 62	3844

$\Sigma T_i = 6475$

$\therefore \bar{T} = 925$

$\Sigma(T_i-\bar{T})^2 = 18660$

Standard deviation $= (18660/6)^{\frac{1}{2}} = 55.8 \times 10^{-8}$ second

Mean error $= 56/\sqrt{7} = 22 \times 10^{-8}$ second

$\therefore$ Period $= 1.00555925 \pm 22 \times 10^{-8}$ seconds

Tokyo A1, Pendulum No. 3-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
T1A	13.9.62	1.00556476			
		457	467	- 10	100
T2A	13.9.62	502			
		491	497	+ 20	400
T3A	13.9.62	445			
		421	433	- 44	1936
T4A	13.9.62	488			
		480	484	+ 7	49
T5A	15.9.62	508			
		466	487	+ 10	100
T6A	15.9.62	472			
		532	502	+ 25	625
T7A	17.9.62	497			
		445	471	- 6	36

$\sum T_i = 3341$

$\therefore \bar{T} = 477$

$\sum (T_i-\bar{T})^2 = 3246$

Standard deviation = $(3246/6)^{\frac{1}{2}} = 23.3 \times 10^{-8}$ second

Mean error = $23/\sqrt{7} = 9 \times 10^{-8}$ second

$\therefore$ Period = $1.00556477 \pm 9 \times 10^{-8}$ seconds.

Tokyo A2, Pendulum No. 2-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
T8A	5.10.62	1.00555921			
		868	895	+ 43	1849
T9A	6.10.62	811			
		754	783	- 69	4761
T10A	6.10.62	807			
		792	800	- 52	2704
T11A	6.10.62	811			
		819	815	- 37	1369
T12A	6.10.62	873			
		849	861	+ 9	81
T13A	8.10.62	855			
		959	957	+ 105	11025
T14A	8.10.62	837			
		865	851	- 1	1

$\sum T_i = 5962$

$\therefore \bar{T} = 852$

$\sum (T_i-\bar{T})^2 = 21790$

Standard deviation = $(21790/6)^{\frac{1}{2}} = 60.3 \times 10^{-8}$ second

Mean error = $60/\sqrt{7} = 23 \times 10^{-8}$ second

$\therefore$ Period = $1.00555852 \pm 23 \times 10^{-8}$ seconds

Tokyo A2, Pendulum No. 3-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
T8A	5.10.62	1.00556416			
		383	400	- 19	361
T9A	6.10.62	395			
		399	397	- 22	484
T10A	6.10.62	504			
		390	447	+ 28	784
T11A	6.10.62	391			
		390	391	- 28	784
T12A	6.10.62	415			
		403	409	- 10	100
T13A	8.10.62	383			
		529	456	+ 37	1369
T14A	8.10.62	416			
		447	432	+ 13	169

$\sum T_i = 2932$

$\therefore \bar{T} = 419$

$\sum (T_i-\bar{T})^2 = 4051$

Standard deviation = $(4051/6)^{\frac{1}{2}} = 26.0 \times 10^{-8}$ second

Mean error = $26/\sqrt{7} = 10 \times 10^{-8}$ second

.. Period = $1.00556419 \pm 10 \times 10^{-8}$ seconds.

Melbourne A, Pendulum No. 2-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
M1A	24.1.63	1.00545411			
		462	437	+ 53	2809
M5A	12.2.63	387			
		337	362	- 22	484
M7A	12.2.63	329			
		352	341	- 43	1849
M9A	12.2.63	399			
		319	359	- 25	625
M10A	13.2.63	469			
		397	433	+ 49	2401
M11A	13.2.63	403			
		327	365	- 19	361
M12A	13.2.63	396			
		445	421	+ 37	1369
M13A	13.2.63	417			
		447	432	+ 48	2304
M14A	13.2.63	401			
		440	421	+ 37	1369
M18A	8.3.63	371			
		329	350	- 34	1156
M19A	8.3.63	311			
		360	336	- 48	2304
M20A	8.3.63	386			
		319	353	- 31	961

$\Sigma T_i = 4610$

$\therefore \bar{T} = 384$

$\Sigma (T_i-\bar{T})^2 = 17992$

Standard deviation $= (17992/11)^{\frac{1}{2}} = 40.4 \times 10^{-8}$ second

Mean error $= 40/\sqrt{12} = 12 \times 10^{-8}$ second

$\therefore$ Period $= 1.00545384 \pm 12 \times 10^{-8}$ seconds

Melbourne A, Pendulum No. 3-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
M1A	24.1.63	1.00546018			
		6064	6041	+ 76	5776
M5A	12.2.63	5937			
		5972	5955	- 10	100
M7A	12.2.63	5960			
		5995	5978	+ 13	169
M9A	12.2.63	6021			
		5974	5998	+ 33	1089
M10A	13.2.63	5998			
		5959	5979	+ 14	196
M11A	13.2.63	5993			
		5864	5929	- 36	1296
M12A	13.2.63	6006			
		6014	6010	+ 45	2025
M13A	13.2.63	5969			
		5897	5933	- 32	1024
M14A	13.2.63	5940			
		5962	5951	- 14	196
M18A	8.3.63	5972			
		5866	5919	- 46	2116
M19A	8.3.63	5895			
		5919	5907	- 58	3364
M20A	8.3.63	6015			
		5942	5979	+ 14	196

$\sum T_i = 71529$

$\therefore \bar{T} = 5965$

$\sum (T_i - \bar{T})^2 = 17547$

Standard deviation $= (17547/11)^{\frac{1}{2}} = 39.9 \times 10^{-8}$ second

Mean error $= 40/\sqrt{12} = 12 \times 10^{-8}$ second

$\therefore$ Period $= 1.00545965 \pm 12 \times 10^{-8}$ seconds

Tokyo B, Pendulum No. 2-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
T2B	12.7.63	1.00555856 851	5854	- 58	3364
T3B	13.7.63	875 887	881	- 31	961
T4B	15.7.63	831 811	821	- 91	8281
T5B	15.7.63	836 876	856	- 56	3136
T6B	15.7.63	6117 6008	6063	+ 151	22801
T7B	15.7.63	- -			
T8B	15.7.63	- -			
T9B	16.7.63	- -			
T10B	16.7.63	- -			
T11B	16.7.63	- 6056	(6056)	+ (144)	(20736)
T12B	17.7.63	6102 5978	6040	+ 128	16384
T13B	17.7.63	6108 6141	6125	+ 213	45369
T14B	19.7.63	- 6080	(6080)	+ (168)	(28224)
T18B	19.7.63	5758 769	5764	- 148	21904
T19B	19.7.63	795 732	764	- 148	21904
T20B	19.7.63	808 838	823	- 89	7921
T21B	19.7.63	875 887	881	- 31	961

$\sum T_i$ = 10940 (the average of the two values in brackets has been included)

$\therefore \bar{T}$ = 912

$\sum (T_i - T)^2$ = 177468 (the average of the two values in brackets has been included)

Standard deviation = $(177468/11)^{\frac{1}{2}}$ = 127.0 x 10^{-8} second

Mean error = $127/\sqrt{12}$ = 37 x 10^{-8} second

$\therefore$ Period = 1.0055912 $\pm$ 37 x 10^{-8} seconds

Tokyo B, Pendulum No. 3-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
T2B	12.7.63	1.00556375 453	414	+ 9	81
T3B	13.7.63	339 360	350	- 55	3025
T4B	15.7.63	319 325	322	- 83	6889
T5B	15.7.63	190 314	252	- 153	23409
T6B	15.7.63	454 384	419	+ 14	196
T7B	15.7.63	412 392	402	- 3	9
T8B	15.7.63	413 281	347	- 58	3364
T9B	16.7.63	449 457	453	+ 48	2304
T10B	16.7.63	501 524	513	+ 108	11664
T11B	16.7.63	- 527	(527)	+ (122)	(14884)
T12B	17.7.63	472 488	480	+ 75	5625
T13B	17.7.63	490 513	502	+ 97	9409
T14B	17.7.63	- 533	(533)	+ (128)	(16384)
T18B	19.7.63	313 285	299	- 106	11236
T19B	19.7.63	371 344	358	- 47	2209
T20B	19.7.63	380 428	404	- 1	1
T21B	19.7.63	462 408	435	+ 30	900

$\sum T_i$ = 6480 (the average of the two values in brackets has been included)

$\therefore \bar{T}$ = 405

$\sum (T_i-\bar{T})^2$ = 95955 (the average of the two values in brackets has been included)

Standard deviation = $(95955/15)^{\frac{1}{2}} = 80.0 \times 10^{-8}$ second

Mean error = $80/\sqrt{16}$ = 20×10^{-8} second

$\therefore$ Period = $1.00556405 \pm 20 \times 10^{-8}$ seconds

Melbourne B, Pendulum No. 3-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
M1B	20.1.64	1.00545910			
		874	892	- 26	676
M2B	20.1.64	919			
		969	944	+ 26	676
M3B	21.1.64	945			
		935	940	+ 22	484
M4B	21.1.64	883			
		898	891	- 27	729
M5B	21.1.64	952			
		874	913	- 5	25
M6B	21.1.64	912			
		893	903	- 15	225
M7B	21.1.64	878			
		936	907	- 11	121
M8B	21.1.64	882			
		899	891	- 27	729
M9B	21.1.64	959			
		880	920	+ 2	4
M10B	21.1.64	925			
		848	887	- 31	961
M11B	21.1.64	941			
		959	950	+ 32	1024
M12B	21.1.64	907			
		908	908	- 10	100
M13B	22.1.64	916			
		-	(916)	- (2)	(4)
M14B	22.1.64	932			
		905	919	+ 1	1
M15B	22.1.64	900			
		984	942	+ 24	576
M16B	22.1.64	874			
		905	890	- 28	784
M17B	22.1.64	6007			
		5968	988	+ 70	4900

$\Sigma T_i = 15601$

$\therefore \bar{T} = 918$

$\Sigma(T_i-\bar{T})^2 = 12019$

Standard deviation = $(12019/16)^{\frac{1}{2}} = 27.4 \times 10^{-8}$ second

Mean error = $27/\sqrt{16} = 7 \times 10^{-8}$ second

$\therefore$ Period = $1.00545918 \pm 7 \times 10^{-8}$ seconds

Melbourne B, Pendulum No. 2-1

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
M1B	20.1.64	1.00545327 335	5331	- 289	83521
M2B	20.1.64	374 453	414	- 206	42436
M3B	21.1.64	444 497	471	- 149	22201
M4B	21.1.64	420 396	408	- 202	40804
M5B	21.1.64	486 446	446	- 154	23716
M6B	21.1.64	493 498	496	- 124	15376
M7B	21.1.64	480 508	494	- 126	15876
M8B	21.1.64	501 540	521	- 99	9801
M9B	21.1.64	540 566	553	- 67	4489
M10B	21.1.64	691 603	647	+ 33	1089
M11B	21.1.64	714 710	712	+ 92	8464
M12B	21.1.64	745 725	735	+ 115	13225
M13B	22.1.64	737 -	(737)	(117)	(13689)
M14B	22.1.64	810 756	783	+ 163	26569
M15B	22.1.64	952 966	959	+ 339	114921
M16B	22.1.64	912 899	906	+ 286	81796
M17B	22.1.64	949 874	912	+ 292	85264

$\sum T_i = 10545$

$\therefore \bar{T} = 620$

$\sum (T_i-\bar{T})^2 = 603237$

Standard deviation $= (603237/16)^{\frac{1}{2}} = 194.2 \times 10^{-8}$ second

Mean error $= 194/\sqrt{16} = 49 \times 10^{-8}$ second

$\therefore$ Period $= 1.00545620 \pm 49 \times 10^{-8}$ seconds

Melbourne B, Pendulum No. 2-1 (Alternative treatment of data)

Record	Date	T	T_i	$(T_i-\bar{T})$	$(T_i-\bar{T})^2$
M1B	20.1.64	1.00545327			
		335	331	- 149	22201
M2B	20.1.64	374			
		453	414	- 66	4356
M3B	21.1.64	444			
		497	471	- 9	81
M4B	21.1.64	420			
		396	408	- 72	5184
M5B	21.1.64	486			
		446	466	- 14	196
M6B	21.1.64	493			
		498	496	+ 16	256
M7B	21.1.64	480			
		508	494	+ 14	196
M8B	21.1.64	501			
		540	521	+ 41	1681
M9B	21.1.64	540			
		566	553	+ 73	5329
M10B	21.1.64	691			
		603	647	+ 167	27889

$\Sigma T_i = 4801$

$\therefore \bar{T} = 480$

$\Sigma (T_i-\bar{T})^2 = 67369$

Standard deviation $= (67369/9)^{\frac{1}{2}} = 86.5 \times 10^{-8}$ second

Mean error $= 87/\sqrt{10} = 27 \times 10^{-8}$ second

$\therefore$ Period $= 1.00545480 \pm 27 \times 10^{-8}$ seconds.

APPENDIX B

Treatment of the data using the least-squares method of adjustment

The following determinations of the most probable values of the periods and their standard errors follow the methods given by Topping (1962, pp 96-112).

The analysis follows from the summary of results in Table 1 (page 3). It is to be noted that the numbers shown in the "Number of pairs of observations" column include records where only one set of readings is contained on the record.

If we denote the period at Melbourne by T_M

and the period at Tokyo by T_T

we have from Table 1:

For Pendulum 3-1,

Mean T_M = 1.005459 42 seconds

Mean T_T = 1.005564 34 seconds

$T_M - T_T$ = -.000104 92 second

For Pendulum 2-1,

Mean T_M = 1.005455 02 seconds

Mean T_T = 1.005558 96 seconds

$T_M - T_T$ = -.000103 94 second

Taking these (approximate) values for T_T (3) and T_T (2) (i.e. for pendulums 3-1 and 2-1 respectively) and for $(T_M - T_T)$ and denoting the residuals that must be added to these values to bring them to their observed values as X, Y, Z respectively, we have the following conditional equations :

$$T_T\,(3) = 1.005564\ 34 + X\times 10^{-8}$$

$$T_T\,(2) = 1.005558\ 96 + Y\times 10^{-8}$$

$$T_M - T_T = -0.000104\ 43 + Z\times 10^{-8}$$

The residuals may be expressed in the form :

$$aX + bY + cZ = k,$$

where the values of a, b, c, and k can be found by comparing the above conditional equations with the data in Table 1. Weights are proportional to the reciprocals of the squares of the mean errors. The following observational equations are obtained :

X		$= +43\times 10^{-8}$	(weight 14)
X		$= -15\times 10^{-8}$	(10)
X	+Z	$= -26\times 10^{-8}$	(8)
X		$= -29\times 10^{-8}$	(3)
X	+Z	$= -73\times 10^{-8}$	(20)

Y		$= +29\times10^{-8}$	(2)
Y		$= -44\times10^{-8}$	(2)
Y	+Z	$= -69\times10^{-8}$	(8)
Y		$= +16\times10^{-8}$	(1)
Y	+Z	$= +167\times10^{-8}$	(0)

i.e.

a	b	c	k	w
1	0	0	+43	14
1	0	0	-15	10
1	0	1	-26	8
1	0	0	-29	3
1	0	1	-73	20
0	1	0	+29	2
0	1	0	-44	2
0	1	1	-69	8
0	1	0	+16	1
-0	1	1	+167	0

now:

waa	wab	wac	wak	wbb	wbc	wbk	wcc	wck
14	-	0	602	0	0	0	0	0
10	-	0	-150	0	0	0	0	0
8	-	8	-208	0	0	0	8	-208
3	-	0	-87	0	0	0	0	0
20	-	20	-1460	0	0	0	20	-1460
0	-	0	0	2	0	58	0	0
0	-	0	0	2	0	-88	0	0
0	-	0	0	8	8	-552	8	-552
0	-	0	0	1	0	16	0	0
0	-	0	0	0	0	0	0	0
55	0	28	-1303	13	8	-566	36	-2220

So the normal equations are :

$$\begin{aligned} 55X \quad\quad\quad &+28Z = -1,303 \\ 13Y &+ 8Z = -\ 566 \\ 28X + 8Y &+36Z = -2,220 \end{aligned}$$

$$\Delta = \begin{vmatrix} 55 & 0 & 28 \\ 0 & 13 & 8 \\ 28 & 8 & 36 \end{vmatrix} = 12,028$$

$$X = \frac{1}{\Delta}\begin{vmatrix} -1,303 & 0 & 28 \\ -\ 566 & 13 & 8 \\ -2,220 & 8 & 36 \end{vmatrix} = \frac{154,884}{12,028} = 12.877$$

$$Y = \frac{1}{\Delta}\begin{vmatrix} 55 & -1,303 & 28 \\ 0 & -\ 566 & 8 \\ 28 & -2,220 & 36 \end{vmatrix} = \frac{7,992}{12,028} = 0.664$$

$$Z = \frac{1}{\Delta}\begin{vmatrix} 55 & 0 & -1,303 \\ 0 & 13 & -\ 566 \\ 28 & 8 & -2,220 \end{vmatrix} = \frac{-863968}{12,028} = -71.830$$

Thus the most probable values of X, Y, and Z are

$$X = 12.877$$
$$Y = 0.664$$
$$Z = -71.830$$

Determination of the standard errors of the periods

If we take the most probable values of X, Y, and Z to the nearest whole number, and let $x = 13$, $y = 1$, and $z = 72$, the 'errors' in the values of x, y, and z can be expressed in the form :

$$ax + by + cz - k = e$$

ax	by	cz	-k	e	e^2	w	we^2
13	0	0	-43	-30	900	14	12600
13	0	0	+15	+28	784	10	7840
13		-72	+26	-33	1089	8	8712
13			+29	+42	1764	3	5292
13		-72	+73	+14	196	20	3920
0	1		-29	-28	784	2	1568
0	1		+44	+45	2025	2	4050
0	1	-72	+69	-2	4	8	32
0	1		-16	-15	225	1	225
0	1	-72	-167	-238	56,644	0	0
							=44,239

$$Ax = \begin{vmatrix} 13 & 8 \\ 8 & 36 \end{vmatrix} = 404$$

$$Ay = \begin{vmatrix} 55 & 28 \\ 28 & 36 \end{vmatrix} = 1196$$

$$Az = \begin{vmatrix} 55 & 0 \\ 0 & 13 \end{vmatrix} = 715$$

The standard error of x is

$$\alpha_x = \left(\frac{Ax\ we^2}{\Delta(n-3)}\right)^{\frac{1}{2}} = \left(\frac{404 x 44,239}{12,028(10-3)}\right)^{\frac{1}{2}}$$

$$= 14.57 \approx 15$$

Similarly, $\alpha_y \approx 25$ and $\alpha_z \approx 20$

Therefore the adjusted values become :

$T_T(3) = 1.00556447 \pm 15 \times 10^{-8}$ seconds

$T_T(2) = 1.00555897 \pm 25 \times 10^{-8}$ seconds

$T_M - T_T = -0.00010515 \pm 20 \times 10^{-8}$ second

Determination of the gravity interval

$$g_M - g_T = \frac{-2g_T}{T_T}(T_M - T_T) + \frac{3g_T}{T_T^2}(T_M - T_T)^2 \qquad \text{.........(1)}$$

Take $T_T = 1.00556 \pm .00001$ seconds (i.e. $dT_T = .00001$)

and $g_T = 979.9770$ gal

$$\frac{2g_T}{T_T} = \frac{2x979.9770}{1.00556} = 1,948.72$$

$$|\text{Error}| = \frac{2g_T}{T_T^2}(dT_T) \leq .02$$

$$\therefore \frac{2g_T}{T_T} = 1,948.72 \pm .03$$

$$\frac{3g_T}{T_T^2} = \frac{2,923.0786}{(1.00556)^2} = 2,906.92$$

$$|\text{Error}| = \frac{6g_T}{T_T^3}(dT_T) \leq .06$$

$$\therefore \frac{3g_T}{T_T^2} = 2,906.92 \pm .07$$

Substituting these values in equation (1) we have :

$g_M - g_T = (+205.29 \pm E)$ milligals

$$\begin{aligned}|E| = {} & (+1,948.72) \text{ x } 20\times10^{-8} \\ & +(.03) \text{ x } (0.00010515) \\ & +(2,906.92) \text{ x } 2 \text{ x } (0.00010515) \text{ x } 20 \text{ x } 10^{-8} \\ & +(0.00010515)^2 \text{ x } (0.07)\end{aligned}$$

$\leq$ .0008 gal

$\therefore$ $\underline{g_M - g_T = + 205.3 \pm 0.9 \text{ mgal}}$

APPENDIX C

Treatment of the data for each pendulum independently

σ = standard deviation

n = number of pairs of observations

n/σ^2 = weight

Tokyo

Pendulum No. 2-1

Station	Date	n	T (s)	σ (10^{-8} s)	Weight
A1	Sept. '62	7	1.005 559 25	57	.00216
A1	Oct. '62	7	558 52	60	.00194
B	July '63	12	559 12	127	.00074
		26			.00484

$\bar{T}$ (weighted) = 1.005 559 07 seconds
σ (weighted) = 66 x 10^{-8} second

Pendulum No. 3-1

A1	Sept. '62	7	1.005 564 77	23	.01323
A2	Oct. '62	7	564 19	26	.01036
B	July '63	16	564 05	80	.00250
		30			.02609

$\bar{T}$ (weighted) = 1.005 564 50 seconds
σ (weighted) = 28 x 10^{-8} second

Melbourne

Pendulum No. 2-1

A	Jan-Mar.'63	12	1.005 453 84	40	.00750
B	Jan. '64	17	456 20	194	.00452
		29			.01202

$\bar{T}$ (weighted) = 1.005 454 70 seconds
σ (weighted) = 97 x 10^{-8} second

Pendulum No. 3-1

A	Jan-Mar.'63	12	1.005 459 65	40	.00750
B	Jan. '64	17	459 18	27	.02332
		29			.03082

$\bar{T}$ (weighted) = 1.005 459 30 seconds
σ (weighted) = 30 x 10^{-8} second

Period differences

Pendulum No. 2-1

$T_M - T_T$ = -0.000 104 37 second; σ = 118 x 10^{-8} second; n = 55

Pendulum No. 3-1

$T_M - T_T$ = -0.000 105 20 second; σ = 42 x 10^{-8} second; n = 59

Calculation of gravity interval

Pendulum No. 2-1

$$g_M = 979{,}979.0 \text{ milligals}$$

$$T_M = 1.005\ 454\ 70 \text{ seconds}$$

$$g_M - g_T = \frac{-2g_M}{T_M}(T_M - T_T) + \frac{3g_M}{T_M^2}(T_M - T_T)^2$$

$$\frac{2g_M}{T_M} = \frac{2(979{,}979.0)}{1.005\ 454\ 70} = 1949.325016$$

$$\frac{3g_M}{T_M^2} = \frac{3(997{,}979.0)}{(1.00545470)^2} = 2908.124577$$

$$\therefore g_M - g_T = -(1949.325016)(-.00010437) + (2908.124577)(-.00010437)^2$$

$$= +\ .203451 + .000032$$

$$= \underline{203.48 \text{ milligals}}$$

$$\sigma = +\ (1949.325016)(118 \times 10^{-8}) + (2908.124577)(118 \times 10^{-8})^2 \text{ gals}$$

$$= 230{,}020.352 \times 10^{-5} + 0.405 \times 10^{-5} \text{ milligals}$$

$$\leq \underline{2.4 \text{ milligals}}$$

Pendulum 3-1

$$g_M = 979{,}979.0 \text{ milligals}$$

$$T_M = 1.005\ 459\ 30 \text{ seconds}$$

$$g_M - g_T = \frac{-2g_M}{T_M}(T_M - T_T) + \frac{3g_M}{T_M^2}(T_M - T_T)^2$$

$$\frac{2g_M}{T_M} = \frac{2(979{,}979.0)}{1.005\ 459\ 30} = 1949.316098$$

$$\frac{3g_M}{T_M^2} = \frac{3(979{,}979.0)}{(1.005\ 459\ 30)^2} = 2908.097968$$

$$\therefore g_M - g_T = -(1949.316098)(-.00010520) + (2908.097968)(.00010520)^2$$

$$= +\ 0.20506805 + .000032184$$

$$= \underline{205.10 \text{ milligals}}$$

$$\sigma = +(1949.316098)(42 \times 10^{-8}) + (2908.097968)(42 \times 10^{-8})^2 \text{ gals}$$

$$= 81{,}871.276 \times 10^{-5} + .051 \times 10^{-5} \text{ milligals}$$

$$\leq \underline{0.9 \text{ milligal}}$$

These results can be amalgamated by taking the weights as inversely proportional to the squares of the estimates of the standard deviations.

For Pendulum 2-1, weight $(w_1) = \frac{1}{(2.4)^2} = \frac{1}{5.76} = 0.1736$

For Pendulum 3-1, weight $(w_2) = \frac{1}{(0.9)^2} = \frac{1}{0.81} = 1.2346$

The weighted mean = 204.9 milligals

$$\text{Estimated } \sigma = \frac{1}{\sqrt{w_1 + w_2}} = \frac{1}{\sqrt{1.4082}} = 0.084$$

$$\approx 0.9$$

$\therefore$ $\underline{g_M - g_T = 204.9 \pm 0.9 \text{ milligals}}$

APPENDIX D

Miscellaneous data

Details of gravity stations (see Plate 1)

Pendulum station, Tokyo.

The gravity station is in the basement of the Geographical Survey Institute Building, Kamemeguro, Tokyo, Japan.

Latitude : $35^{\circ}38.6'$N
Longitude : $139^{\circ}41.3'$E
Elevation : 28.04 m. A.S.L.

Pendulum station, Melbourne.

The gravity station (N.G.B.S.) is in one of the ground floor instrument store rooms in the Bureau of Mineral Resources Geophysical Laboratory, Gordon Street, Footscray, Victoria.

Latitude : $37^{\circ}47.2'$S
Longitude : $144^{\circ}53.5'$E
Elevation : 43.89 m. A.S.L.

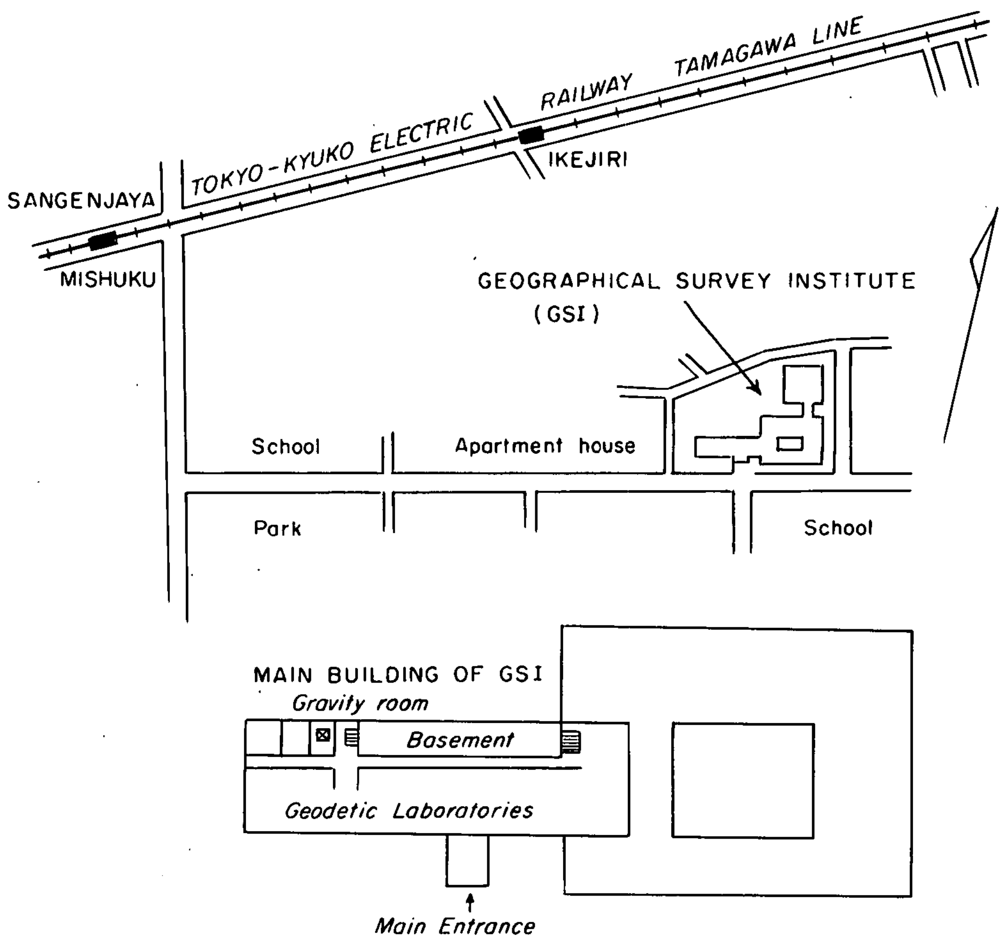
Personnel

Stations	Dates	Observers
Tokyo A1	13/9/62-17/9/62	Y. Fuzii and W. Langron
Tokyo A2	5/10/62-8/10/62	Y. Fuzii and W. Langron
Melbourne A	24/1/63-8/3/63	W. Langron and W. Burch
Tokyo B	12/7/63-19/7/63	Y. Fuzii
Melbourne B	20/1/64-22/1/64	W. Langron, T. Lai, and J. Shirley.

Project Files

The data are filed in the general drawers on GSI pendulum data.

PLATE I



GRAVITY STATION
GEOGRAPHICAL SURVEY INSTITUTE
TOKYO, JAPAN

GEOPHYSICAL BRANCH, BUREAU OF MINERAL RESOURCES, GEOLOGY AND GEOPHYSICS G65-173

TO ACCOMPANY RECORD No. 1966/109

PLATE 2

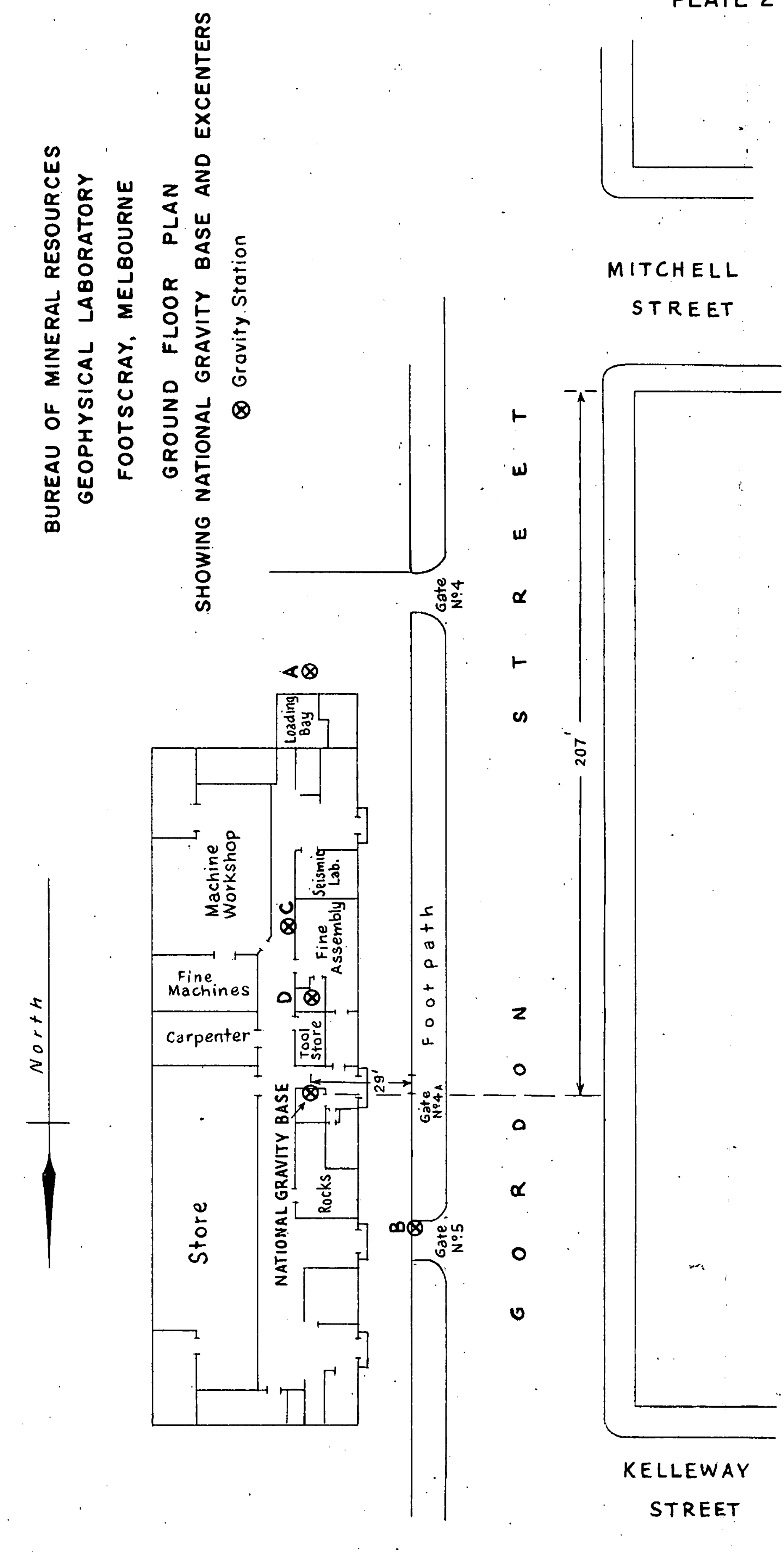


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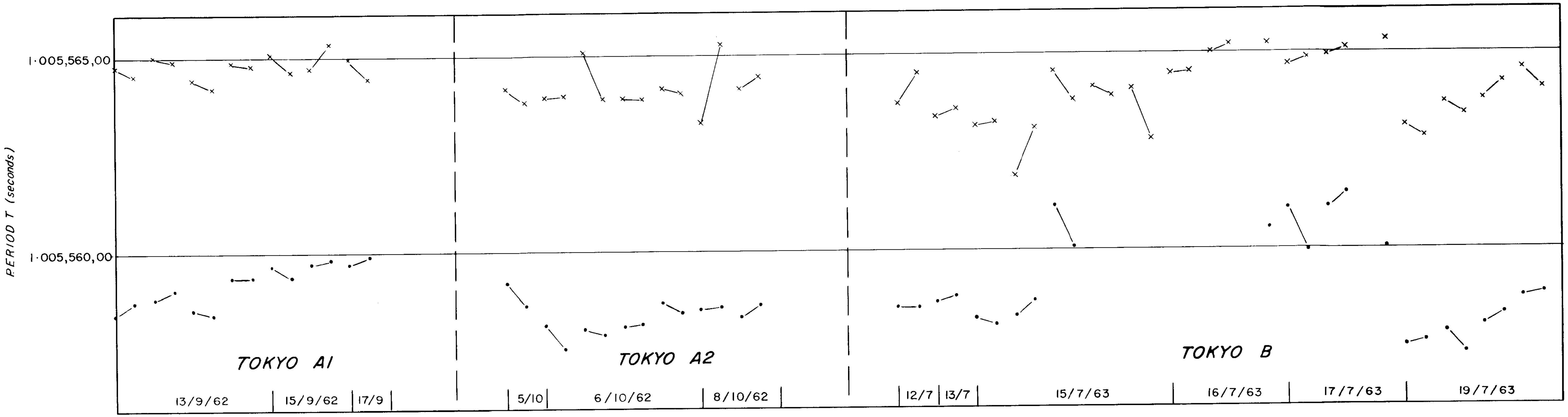
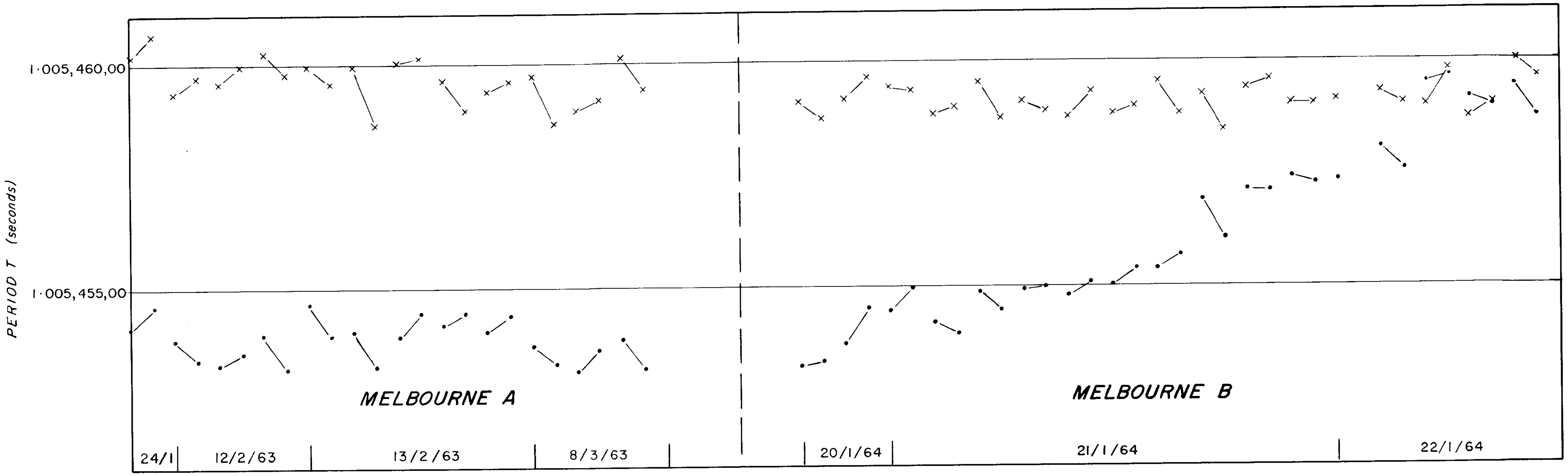
PLATE 3





Pendulum No. 2-1

Pendulum No. 3-1

PENDULUM GRAVITY MEASUREMENTS
TOKYO AND MELBOURNE 1962-1964

PLOT OF PENDULUM PERIODS

TO ACCOMPANY RECORD No. 1966/109

Geophysical Branch, Bureau of Mineral Resources, Geology and Geophysics

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