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DEPARTMENT OF NATIONAL DEVELOPMENT

BUREAU OF MINERAL RESOURCES, GEOLOGY AND GEOPHYSICS

RECORD No. 1966/153

TOTAL MAGNETIC INTENSITY ANOMALIES DUE TO TWO - DIMENSIONAL MODELS

by

G.B. CLARKE I. WIDDOWS and J.H. QUILTY

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ILLUSTRATIONS

SUMMARY

Total magnetic intensity anomalies due to idealised, inductively magnetised rock structures are presented in profile form suitable for comparison with observed anomalies. The models studied are the dyke, horizontal block, rectangular slab, simple fault, buried contact, and dipole.

1. INTRODUCTION

In 1957, a study of the total magnetic intensity anomalies due to a set of idealised magnetised rock structures was carried out in the Bureau of Mineral Resources. The study involved calculating the anomalies due to models simulating the structures and presenting them in a profile form suitable for comparison with observed anomalies.

The magnetic profiles are intended for use qualitatively, by allowing the reader to become familiar with the form of anomalies produced by bodies in a variety of shapes, orientations, and magnetic inclinations, and quantitatively in that methods of depth estimates may be obtained in an empirical manner from them. It should be noted that the profiles are based on inductively magnetised models only.

The models chosen for calculation were, with one exception (the dipole), of the two-dimensional prismatic type, being regularly shaped models with vertical sides, such as blocks, horizontal slabs, vertical sheets, or slabs, and combinations of these to simulate fault structures. Some sloping-sided models were also studied. The two-dimensional criterion required that the horizontal dimension perpendicular to the profile be large in comparison with the depth of the surface of the prism below the level of observation.

This Record describes the basis of the mathematical treatment used and shows the anomaly profiles obtained from the study with explanatory notes on notation and usage. G. B. Clarke, I. Widdows, and J. H. Quilty initiated the work, the major part of which was carried out by I. Widdows.

2. THEORETICAL TREATMENT

Notation

The following sympols are used in this Record :

- V – vertical component of the Earth's field T
- H – horizontal component of the Earth's field T
- ΔZ – vertical component of induced field
- ΔF – horizontal component of induced field
- i – angle of inclination of Earth's field
- β – strike angle of body measured anticlockwise with respect to Earth's magnetic east.
- k – magnetic susceptibility of body
- h – depth of top of body from the level where anomaly profile is obtained

Subscript r is used to denote any physical quantity that is normalised with respect to depth, e.g. $x_r = x/h$; $b_r = b/h$, etc.

Other symbols are explained in the figures and plates.

Component of magnetic field measured by airborne magnetometer

The airborne magnetometer measures the total magnetic field, which, in general, is the resultant of the Earth's normal field and the anomalous field due to local geological structure.

Consider a body (Figure 1) magnetised either by induction in the Earth's present magnetic field or permanently magnetised by some previous magnetic field. The direction of magnetisation (polarisation) may or may not coincide with the direction of the Earth's present field.

As the airborne magnetometer passes over the vody, at any point P its detector coil is aligned by an orienter mechanism in the direction of T_{total}, which is the resultant of the vectors T (the Earth's normal field vector in the region) and ΔT_A (the anomaly field vector at P).

The anomaly recorded by the magnetometer is the difference (T_{total} – T). Since the intensity of T is approximately one-half gauss, which equals 50,000 gammas (the value varies over the Earth's surface between 25,000 and 70,000 gammas), and ΔT_A is normally only a few hundred gammas, the direction of T_{total} nearly equals that of T. Hughes and Pondrom (1947) have shown that $T_{total} - T$ is approximately equal to the component of the anomalous field in the direction of the Earth's normal field.

In calculating the anomaly profiles, it has been assumed that the anomaly is equal to the component of the anomalous field in the direction of the Earth's normal field. The error involved in this assumption is very small.

Calculation of the anomaly in total magnetic intensity due to a two-dimensional prism

Consider a two-dimensional prism magnetised by induction in the Earth's magnetic field (Figure 2). The magnetisation is uniform throughout the body. The Earth's magnetising vector is along T. The effect may be considered as producing an infinite number of mangetic dipoles in the body, the poles of opposite sign cancelling each other within the body. The polarisation on the surfaces of the body, however, are retained. The net magnetisation of the body is therefore equal to the surface polarisation over its boundaries.

The same result is obtained in mathematical terms by applying Green's Theorem to Poisson's Equation. It can be shown that for a uniformly magnetised body, the volume integral can be reduced to a surface integral along the boundary of the magnetic body. The surface polarisation is to be taken normal to the boundary (Gulatee, 1938).

The magnetic intensity due to a single polarised plane is first derived in the following treatment and then the anomaly in total intensity due to a prism is calculated by summing the horizontal and vertical components of intensity due to the plane surfaces and resolving them in the direction of the Earth's normal field.

Consider the plane AB (Figure 2), infinite in a direction perpendicular to the plane of the paper, with surface polarisation I per unit length normal to the plane. The magnetic intensity at P due to an infinite strip of width dx is 2Idx/r along r.

The vertical component of intensity ΔZ_{AB} due to the total width AB of the plane is given by :

$$\Delta Z_{AB} = \int 2I\, dx/r.\cos\phi = 2I\phi$$

and the horizontal component of intensity ΔF_{AB} is

$$\Delta F_{AB} = \int 2I\, dx/r.\sin\phi = 2I \log r_A/r_B$$

But $I = kV$

$\therefore \quad \Delta Z_{AB} = 2kV\phi$

and $\Delta F_{AB} = 2kV \log r_A/r_B$

The surface polarisation normal to the vertical sides of the prism equals kH, so that the vertical component of intensity ΔZ_s due to the vertical sides of the plain is

$$\Delta Z_s = 2kH \log r_A/r_B.\cos\beta$$

Similarly,

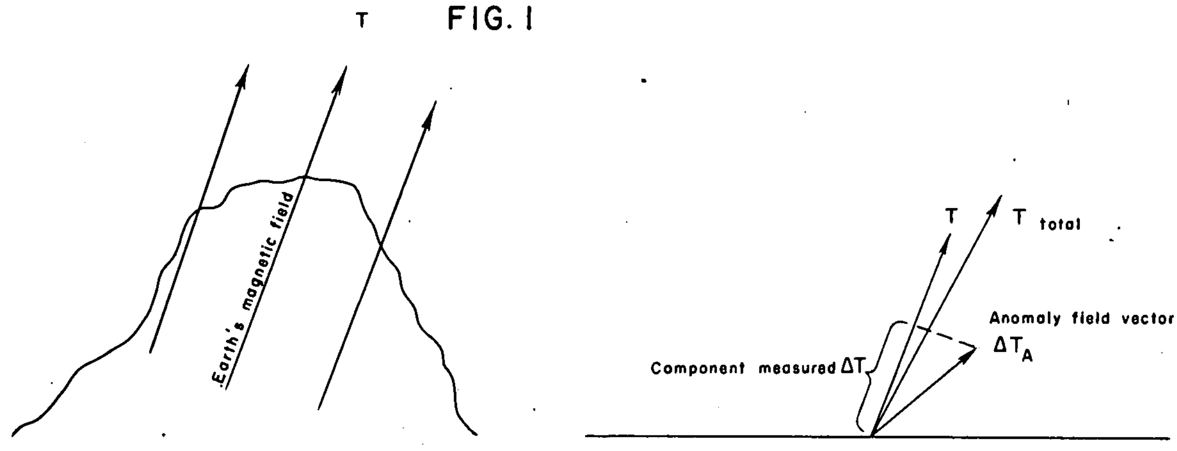
$$\Delta F_s = -2kH\phi \cos\beta$$

Thus the total horizontal and vertical anomalies due to the prism are

$$\Delta Z = 2kV\phi + 2kH \log r_A/r_B.\cos\beta$$

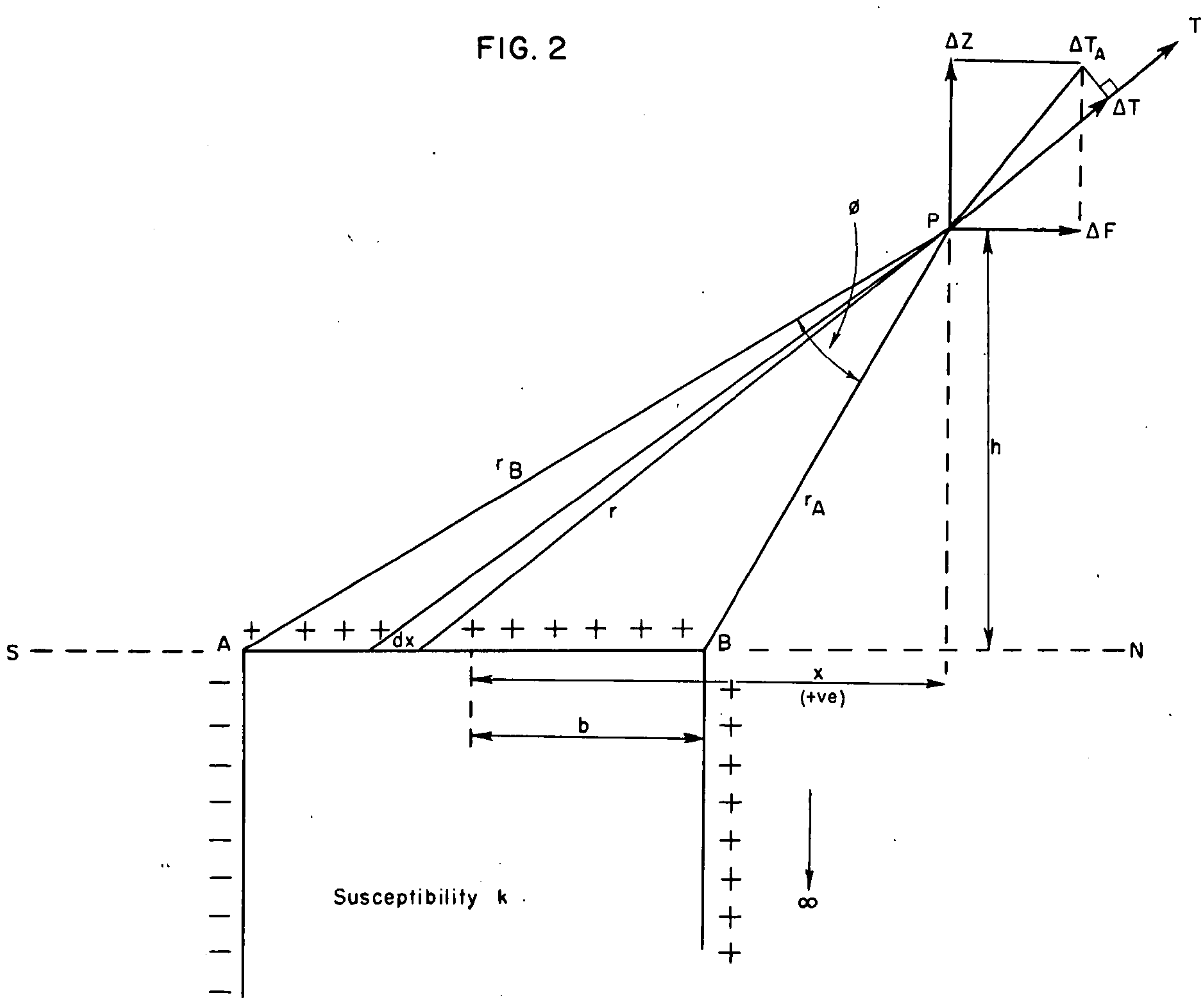
$$\Delta F = 2kV \log r_A/r_B - 2kH\phi \cos\beta$$

FIG. 1



COMPONENT OF MAGNETIC ANOMALY MEASURED BY AIRBORNE MAGNETOMETER

FIG. 2



ANOMALY DUE TO A TWO-DIMENSIONAL PRISM

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Substituting H = T cos i and V = T sin i and expressing r_A, r_B, and ϕ in terms of x_r and b_r, we obtain

$$\Delta Z = kT\Big[2\sin i\,[\arctan(x_r + b_r) - \arctan(x_r - b_r)] + \cos i\,\cos\beta\,\log[1 + (x_r + b_r)^2]/[1 + (x_r - b_r)^2]\Big]$$

$$\Delta F = kT\Big[\sin i\,\log[1 + (x_r + b_r)^2]/[1 + (x_r - b_r)^2] - 2\cos i\,\cos\beta\,[\arctan(x_r + b_r) - \arctan(x_r - b_r)]\Big]$$

The anomalous magnetic intensity ΔT in the direction of the Earth's field at P is given by

$$\Delta T = Z_T + F_T$$

where $Z_T = \Delta Z \sin i$

$F_T = \Delta F \cos i \cos\beta$

The expression for the total intensity anomaly along an axis at right angles to the direction of elongation of the prism is

$$\Delta T/2kT = \sin i \cos i \cos\beta \log[1 + (x_r + b_r)^2]/[1 + (x_r - b_r)^2] + (\sin^2 i - \cos^2 i \cos^2\beta)\,[\arctan(x_r + b_r) - \arctan(x_r - b_r)]$$

When the prism is elongated in the direction of the magnetic meridian, the expression reduces to

$$\Delta T/2kT = \sin^2 i\,[\arctan(x_r + b_r) - \arctan(x_r - b_r)]$$

Expressions were derived in a similar manner for all models in this record, with the exception of the simple dipole, for which the expression is well known.

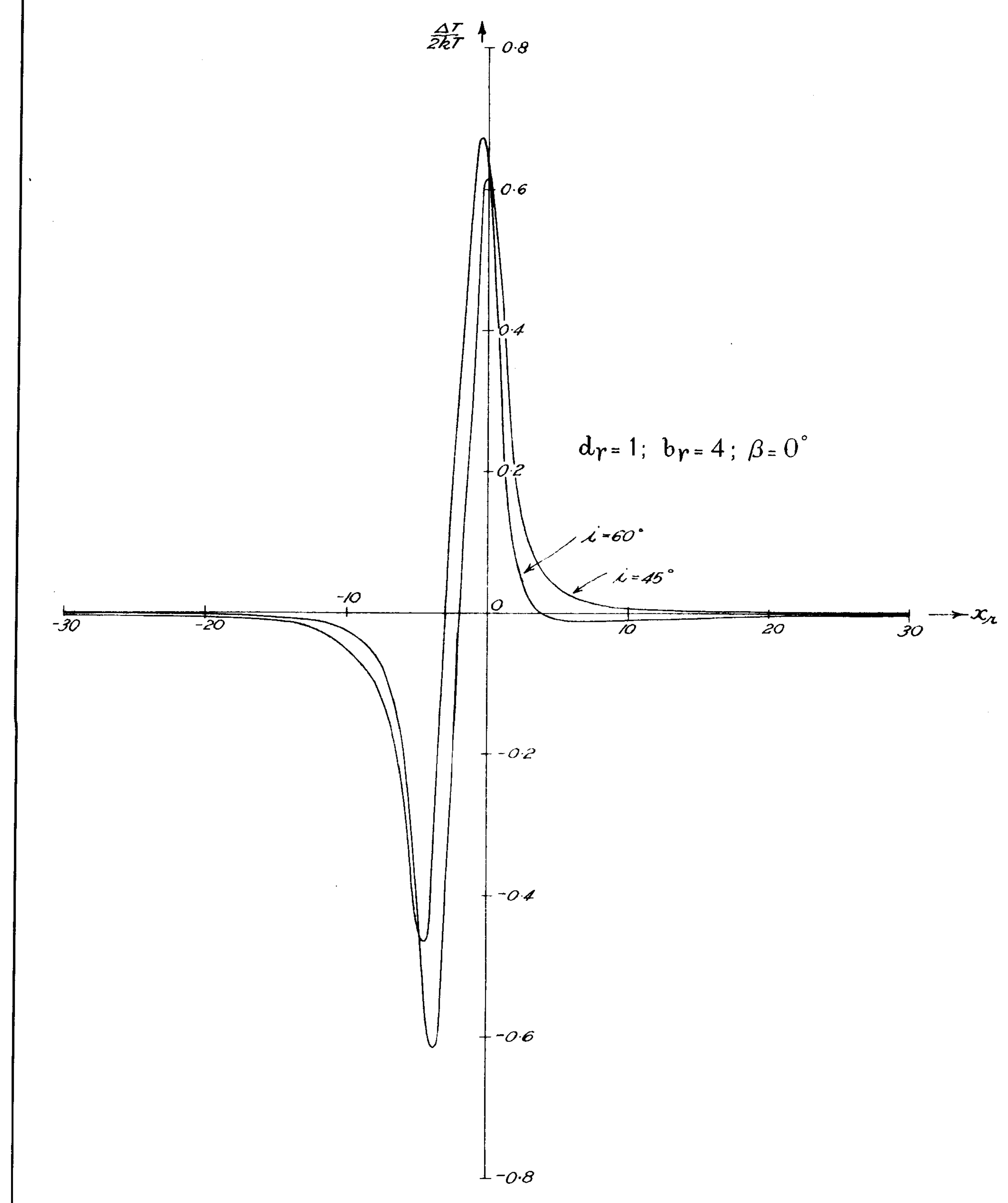
Notes on models and profiles

The profiles are calculated on a horizontal plane in a direction at right angles to the direction of elongation of the model. The sides of the models are vertical (except where shown with dip angle α) and, in some models, extend to infinite depth. In others, the normalised vertical extent is d_r. In the fault model (Plate 4), the throw is d_r and the slab thickness on either side of the fault is l_r. The dipole length is also l_r.

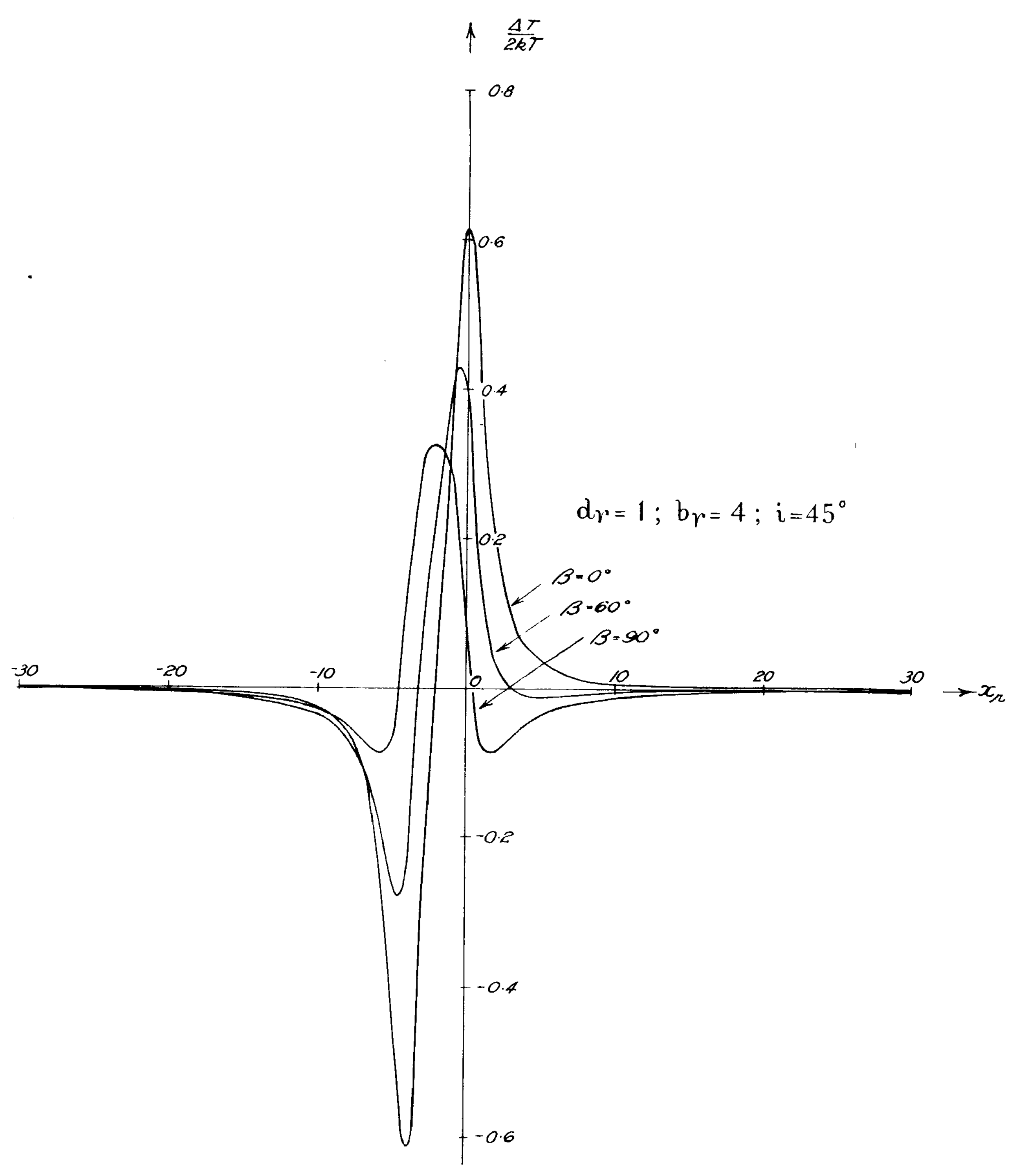
The total intensity anomaly is shown, together with the vertical intensity and horizontal intensity components in some cases. The anomalies have been calculated for variations in the parameters of model dimensions, orientation, and inclination of magnetising field.

The vertical scale of each profile is in units of $\Delta T/2kT$, and $\Delta Z/2kT$ and $\Delta F/2kT$ are shown where appropriate. By substitution of the appropriate values for the Earth's field and the susceptibility of the model, the scale may be readily converted to gammas. The horizontal scale is in units of x_r.

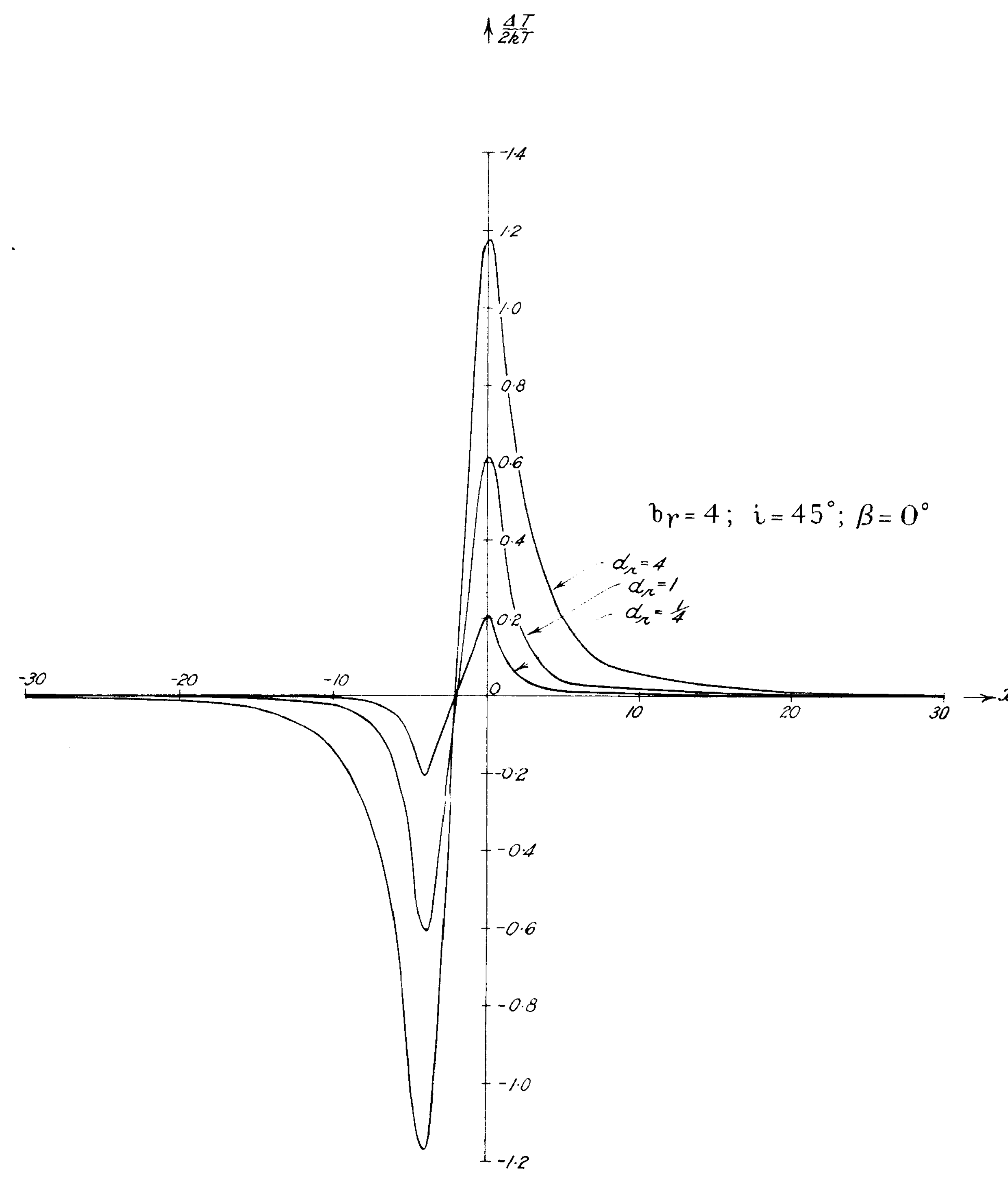
In the model that simulates a geological contact (Plate 5), a slightly different treatment is used, the distances and depth extent of the model being assigned definite values. The results of varying these parameters are also illustrated (Plate 6). The vertical scale in some plates of this model are in units of $\Delta Z/kV$ or $\Delta F/kV$.



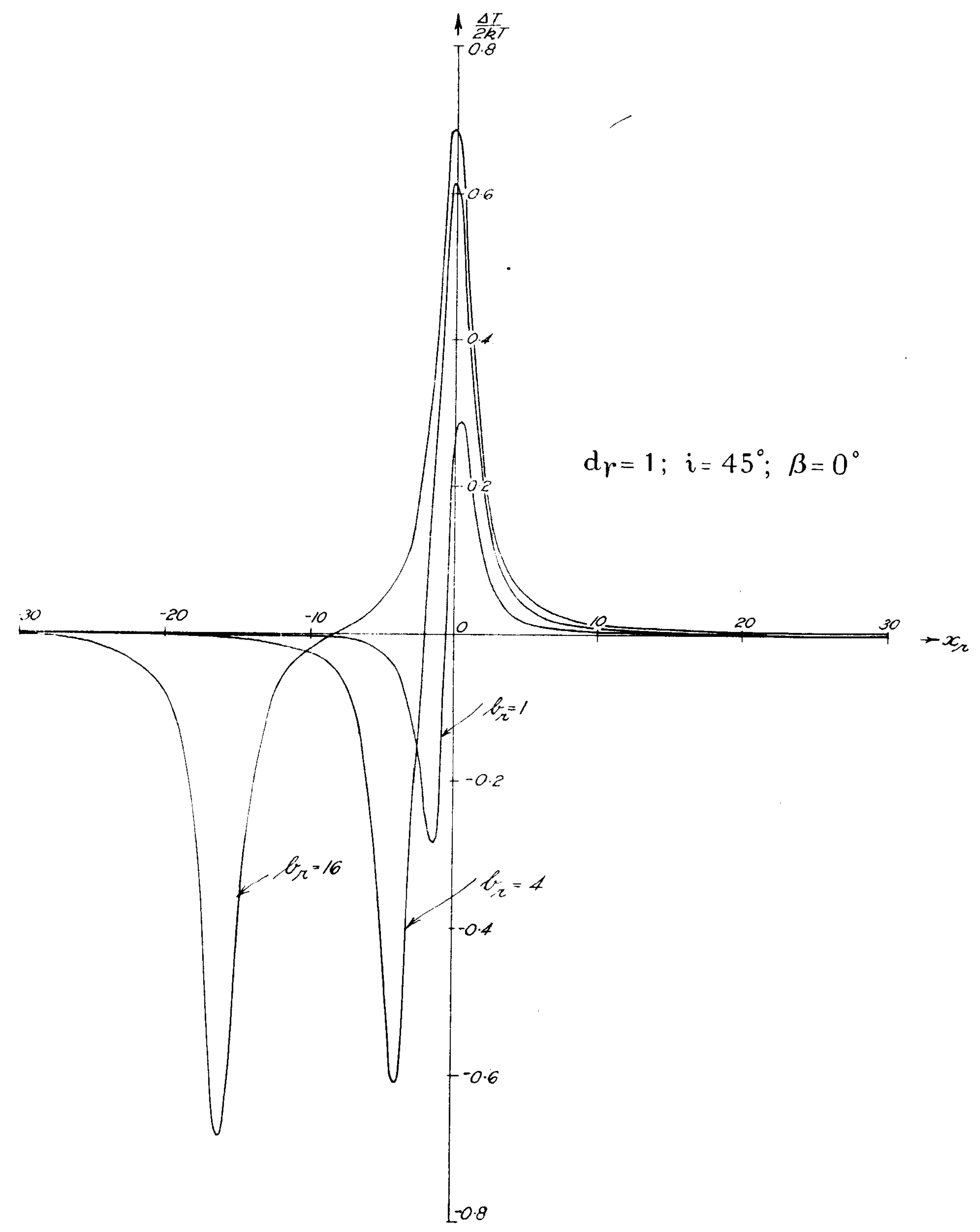
VARIATION WITH INCLINATION



VARIATION WITH STRIKE

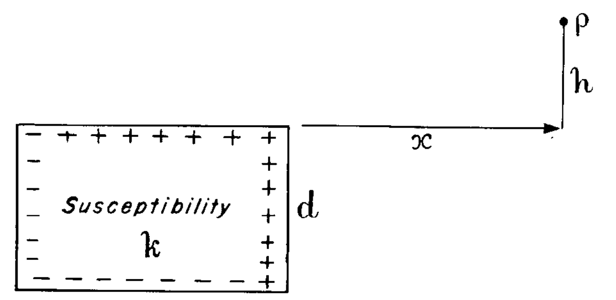


VARIATION WITH d_r



VARIATION WITH b_r

LEGEND



MAGNETIC ANOMALY DUE TO A HORIZONTAL BLOCK

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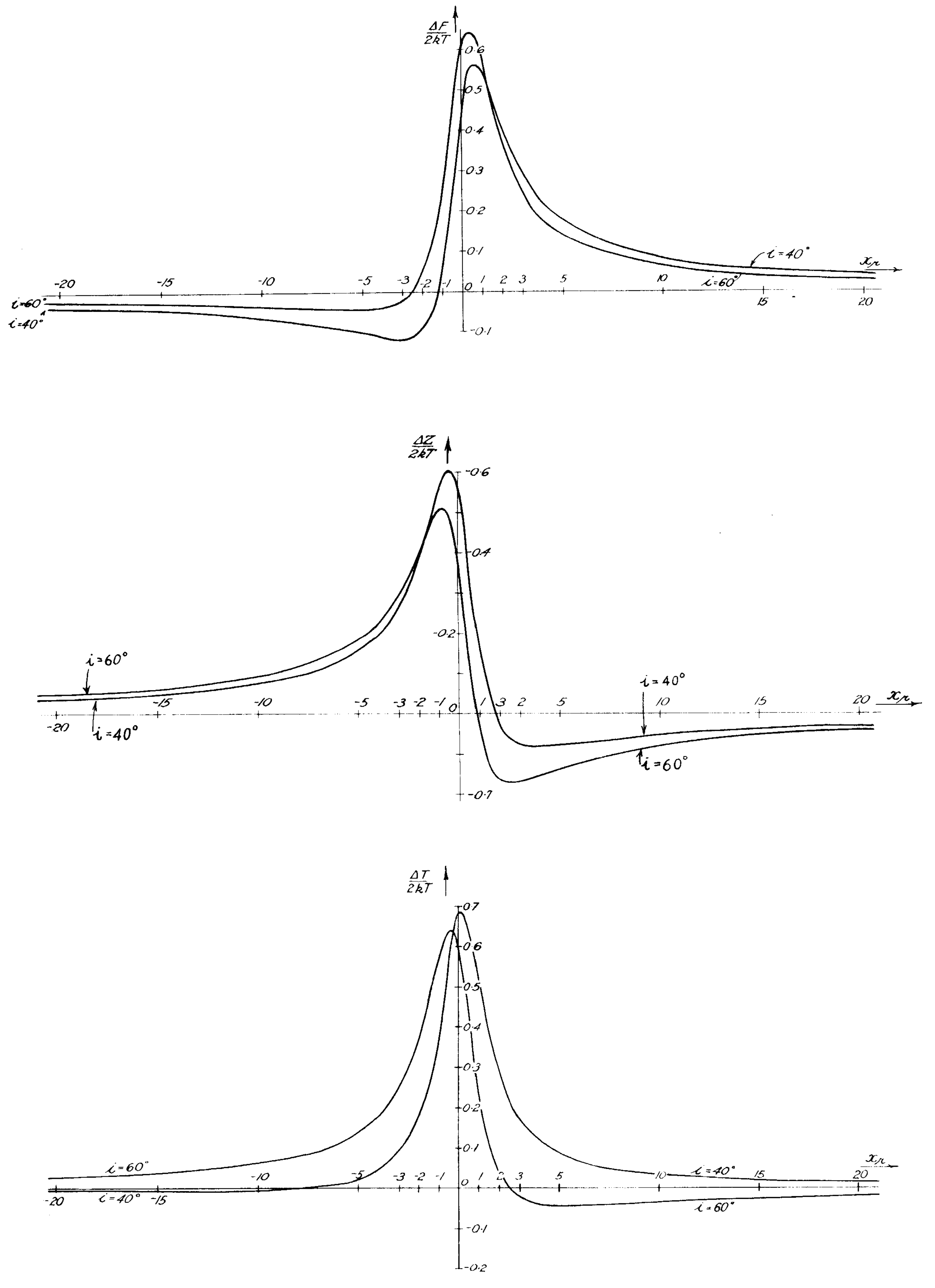
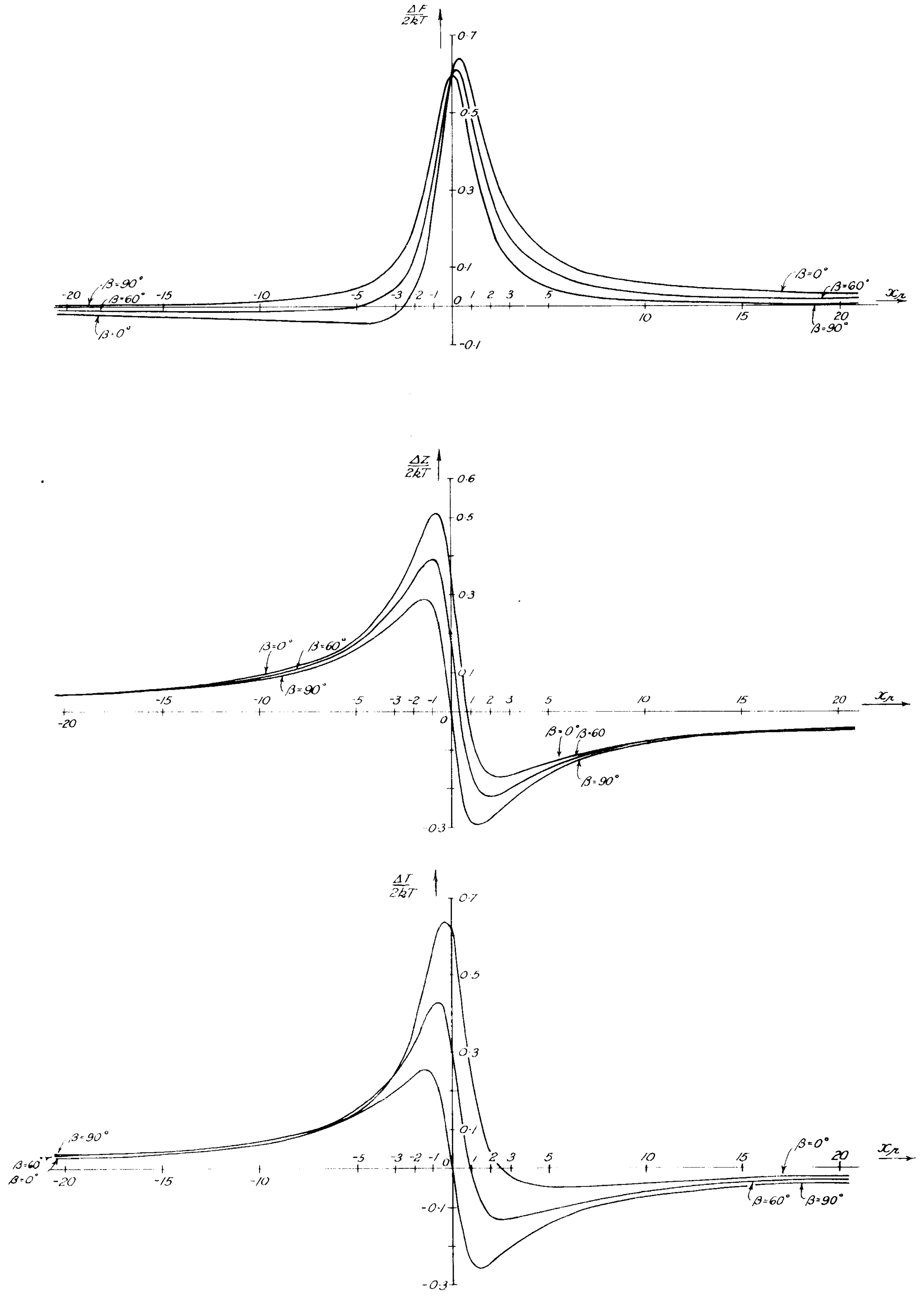
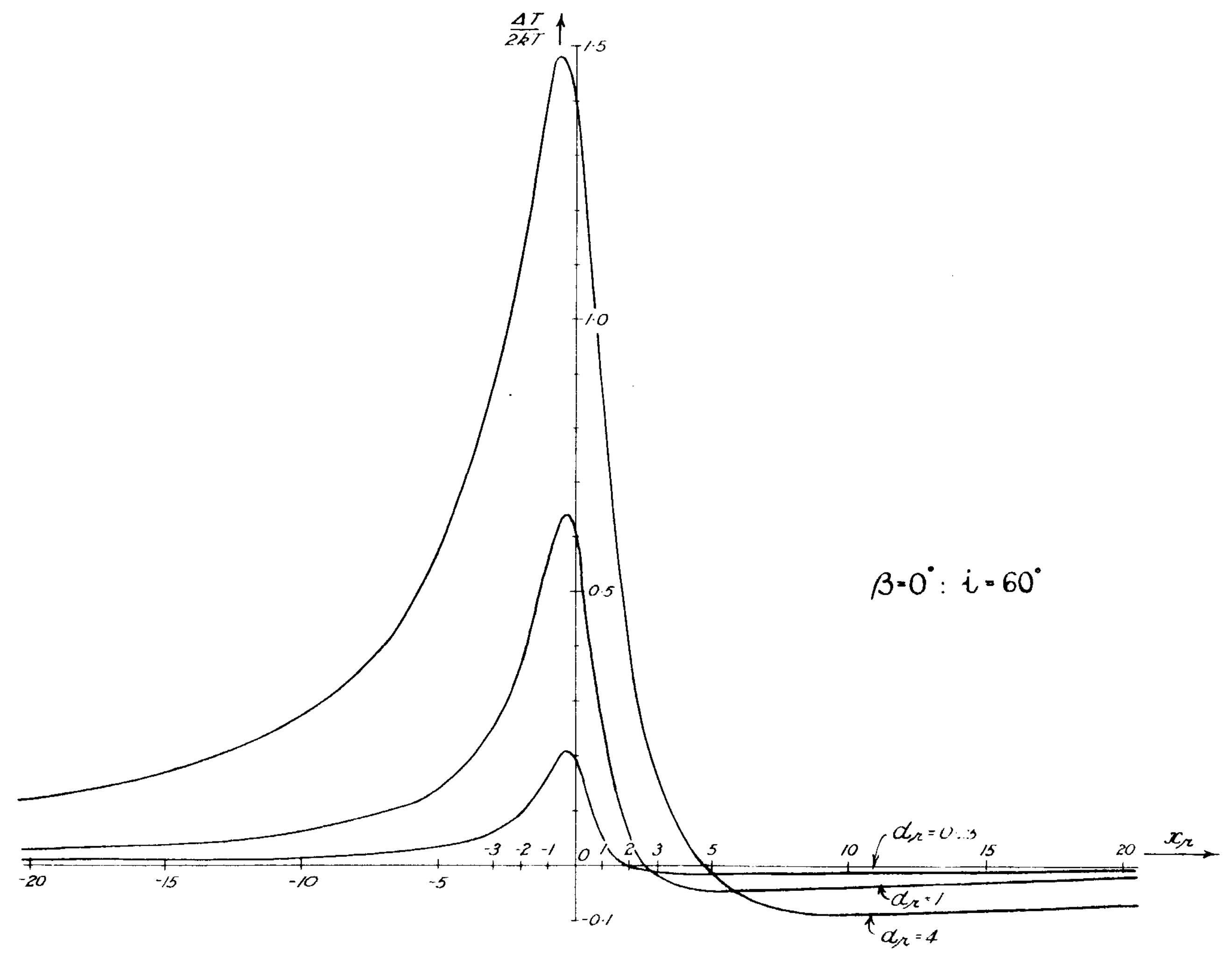
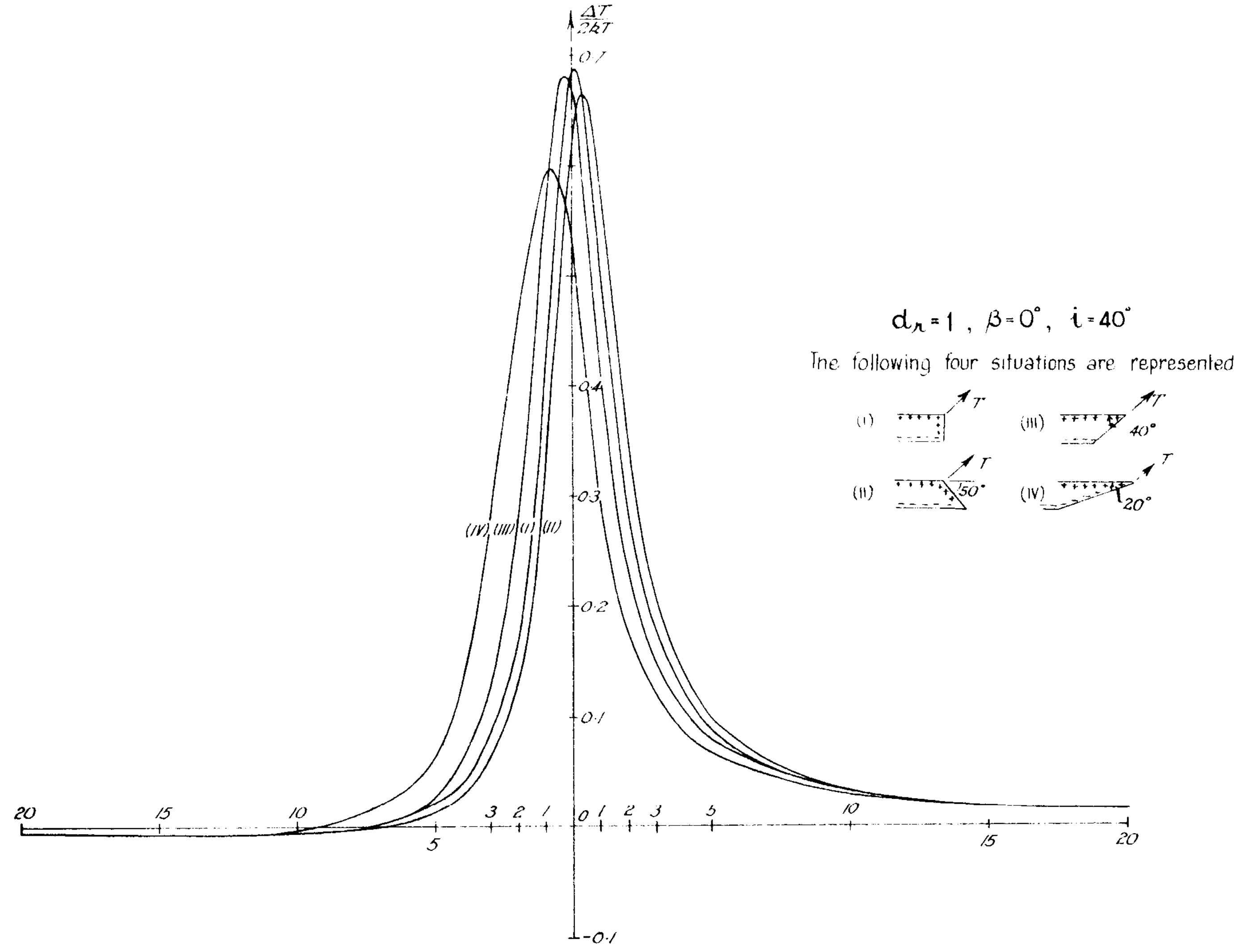

VARIATION WITH INCLINATION
$d_r = 1$, $\beta = 0°$


VARIATION WITH STRIKE
$d_r = 1$, $i = 60°$

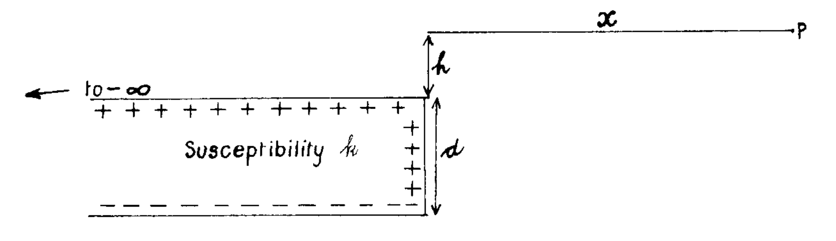


VARIATION WITH d_r



VARIATION WITH SLOPE

LEGEND



MAGNETIC ANOMALY DUE TO A RECTANGULAR SLAB

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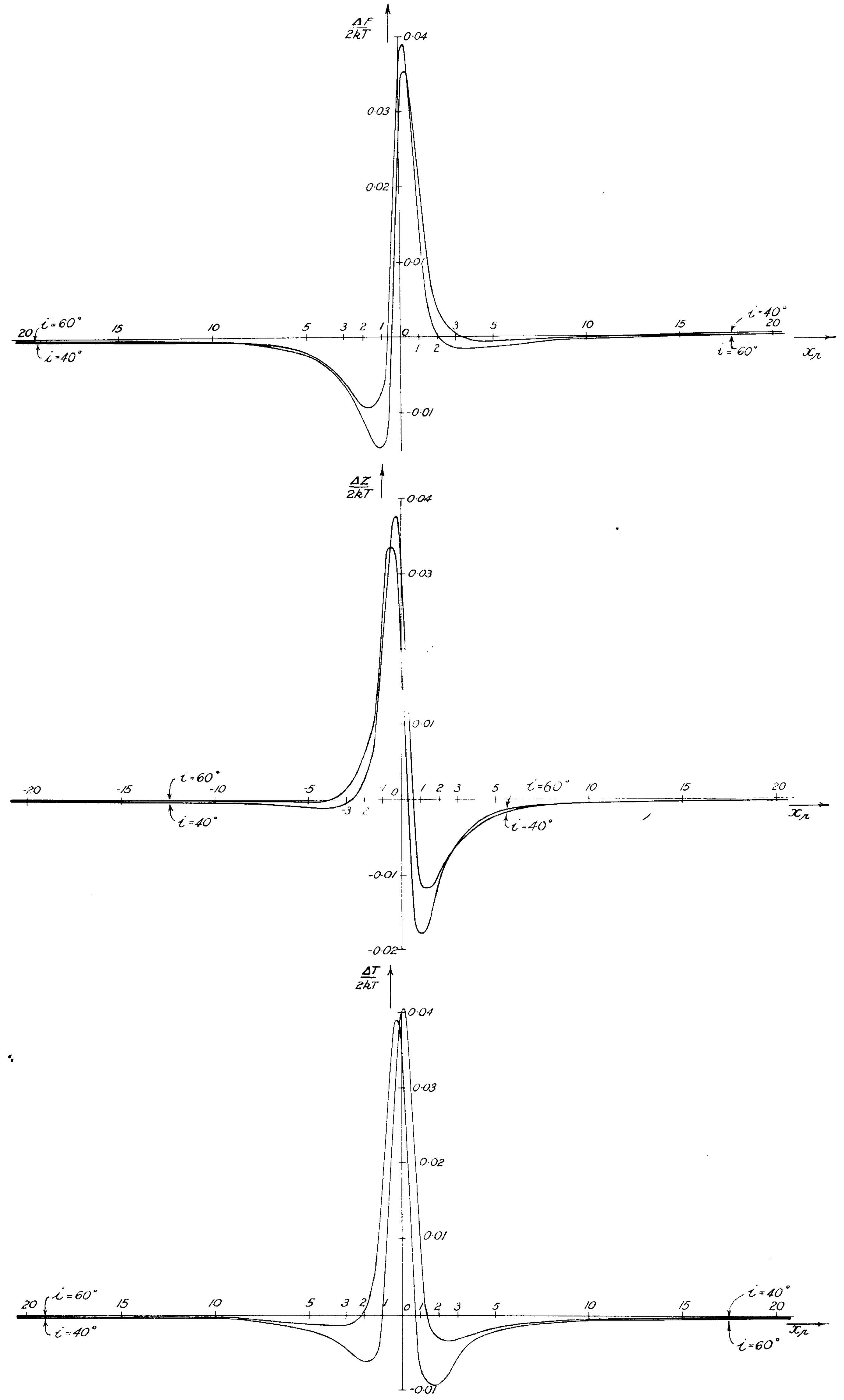
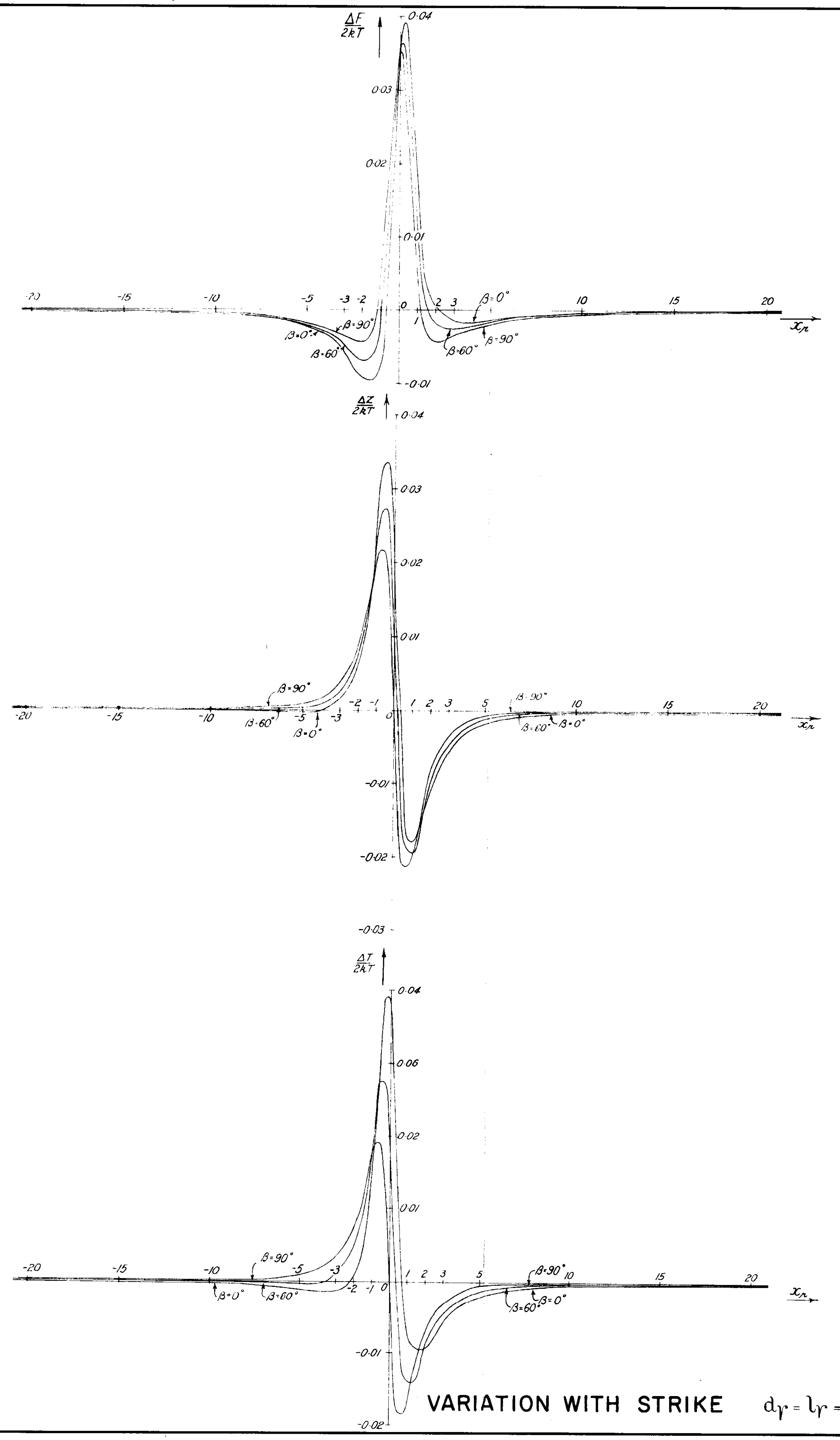
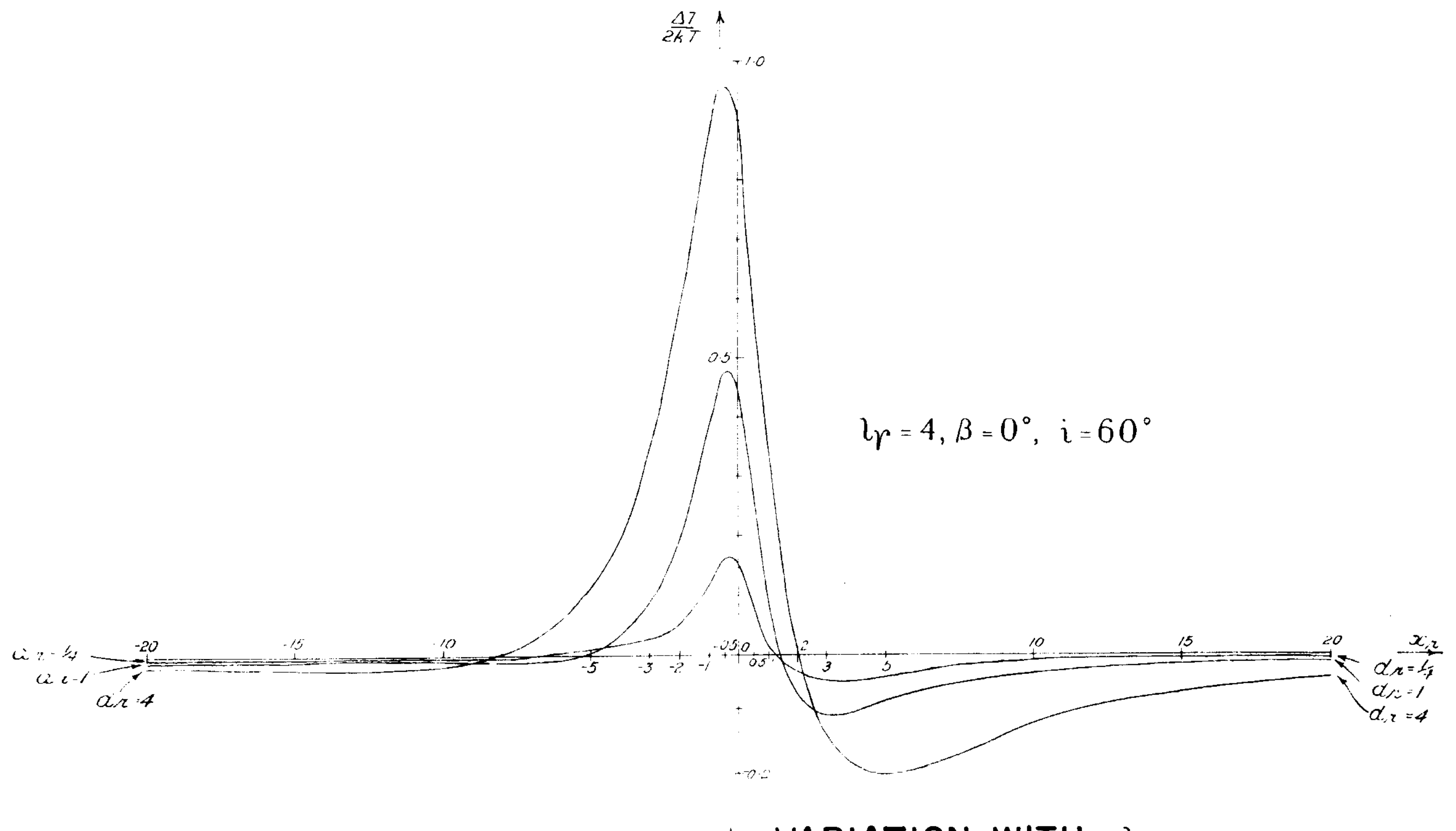
PLATE 4



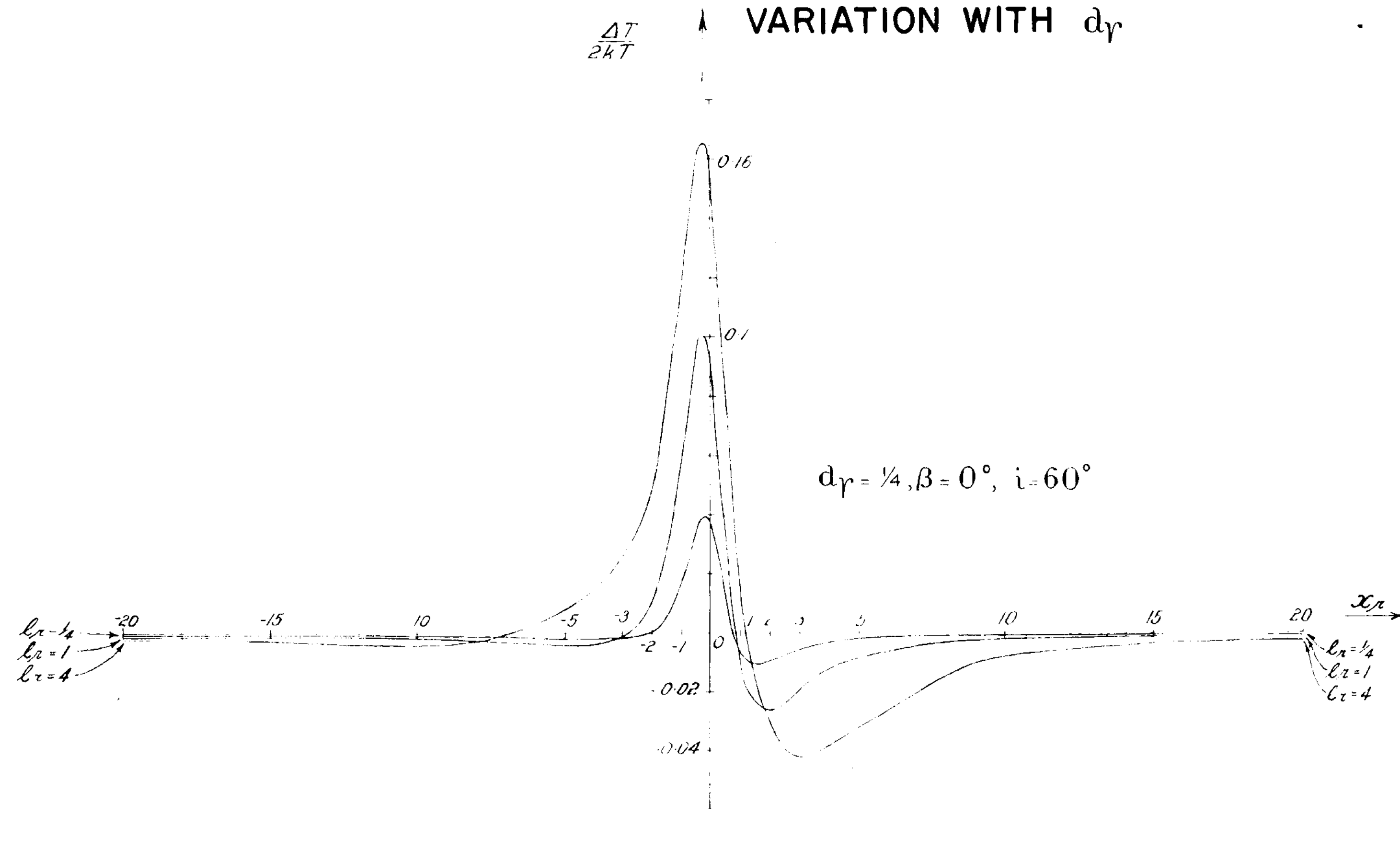
VARIATION WITH INCLINATION $d_r = l_r = 1/4, \beta = 0°$



VARIATION WITH STRIKE $d_r = l_r = 1/4, i = 60°$

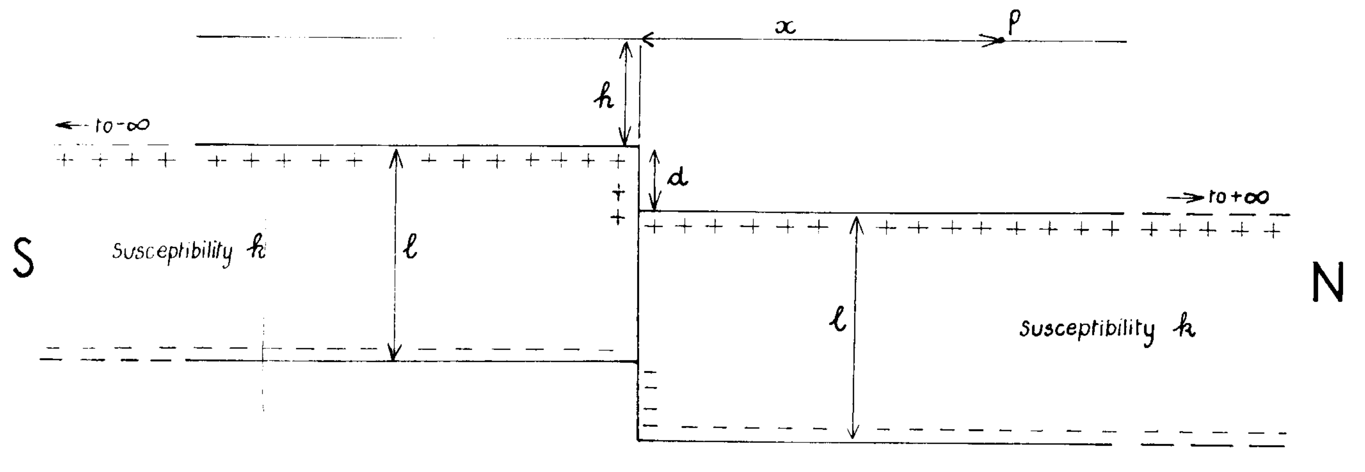


VARIATION WITH d_r



VARIATION WITH l_r

LEGEND



MAGNETIC ANOMALY
DUE TO A SIMPLE FAULT

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PLATE I

VARIATION WITH INCLINATION

$b_r = 0{\cdot}5 \quad \beta = 0$

VARIATION WITH STRIKE

$i = 40°; \ b_r = 0{\cdot}5$

VARIATION WITH b_r

$i = 60° \quad \beta = 0$

VARIATION WITH ANGLE OF DIP

$i = 60°; \ \beta = 0; \ b_r = 0{\cdot}5$

MAGNETIC ANOMALY DUE TO A DYKE

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PLATE 5

i = 45°
VARIATION WITH h $k_1 = \frac{1}{2} k_2$
$\beta = 0$

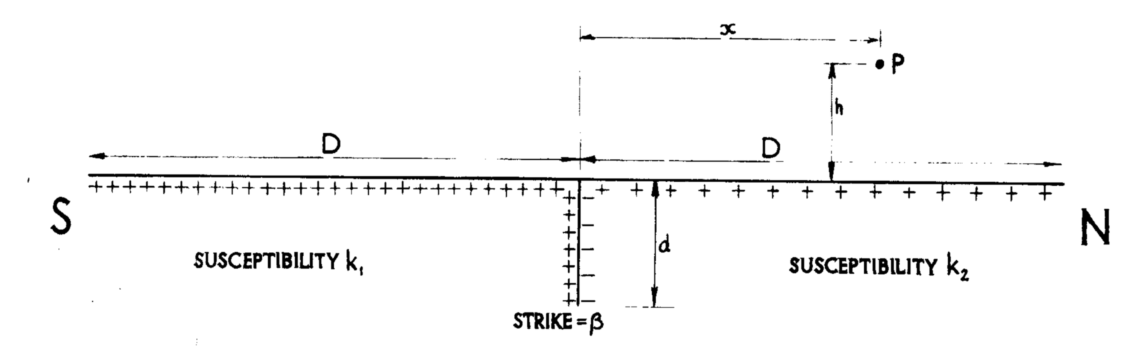
i = 45°
VARIATION WITH h $k_1 = 2 k_2$ $\beta = 0$

i = 45°
VARIATION WITH h $k_1 = 50 k_2$ $\beta = 0$

i = 60°
VARIATION WITH h $k_1 = 50 k_2$
$\beta = 0$

i = 60°
VARIATION WITH h $k_1 = 50 k_2$
$\beta = 60°$

LEGEND

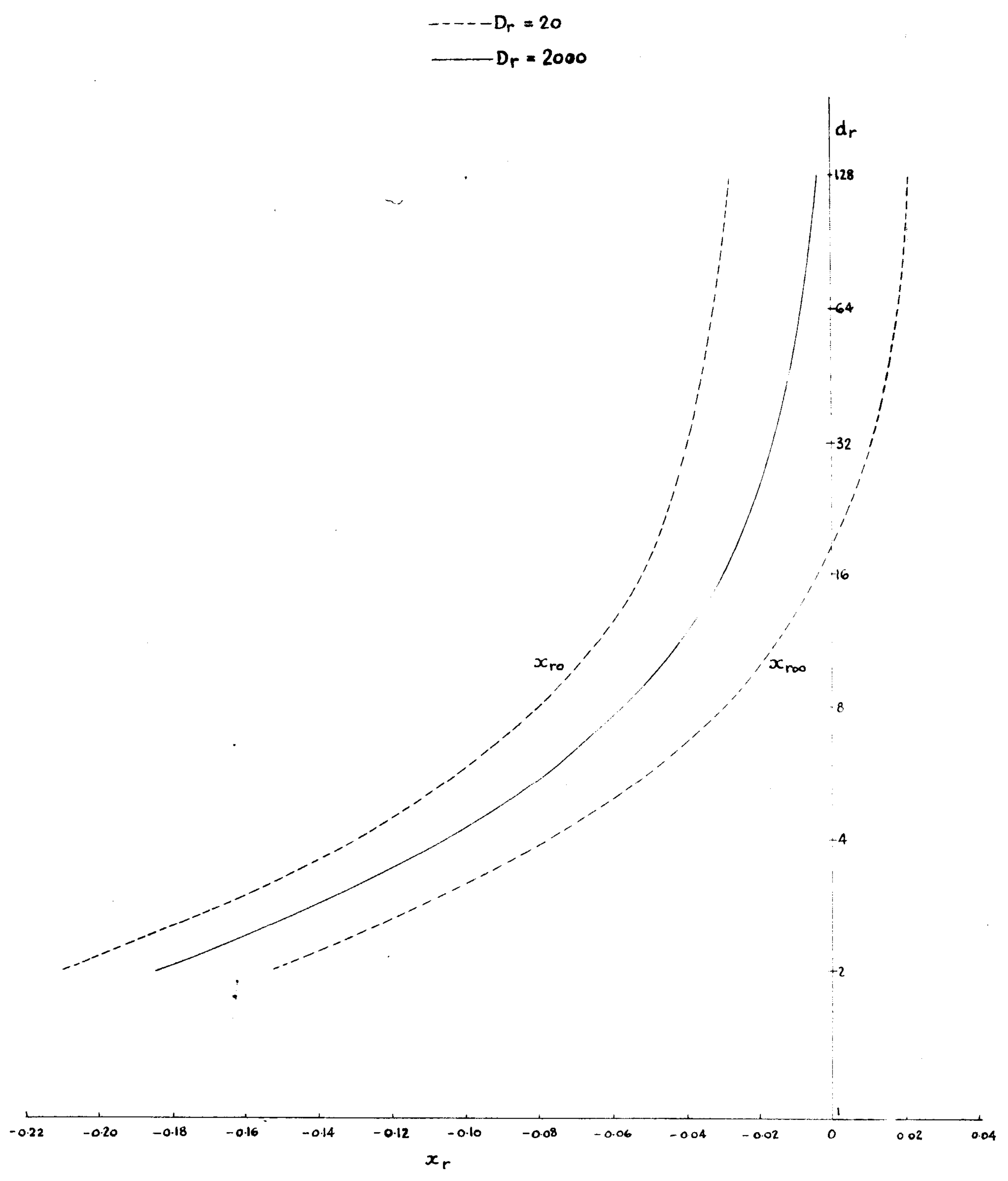


IN ALL PROFILES DRAWN D = 500m., d = 15m., x = -30 --- + 30m.,
h = 0·5m., 1·5m., and 4·0m..

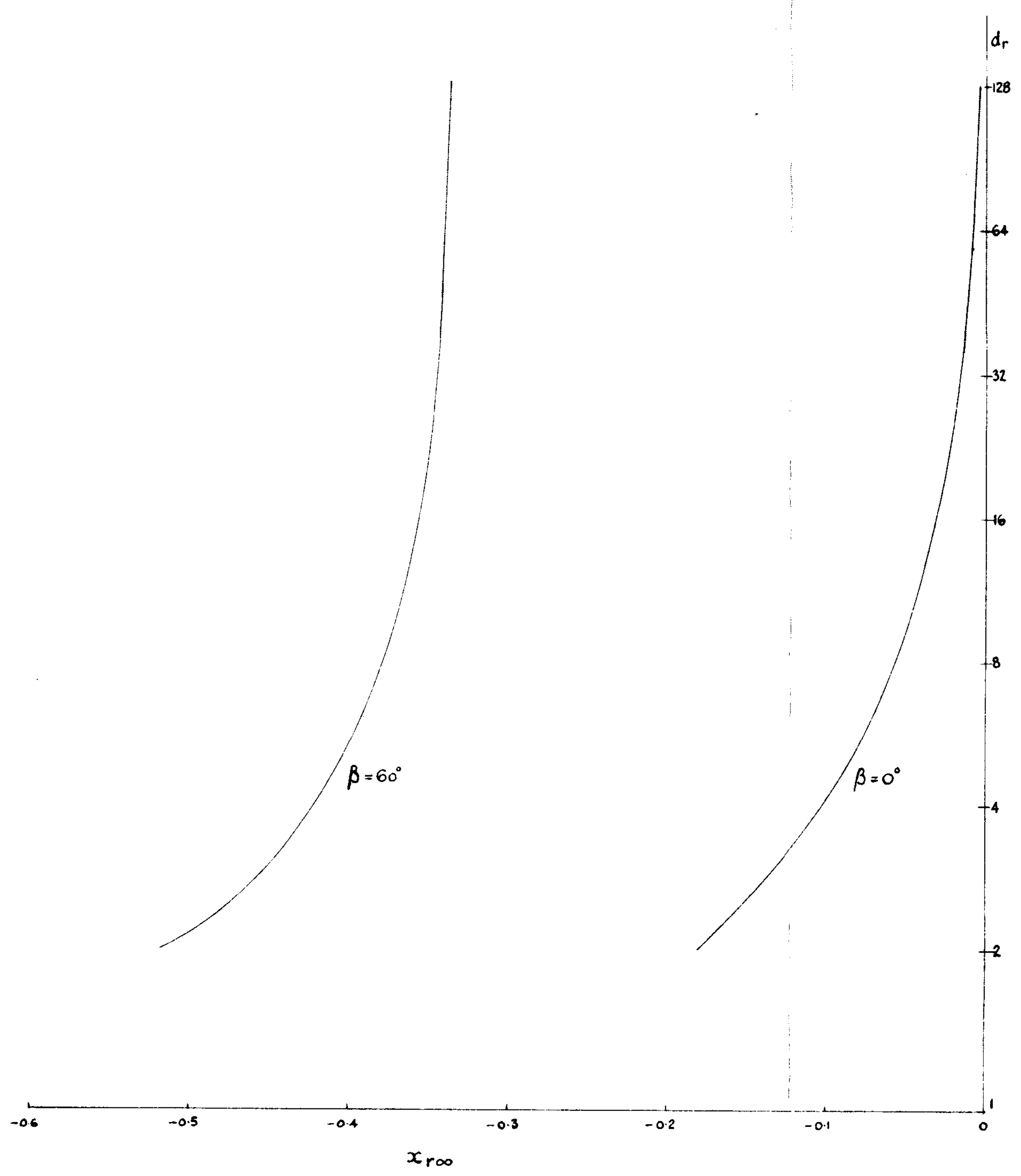
MAGNETIC ANOMALY DUE TO A BURIED CONTACT

PLATE 6

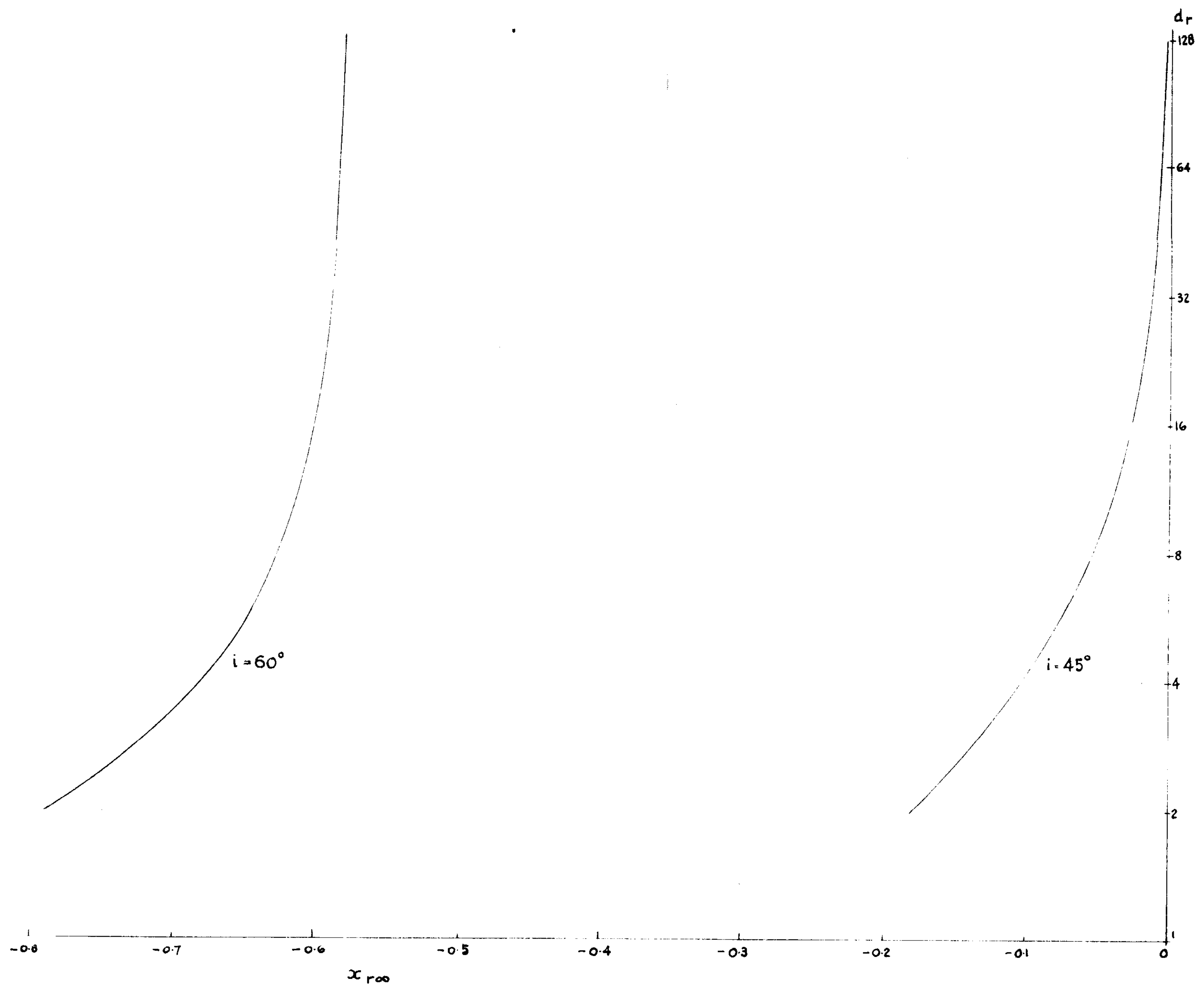
x_{ro} and $x_{r\infty}$ vs. d_r for $i = 45°$, $\beta = 0°$



$x_{r\infty}$ vs. d_r for $i = 45°$, $D_r = 2000$



$x_{r\infty}$ vs. d_r for $\beta = 0°$, $D_r = 2000$



LEGEND

$x_r = \frac{x}{h}$, $D_r = \frac{D}{h}$, $d_r = \frac{d}{h}$

x_{ro} = value of x_r for $S = \frac{k_2}{k_1} = 0$

$x_{r\infty}$ = value of x_r for $S = \frac{k_2}{k_1} = \infty$

x HERE REFERS TO THE POSITION OF THE MAIN TURNING POINT

MAGNETIC ANOMALY
DUE TO A BURIED CONTACT

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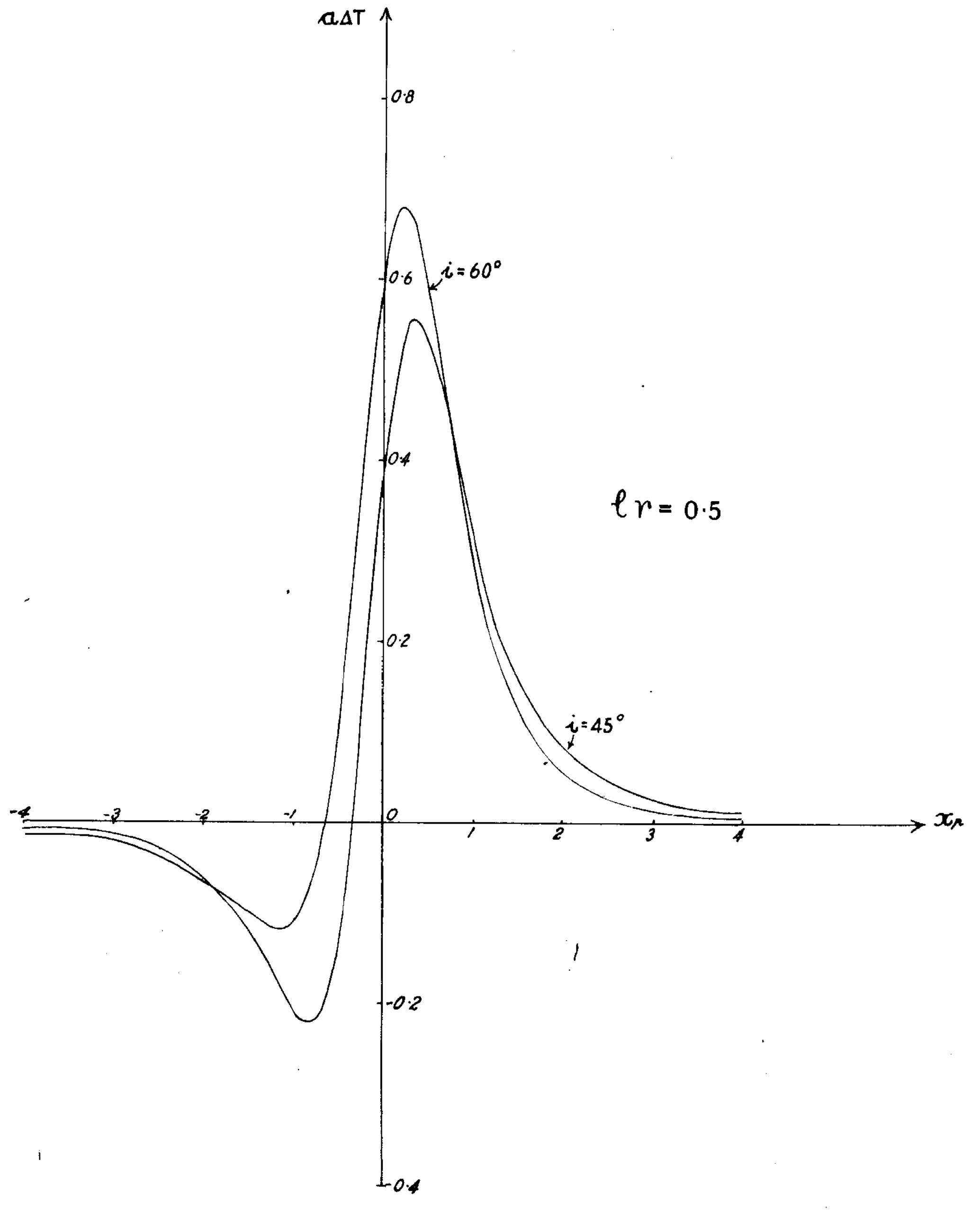
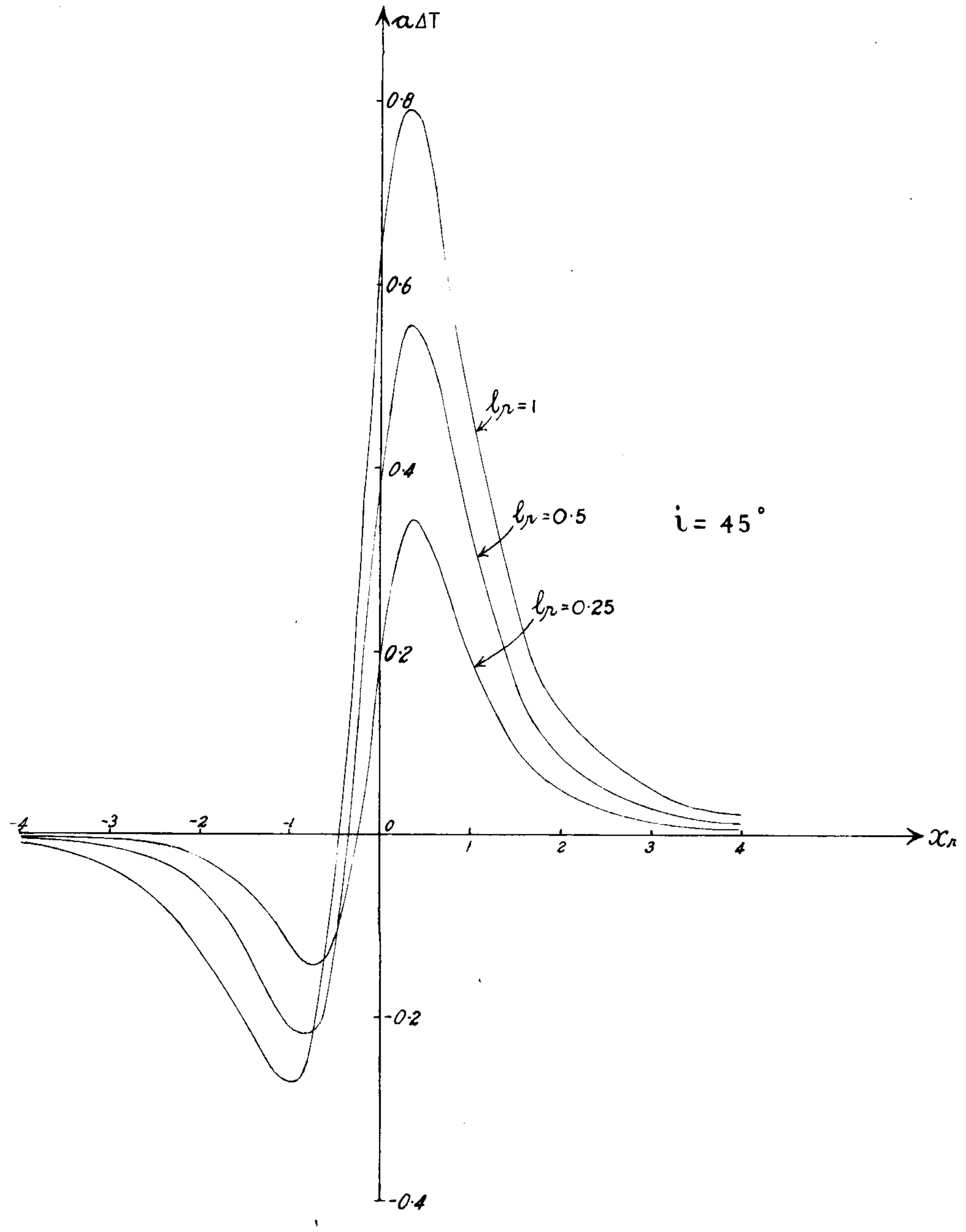
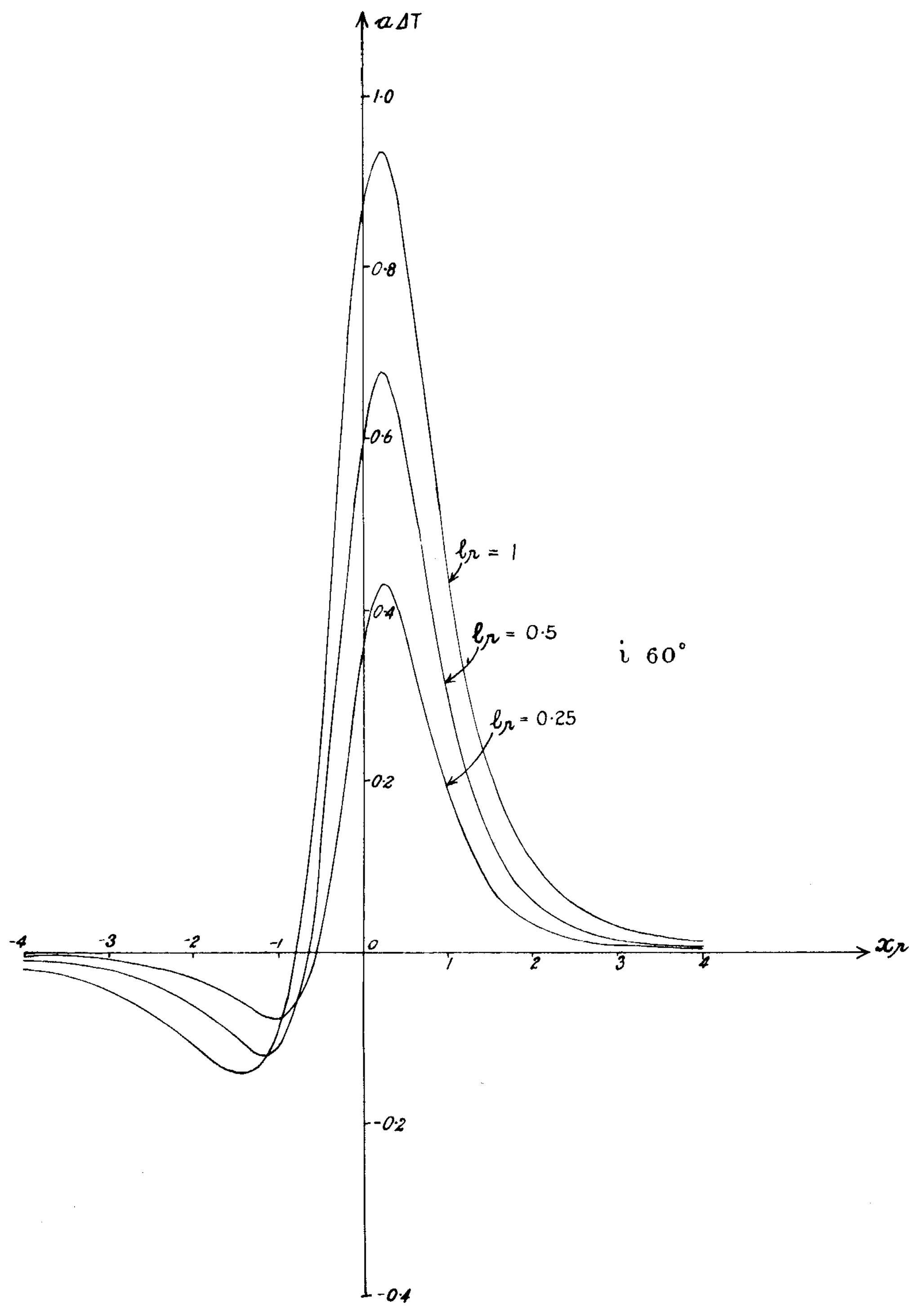


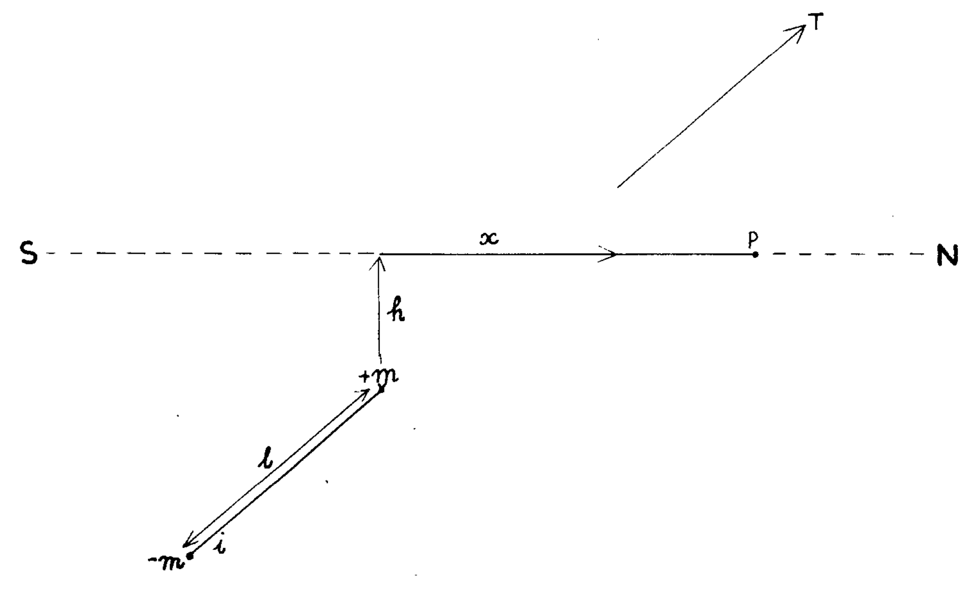
VARIATION WITH INCLINATION





VARIATION WITH ℓr

LEGEND



$l_r = \frac{l}{h}$; $x_r = \frac{x}{h}$; $d_r = \frac{d}{h}$;

MAGNETIC ANOMALY

DUE TO A DIPOLE