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DYNAMIC CALIBRATION OF THE PORT MORESBY RAPID - RUN MAGNETOGRAPH

by

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SUMMARY

The magnetograph has high sensitivity for this type of instrument, but it is not high enough satisfactorily to record pulsations with periods less than about thirty seconds. The resonance periods for H, D, and Z are respectively 2.1 seconds, 10.0 seconds, and 9.0 seconds. Response to short-period pulsations decreases rapidly below the resonance period. The damping on the three components is low and consequently near resonance there is a sharp increase in the ratio of dynamic to static sensitivity.

Methods are suggested whereby the equipment could be modified so that recording of geomagnetic pulsations would be improved.

1. INTRODUCTION

The La Cour rapid-run magnetograph records three components (H, D, and Z) of the Earth's magnetic field (Laursen, 1943). All records are made photographically with a paper speed of 180 mm/hr. An image consisting of many spots for each element is focussed on the recording drum but only one spot is selected by a lens carriage system, which is stepped across the record each revolution of the drum. Thus a record of the variations in the three components of the Earth's field is recorded for a whole day on one piece of photographic paper. Slow field changes are recorded in a dissected form and the absolute value of the field elements cannot be readily reduced from these.

In the Port Moresby equipment a 100 dyne-cm/oersted control magnet is mounted between the D and Z variometers to increase the D and Z sensitivities. A further modification consists of small copper damping blocks mounted on adjustable supports under the H and D magnets. All baseline mirrors have been removed.

Because rapid variations in the magnetic field are of low amplitude, the sensitivity of the rapid-run instrument has to be greater than that of the normal-run. The steady state response to a pulsation of given amplitude and frequency is a forced oscillation of the same frequency. The phase difference between the forced oscillation and the original pulsation depends upon the period of the pulsation and the natural period of the variometer. Because the suspended magnet systems have natural periods in the pulsation period range, resonance will occur when pulsations of this period occur. The sensitivity of the undamped system is therefore very dependent upon the period. An ideal apparatus would have sensitivity independent of the period; this can be achieved approximately by suitably damping the system. The usual method of damping is to use the induced currents (which oppose the motion) in a mass of conductor, copper or silver, placed near the magnet.

Some details of the response of the above type of variometer to pulsations are considered in the appendix.

2. CALIBRATION

A Hewlett Packard low frequency oscillator (Model 202A) was used to simulate the pulsation by passing the output of the oscillator at various frequencies and constant amplitude through a Helmholtz coil placed symmetrically about the magnet of the component being tested.

Except for the longer periods, which required longer recording time to obtain sufficient cycles for a reliable result, it was necessary to record each period setting for only a few minutes. Periods ranging from one second to 125 seconds were used.

The Hewlett Packard oscillator was calibrated using a seismic pen recorder with accurate timing and a chart speed of 60 mm/min. The accuracy of the frequency dial was found to be very good and no correction (considering the accuracy of this experiment) had to be made.

The damping ratios were determined by deflecting the spots and then recording the decaying oscillations.

It was suspected that the H magnet was affecting the D scale value. This was investigated and it was found that the D scale value was independent of H (a change in H of 75 gammas gave less than 0.04 gamma/mm change in D scale value).

3. DISCUSSION

Frequency response

Plate 1 shows the frequency response curves for H, D, and Z. Characteristic of the three graphs is the large ratio of dynamic to static sensitivity near resonance (static and dynamic sensitivity should be approximately equal at longer periods) and the rapid decrease in response at frequencies higher than the natural period of each. These particular instruments are therefore only useful in recording pulsations of periods from close to the natural period of the instruments to infinite period. The variometers must therefore be designed to have as small a natural period as practicable to obtain their maximum usefulness. The natural periods are :

H : 2.1 seconds

D : 10.0 seconds

Z : 9.0 seconds

The recording method (photographic paper moving at 180 mm/hr) sets a limit of period resolution of about 4 seconds, depending upon the spot focus and intensity.

Damping

The large ratio of dynamic to static sensitivity shown by the frequency response curves indicates that the damping of the three components is very small. The damping constant α , calculated from the logarithmic decrement and equation 6 (see appendix), for each component is given below :

H : 0.10

D : 0.075

Z : 0.12

Thellier (1957) concluded that a value of α of about 0.5 is the best practical value. With $\alpha = 0.5$, the frequency response is almost flat from infinite period to the natural period. All three Port Moresby components are operating in a far from ideal condition. Damping blocks underneath the H and D magnets are set as close as practicable, but extra damping could be achieved if a copper sheet were fixed above the magnets. In the case of Z, damping blocks could be placed on either side of the magnet to give more damping.

Phase

The phase lag, calculated from equation 7 (see appendix), of the recorded oscillation from the original pulsation depends upon both the period and the damping. The phase lag plotted against period for H, D, and Z is shown in Plate 2. As the damping is small the phase lag for periods greater than about twice the natural period is negligible.

Sensitivity

The static sensitivity of each component is as follows :

H : 1.2 gammas/mm

D : 0.3 gammas/mm (0.03 minute/mm)

Z : 0.4 gammas/mm

In Plate 3 the maximum H amplitude of pulsations as recorded at Brisbane is plotted against period (Wilkie, 1962). Brisbane has a geomagnetic latitude of 36°S and approximately the same longitude as Port Moresby. Because the amplitude of pulsations diminishes from the poles towards the equator (Jacobs & Sinno, 1960), we would expect the amplitude of pulsations to be less at Port Moresby, which is at 18.7°S geomagnetic latitude. Note that Plate 3 uses the maximum amplitude; the mean amplitude would be less than a fifth of these values. Assuming 0.5 mm to be the required mean amplitude for pulsations, the following sensitivities would be necessary for the indicated period ranges :

10 - 20 seconds : 0.2 gamma/mm

20 - 30 seconds : 0.4 gamma/mm

30 - 40 seconds : 0.8 gamma/mm

40 - 70 seconds : 2.6 gammas/mm

70 -100 seconds : 3.4 gammas/mm

The amplitude of D and Z pulsations is in general similar to or smaller than H. Thus we see that D and Z have sensitivities comparable with the above, but H is too insensitive to record the shorter period pulsations (excluding periods close to the natural period). D and Z sensitivities could be at least doubled to give satisfactory recording amplitudes.

The required H sensitivity could be achieved by using a supplementary magnet aligned east-west opposing the H variometer magnet field.

H scale value $S_H = \epsilon\left(\frac{k}{M} + f_E\right) \times 10^5$ gammas/mm (McComb, 1952).

where ϵ = 1/2R = optical scale value in radians/mm,
R = distance from variometer lens to drum in millimetres,
k = fibre torsion constant in dyne-cm/radian twist,
M = magnetic moment of the variometer magnet in dyne-cm/oersted,
f_E = the magnetic east field in gammas at the centre of the H variometer due to all magnets of the magnetograph except the H recording magnet.

R = 1400 k = 0.155 M = 4.6

If the above values and the present H scale value of 1.2 gammas per millimetre are substituted in the formula, f_E is found to be negligible. Using the above formula it can now be calculated that to increase the sensitivity to within the range 0.1 to 0.5 gamma per millimetre, f_E would have a value between -3000 and -2000 gammas. This could be achieved using a 10 dyne-cm/oersted magnet with its north end west (the recording magnet has its north end east), between 8.6 and 10 cm from the H variometer magnet (McComb, 1952, p.217). The supplementary magnet could give a field of about 10 gammas at the D variometer magnet, which would cause a small deflection. The D magnet could then be restored to the magnetic meridian by a small amount of fibre torsion.

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APPENDIX

Basic variometer dynamic response theory

Let D be the declination,
δ the pulsation amplitude, and
ω the angular frequency (pulsation).

Then $D = D_o + \delta \sin\omega t$, where D_o is the mean declination.

Let Θ be the angular displacement of the magnet from the mean meridian.

In the general case with any fibre torsional constant C and opposing field H_o, the differential equation for the motion of the magnet is :

$$I\frac{d^2\Theta}{dt^2} + F\frac{d\Theta}{dt} + (C + MH - MH_o)\Theta = MH\,\delta\sin\omega t \quad \ldots\ldots\ldots\ldots(1)$$

where I is the moment of inertia of the moving system about the axis of rotation,
M is the magnetic moment of the magnet, and
F is the damping constant that results from air friction and Foucault currents induced in surrounding conductors by the motion of the magnet.

We now define the following parameters of the system :

$$\omega_o^2 = \frac{C + MH - MH_o}{I} ; \quad \frac{F}{I} = 2\alpha\omega_o ; \quad \frac{MH}{C + MH - MH_o} = S ; \quad S\delta = a_o$$

where ω_o is the frequency at which the system oscillates for zero damping,
α is the damping constant,
S is the static sensitivity, and
a_o is the deviation of the magnet corresponding to a static change δ of the declination, i.e. it is the value Θ would take for a constant value of D equal to $D_o + \delta$.

Equation (1) becomes :

$$\frac{d^2\Theta}{dt^2} + 2\alpha\omega_o\frac{d\Theta}{dt} + \omega_o^2 = \omega_o^2 a_o \sin\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Equations for the magnet motion in the case of H and Z can be put in the same form.

First consider solutions of this differential equation that represent free natural oscillations of the system. The external force on the system is represented by $\omega_o^2 a_o \sin \omega t$ and the integral of

$$\frac{d^2\theta}{dt^2} + 2\alpha\omega_o\frac{d\theta}{dt} + \omega_o^2\theta = 0 \quad \ldots\ldots\ldots\ldots.(3)$$

will represent free or natural oscillations. These may be either the principal oscillation or any motion represented by any number of terms from the complementary function.

If we substitute e^{st} for θ and divide out by e^{st} we obtain a quadratic equation determining the quantity s. Let us denote the roots by s_1 and s_2 and we have :

$$\theta = C_1e^{s_1t} + C_2e^{s_2t}$$

where C_1 and C_2 are arbitrary constants.

The nature of the solution depends on the nature of the roots. For real roots we have aperiodic motion where the effect of damping is so great that it prevents the restoring forces from setting up oscillatory motions.

For the case of complex roots, say $-a \pm ib$,

$$\theta = e^{-at}(C_1\sin bt + C_2\cos bt).$$

Roots are $\omega_o\ (-\alpha \pm \sqrt{\alpha^2 - 1})$

Thus we have $a = \omega_o\alpha$ and $b = \omega_o\sqrt{1-\alpha^2}$ and the complementary function representing free or natural oscillation becomes

$$\theta = e^{-\omega_o\alpha t}\left(C_1\sin(\omega_o\sqrt{1-\alpha^2})t + C_2\cos(\omega_o\sqrt{1-\alpha^2})t\right)\ldots.(4)$$

Let $C_1 = Y\cos\omega_o\gamma$ and $C_2 = Y\sin\omega_o\gamma$, where the arbitrary constants are now Y and γ.

$$\theta = Ye^{-\omega_o\alpha t}\sin\omega_o\sqrt{1-\alpha^2}\,\gamma\,(t+\gamma)\ \ldots\ldots\ldots\ldots\ (5)$$

Y is called the initial amplitude and $\omega_o\sqrt{1-\alpha^2}\gamma$ the phase. The amplitude is given by

$$Ye^{-\omega_o\alpha t} \quad \ldots\ldots\ldots\ldots\ (6)$$

and decreases exponentially with increase in t. The term $e^{-\omega_o\alpha t}$ becomes negligible after approximately five seconds for the Port Moresby instruments.

Now consider the case when a physical torque is impressed upon the above system, e.g. sinusoidal magnetic pulsations; the differential equation of the motion is then represented by equation (2).

The complete solution is the sum of the complementary function and a particular integral. The particular integral represents the effects of the external impressed forces that produce the forces oscillations.

Any particular integral represents the forced vibrations, but one is more convenient than others.

Let $\Theta = A \cos \omega t + B \sin \omega t$ be this particular integral.

Substitute in equation (2) and solve for A and B.

$$B = \frac{\omega_0^2 a_o (\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + 4\alpha^2 \omega_0^2 \omega^2}$$

$$A = -\frac{2\omega_0^3 a_o \alpha \omega}{(\omega_0^2 - \omega^2)^2 + 4\alpha^2 \omega_0^2 \omega^2}$$

It is convenient to collect these terms under the symbols R, $\cos \epsilon$, and $\sin \epsilon$ so that

$$\frac{\omega_0^2 - \omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\alpha^2 \omega_0^2 \omega^2}} = \cos \epsilon, \qquad \frac{2\alpha \omega_0 \omega}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\alpha^2 \omega_0^2 \omega^2}} = \sin \epsilon$$

$$\epsilon = \tan^{-1} \frac{2\alpha \omega_0 \omega}{\omega_0^2 - \omega^2} \qquad \ldots\ldots\ldots\ldots\ldots(7)$$

$$R = \frac{\omega_0^2 a_o}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\alpha^2 \omega_0^2 \omega^2}}$$

So we can now write the particular integral

$\Theta = R (\cos \epsilon \sin \omega t - \sin \epsilon \cos \omega t)$, or

$\Theta = R \sin (\omega t - \epsilon)$

This expression represents the forced oscillation of the system due to the external periodic force. The forced oscillation is not in the same phase as the forcing oscillation, but lags behind by a definite amount ϵ.

The complete solution is then :

$$\Theta = Ye^{-\alpha\omega_0 t}\sin \omega_0 \sqrt{1-\alpha^2}\,(t+\gamma) + R\sin(\omega t - \epsilon)$$

If $D = D_o, \quad t < 0$

$D = D_o + \delta \sin \omega t, \quad t \geqslant 0$

then because Θ and $\frac{d\Theta}{dt}$ both vanish at $t = 0$, Y and γ can be found.

Neglecting the term $\sqrt{1-\alpha^2}$, they are given by

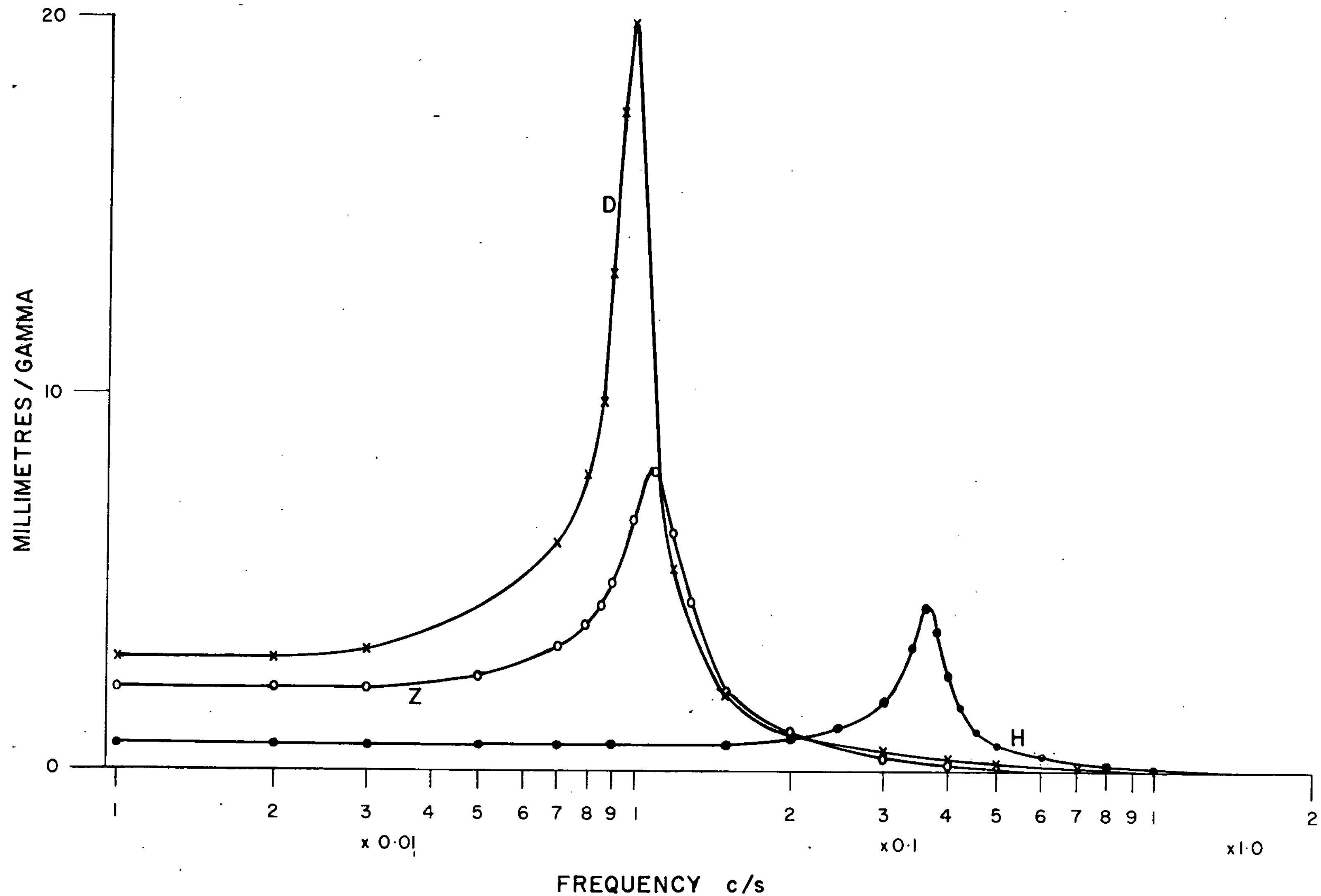
$$\cot \omega_0 \gamma = \alpha - \frac{\omega}{\omega_0} \cot \epsilon$$

$$\frac{Y}{R} = \sqrt{\sin^2 \epsilon + \left(\frac{\omega}{\omega_0} \cos\epsilon + \alpha \sin\epsilon\right)^2}$$

Plate 2 shows that ϵ is either almost 0 or 180° except near the resonance frequency, so usually

$$\frac{Y}{R} = \frac{\omega}{\omega_0}$$

The transient is thus smaller than the steady state response as long as the resonant frequency is higher than the applied frequency.



VARIOMETER FREQUENCY RESPONSE CURVES FOR H, D and Z

PLATE 1

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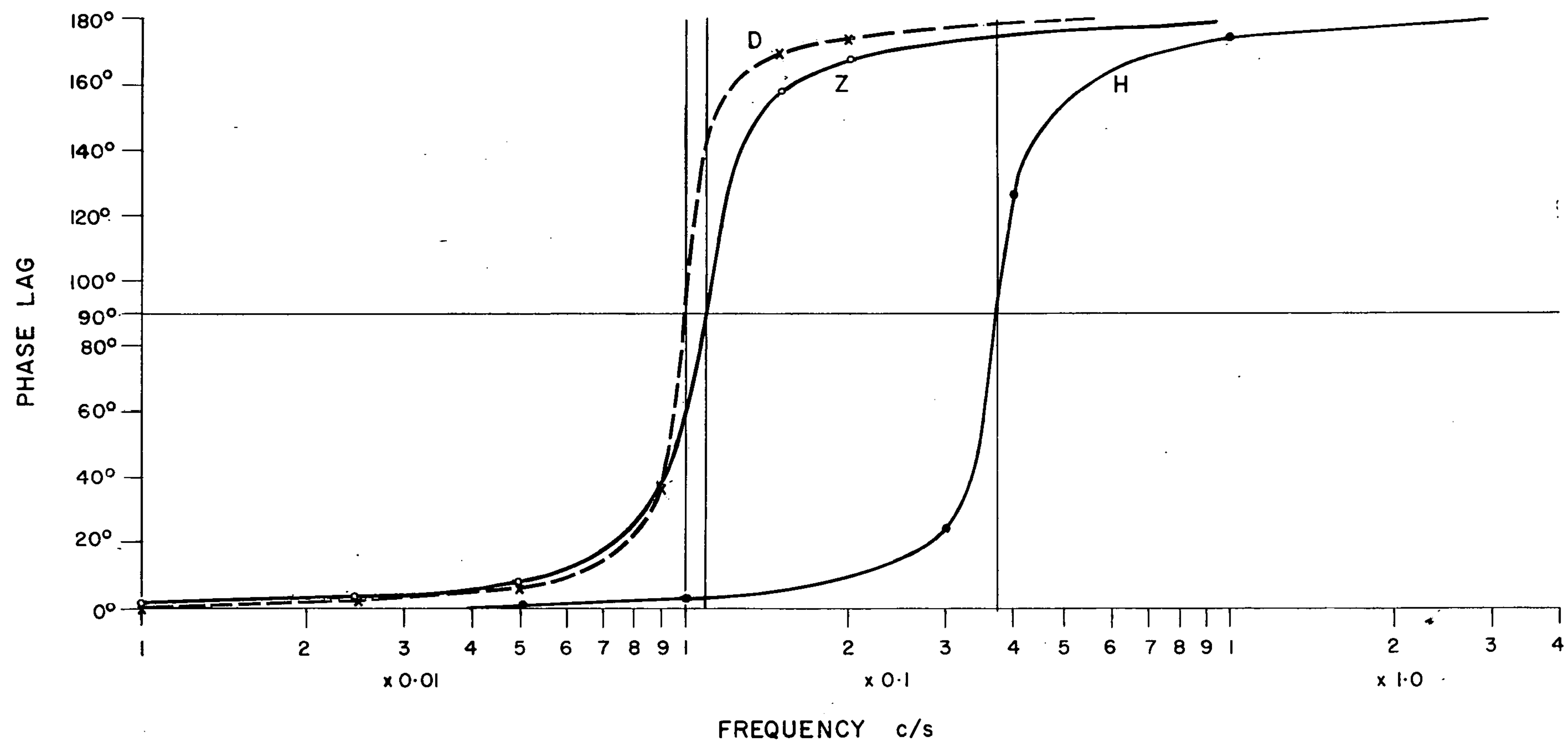

VARIOMETER PHASE LAG DIAGRAMS

PLATE 2

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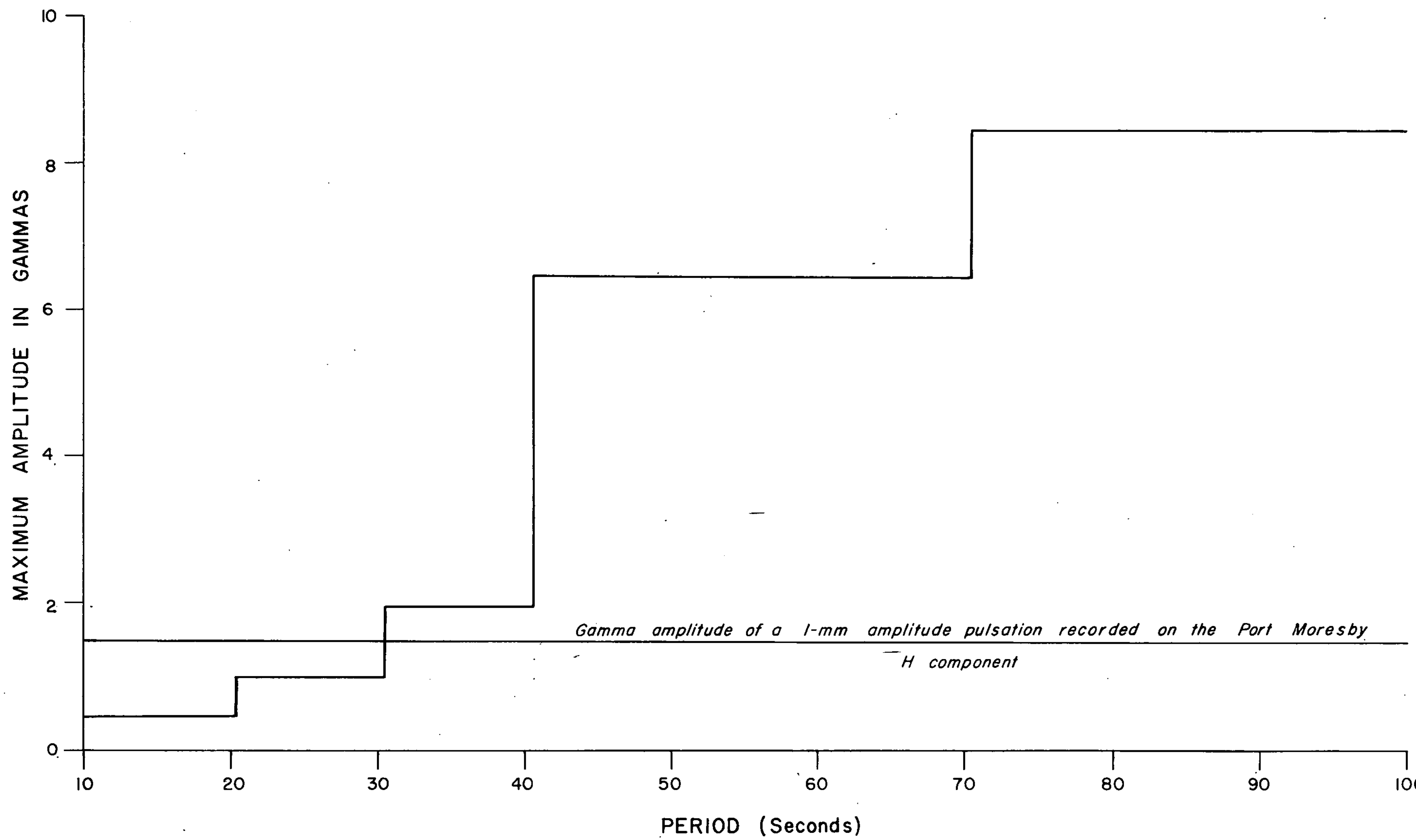
PLATE 3



MAXIMUM H AMPLITUDE VERSUS PERIOD
FOR PULSATIONS RECORDED AT BRISBANE

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