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A METHOD OF CALIBRATING SPRENGNETHER HORIZONTAL SEISMOGRAPHS AND COLUMBIA VERTICAL SEISMOGRAPHS USING A WILLMORE BRIDGE

by

I.E. BLACK

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SUMMARY

Trouble was encountered at Toolangi Observatory in attempting to calibrate the long-period seismographs with the Willmore Bridge because the method presented by Willmore (1959) assumes a knowledge of the seismometer dynamical constants, in particular the mass, centre of mass, and moment of inertia of the seismometer boom. These were not available for the Toolangi instruments.

The method of calibration presented in this record requires a knowledge of the mass and the centre of mass of the seismometer boom in the case of the horizontal seismometer, and only the centre of mass in the case of the vertical seismometer. The moment of inertia of the seismometer boom is not required in either case. The horizontal seismometer is easy to dismantle and the mass of its boom can be found by direct weighing. The centre of mass of the boom can be found by balancing it on a knife edge. The vertical seismometer creates a problem because this instrument cannot be dismantled without great difficulty. The centre of mass of its boom must be determined mathematically from a knowledge of its geometry and the densities of the materials.

1. INTRODUCTION

In 1962 a seismological observatory was established by the Bureau of Mineral Resources at Toolangi (latitude 37° 34' 17" S, longitude 145° 29' 26" E) to replace the Melbourne Seismological Observatory, which was forced to close down because of high background noise caused by increasing industrial activity.

The Toolangi Observatory consists of two Benioff horizontal seismometers and one Benioff vertical seismometer each having a free period of 1.0 second, two Sprengnether horizontal seismometers each having a free period of 15.0 seconds, and one Columbia vertical seismometer with a free period of 15.0 seconds. The short-period seismometers drive galvanometers with a free period of 0.2 second and the long-period seismometers drive galvanometers with a free-period of 90 seconds. Short-period recording is done via a Benioff 60-mm/min three-channel recorder and long-period recording via a Benioff 30-mm/min three-channel recorder.

To calibrate a seismograph the ratio of trace amplitude to ground acceleration must be determined for all frequencies of interest. The method of Willmore (1959) for the Galitzin type of instrument is to determine the amplitude of the trace when a sinusiodal current is fed into the seismometer coil. If the current is the real part of

$$\underline{i} = i_o\, e^{jwt}$$

the force imposed on the coil is the real part of

$$K\, \underline{i}$$

and the torque is the real part of

$$K\, \underline{i}\, l_c$$

where l_c is the distance from the hinge to the coil,
K is a real constant,
$j = \sqrt{-1}$, and all other complex numbers are underlined.

If the ground and frame of the instrument have an acceleration equal to the real part of

$$\underline{f} = f_o\, e^{jwt}$$

the torque imposed on the boom is

$$\int \underline{f}\, y\, dm = \underline{f} \int y\, dm = \underline{f}\, m\, l_m$$

where y is the distance from the hinge to the element of mass dm,
m is the total mass, and
l_m is the distance of the centre of mass from the hinge.

The ground acceleration simulated by the current $\underline{i}$ through the coil is found by equating these two torques:

$$\underline{f} = \underline{i}\, \frac{K\, l_c}{m\, l_m}$$

Of the quantities on the right hand side, l_c, m, and l_m can be measured directly. The following section describes Willmore's method of measuring $\underline{i}$ and K.

2. CALIBRATION OF THE SPRENGNETHER HORIZONTAL SEISMOGRAPH USING A WILLMORE BRIDGE

Under steady state conditions a horizontal sinusoidal ground motion may be represented by the real part of the complex quantity

$$\underline{x} = x_o e^{jwt}$$

where x_o is a real constant,
w is the angular frequency of the motion, and
t is time.

If a seismometer of the hinge type is being driven by this ground motion and steady state conditions have been reached, the angle that the seismometer boom makes with its rest position at time t can be represented by the real part of the complex quantity

$$\underline{\theta} = \underline{\theta}o\, e^{jwt}$$

where $\underline{\theta}o$ is a complex constant since the motion of the boom differs from that of the ground by a constant phase angle.

The differential equation of the motion of the seismometer boom can be written (Bullen, 1947)

$$I\ddot{\underline{\theta}} + \lambda\dot{\theta} + mgl_m\phi\underline{\theta} = -l_m m\ddot{\underline{x}} \qquad \ldots 1$$

where I = the moment of inertia of the seismometer boom about the hinge,
m = the mass of the seismometer boom,
l_m = the distance of the centre of mass of the seismometer boom from hinge,
ϕ = the angle that the seismometer boom makes with the horizontal,
λ = the seismometer damping constant, and
g = the acceleration due to gravity.

This equation is of the same form as that relating condenser charge to applied e.m.f. in the simple resistance, capacitance, inductance series circuit under steady state conditions. The electrical equation is

$$L\ddot{\underline{q}} + R\dot{\underline{q}} + \frac{\underline{q}}{C} = \underline{E} \qquad \ldots 2$$

where the e.m.f. is represented by the real part of the complex quantity

$$\underline{E} = Eo\, e^{jwt}$$

Eo being a real constant, w the angular frequency of the e.m.f., and t the time. The condenser charge at time t is represented by the real part of the complex quantity

$$\underline{q} = \underline{q}o\, e^{jwt}$$

where $\underline{q}o$ is a complex constant since the condenser charge differs from the e.m.f. by a constant phase angle.

The complex impedance of the electrical circuit is

$$\underline{Z} = R + j(wL - \frac{1}{wC})$$

Therefore by analogy we can define complex mechanical impedance from equation (1) as

$$\underline{Z}m = \lambda + j\left(wI - \frac{mgl_m \phi}{w}\right)$$

The calibration takes place in four stages.

Stage 1 (Plate 2a)

The seismometer boom is clamped to the seismometer frame and the seismometer coils are connected to the bridge. The bridge is balanced.

Under this condition a current flows through the seismometer coils producing a force on the coils since they lie in a magnetic field. Let the bridge drive voltage be denoted by the real part of

$$\underline{E}_1 = E_{10}\, e^{jwt}$$

where E_{10} is a real constant and t is the time. Let the current through the coils be denoted by the real part of the complex quantity

$$\underline{i}_{c1} = \underline{i}_{c10}\, e^{jwt}$$

where $\underline{i}_{c10}$ is a complex constant.

The force on the coils is given by the real part of the complex quantity $\underline{p} = K\underline{i}_{c1}$ ... 3

where K is the constant of proportionality between the force and the current. K is real since the force is in phase with the current.

Stage 2 (Plate 2b)

Unclamp the seismometer boom keeping the circuit elements the same. Owing to the fact that the seismometer boom is now moving, a different current will be flowing through the coils than in Stage 1. Let us denote this current by the real part of the complex quantity

$$\underline{i}_{c2} = \underline{i}_{c20}\, e^{jwt}$$

where $\underline{i}_{c20}$ is a complex constant.

The force on the coils at time t will therefore be given by the real part of $K\underline{i}_{c2}$ and the torque by the real part of $K\underline{i}_{c2}\, l_c$, where l_c is the distance of the axis of the coils from the hinge.

Let the displacement of the seismometer boom from its rest position at time t be denoted by the real part of

$$\underline{\theta}_2 = \underline{\theta}_{20}\, e^{jwt}$$

where $\underline{\theta}_{20}$ is a complex constant.

We can therefore write the equation of motion of the boom in complex form as

$$I\ddot{\underline{\theta}}_2 + \lambda\dot{\underline{\theta}}_2 + mg\, l_m\, \phi\, \underline{\theta}_2 = K\, \underline{i}_{c2}\, l_c \qquad \text{... 4}$$

In the electrical analogy, current is given by the real part of

$$\dot{\underline{q}} = \frac{\underline{E}}{\underline{Z}} \text{ by Ohm's law}$$

By analogy we obtain from equation (4) that the angular velocity of the seismometer boom is given by the real part of

$$\dot{\underline{\theta}}_2 = \frac{K\, l_c\, \underline{i}_{c2}}{\underline{Z}_m} \qquad \ldots 5$$

Since the seismometer coils are moving in a magnetic field they are generating en e.m.f. It can be shown that if K is defined by equation (3) then this e.m.f. is given by the real part of

$$\underline{e}_{c2} = -Kl_c\, \dot{\underline{\theta}}_2$$

$$= \frac{-K^2\, l_c^{\,2}\, \underline{i}_{c2}}{\underline{Z}_m} \qquad \ldots 6$$

Stage 3 (Plate 2c)

The conditions are as in Stage 2 but with the bridge drive voltage applied across R_R. Let the current through the coils in this case be $\underline{i}_{c3}$. Then by analogy with equations (5) and (6), the angular velocity of the boom is given by the real part of

$$\dot{\underline{\theta}}_3 = \frac{K\, l_c\, \underline{i}_{c3}}{\underline{Z}_m} \qquad \ldots 7$$

and the e.m.f. generated by the moving coils by the real part of

$$\underline{e}_{c3} = \frac{-K^2\, l_c^{\,2}\, \underline{i}_{c3}}{\underline{Z}_m} \qquad \ldots 8$$

Let the galvanometer deflection in Stages 2 and 3 at time t be given by the real part of

$$\underline{\xi}_2 = \underline{\xi}_{20}\, e^{jwt} \quad \text{and} \quad \underline{\xi}_3 = \underline{\xi}_{30}\, e^{jwt} \quad \text{respectively.}$$

Because the galvanometer deflection is a linear function of the applied e.m.f. for a given frequency, we have at time t

$$\frac{\underline{e}_{c3} + \underline{e}_{s3}}{\underline{\xi}_3} = \frac{\underline{e}_{c2}}{\underline{\xi}_2} \qquad \ldots 9$$

where the e.m.f. across R_R is given by the real part of $\underline{e}_{s3}$.

To obtain this equation we neglect the effect of R_B and R_D because these are large in comparison with R_C. We also ignore the effect of R_R because this is small in comparison with R_C.

By substitution from equations (6) and (8), equation (9) can be written

$$\underline{e}_{s3}\,\frac{\underline{\xi}_2}{\underline{\xi}_3} = -\,\underline{e}_{c3}\,\frac{\underline{\xi}_2}{\underline{\xi}_3} + \underline{e}_{c2}$$

$$= K^2\,l_c^{\,2}\,\frac{\underline{i}_{c3}}{\underline{Z}_m}\,\frac{\underline{\xi}_2}{\underline{\xi}_3} - \frac{K^2\,l_c^{\,2}\,\underline{i}_{c2}}{\underline{Z}_m}$$

Therefore $$\underline{e}_{s3}\,\frac{\underline{\xi}_2}{\underline{\xi}_3} = -\,\frac{K^2\,l_c^{\,2}}{\underline{Z}_m}\left(\underline{i}_{c2} - \underline{i}_{c3}\,\frac{\underline{\xi}_2}{\underline{\xi}_3}\right) \qquad \ldots\ 10$$

Since the bridge was balanced in Stage 1, the deflection of the galvanometer in Stage 2 will be due to the difference in the seismometer coil currents, namely the real part of $\underline{i}_{c2} - \underline{i}_{c1}$, again neglecting the effect of R_B and R_D.

Since the galvanometer deflection is a linear function of the current flowing through it, we have at time t

$$\frac{\underline{i}_{c2} - \underline{i}_{c1}}{\underline{\xi}_2} = \frac{\underline{i}_{c3}}{\underline{\xi}_3} \qquad \ldots\ 11$$

Therefore equation (10) reduces to

$$\underline{e}_{s3}\,\frac{\underline{\xi}_2}{\underline{\xi}_3} = \frac{-K^2\,l_c^{\,2}\,\underline{i}_{c1}}{\underline{Z}_m} \qquad \ldots\ 12$$

Now, from Plate 2c,

$$\underline{e}_{s3} = \frac{\underline{E}_s\,R_R}{R_s + R_R}$$

where the e.m.f. applied across R_s and R_R at time t is the real part of

$$\underline{E}_s = E_{so}\,e^{jwt}$$

Here E_{so} is a real constant.

Also we have, from Plate 2a, with the bridge balanced

$$\underline{i}_{c1} = \frac{\underline{E}_1}{\underline{Z}_c + R_B}$$

where the e.m.f. applied across the bridge at time t is the real part of

$$\underline{E}_1 = E_{10}\,e^{jwt}$$

Here E_{10} is a real constant and $\underline{Z}_c$ is the complex electrical impedance of the seismometer coils.

Substitution for $\underline{e}_{s3}$ and $\underline{i}_{c1}$ in equation (12) yields

$$\frac{\underline{E}_s R_R}{R_s + R_R} \quad \frac{\underline{\xi}_2}{\underline{\xi}_3} = \frac{-K^2 l_c^2 \underline{E}_1}{\underline{Z}_m (\underline{Z}_c + R_B)}$$

Taking the moduli of both sides of this equation and using the fact that $wL_c \ll R_c$, where R_c is the seismometer coil resistance and L_c is the seismometer coil inductance, we obtain

$$\left| \frac{\underline{E}_s R_R}{R_s + R_R} \quad \frac{\underline{\xi}_2}{\underline{\xi}_3} \right| = \frac{-K^2 l_c^2 \underline{E}_1}{(R_c + R_B) \left\{ I^2 \left(w - \frac{w_m^2}{w} \right)^2 + \lambda^2 \right\}^{\frac{1}{2}}} \qquad \dots 13$$

where w_m is the natural angular frequency of the seismometer.

i.e., $$w_m = \sqrt{\frac{mg\, l_m \phi}{I}}$$

or $$T_m = 2\pi \sqrt{\frac{I}{mg\, l_m \phi}} = \frac{1}{f_m}$$

where T_m is the natural period of the seismometer and f_m is its natural frequency.

Equation (13) can now be written

$$\left| \frac{\underline{E}_s R_R}{R_s + R_R} \quad \frac{\underline{\xi}_2}{\underline{\xi}_3} \right| = \frac{K^2 l_c^2 |\underline{E}_1|}{(R_B + R_c) \left[(2\pi I)^2 \left(f - \frac{f_m^2}{f} \right)^2 + \lambda^2 \right]^{\frac{1}{2}}}$$

i.e., $$\frac{|\underline{\xi}_3|}{|\underline{\xi}_2|} = \frac{|\underline{E}_s| R_R \left[(2\pi I)^2 \left(f - \frac{f_m^2}{f} \right) + \lambda^2 \right]^{\frac{1}{2}} (R_B + R_c)}{(R_s + R_R)\ K^2 l_c^2 |\underline{E}_1|} \qquad \dots 14$$

This expression has a minimum where $f = f_m$. Therefore if we plot $|\underline{\xi}_3| / |\underline{\xi}_2|$ against frequency we get a curve with a minimum where $f = f_m$. To obtain the ratio $\frac{|\underline{\xi}_3|}{|\underline{\xi}_2|}$ we compare the amplitudes of deflection in Stages 2 and 3.

If during the calibration we make $\underline{E}_s = \underline{E}_1$, we obtain

$$\left| \frac{\underline{\xi}_3}{\underline{\xi}_2} \right|^2 = \left\{ \frac{R_R (R_B + R_c)}{R_s + R_R} \right\}^2 \frac{1}{K^4 l_c^4} \left[(2\pi I)^2 \left(f - \frac{f_m^2}{f} \right)^2 + \lambda^2 \right]$$

Therefore if we plot $\left|\frac{\xi_3}{\xi_2}\right|^2$ against $\left(f - \frac{f_m^2}{f}\right)$ we get a straight line with gradient

$$\left(\frac{2\pi I \quad R_R (R_B + R_c)}{R_s + R_R}\right)^2 \frac{1}{K^4 \; l_c^4}$$

which we designate by G^2.

Therefore $$\frac{l_c^2 \, K^2}{I} = \frac{2\pi \, R_R \, (R_B + R_c)}{G \, (R_s + R_R)} \qquad \ldots 15$$

Thus K can be determined if we know I.

Stage 4

Willmore's method for a Galitzin type of seismometer requires a knowledge of the moment of inertia of the boom (I) to derive K from equation (15). This section describes a more convenient method of deriving I.

The free period of the seismometer is given by

$$T_m = 2\pi\sqrt{\frac{I}{m \, g \, l_m \, \phi}} \qquad \ldots 16$$

where ϕ is the angle of tilt of the boom below the horizontal. T_m can be determined from the minimum condition of equation (14), but both I and ϕ are unknown. However, changes in ϕ can be measured either by knowing the pitch of the footscrew or by using a calibrated spirit level. Writing ϕ as the sum of the original angle ϕ_o and a measurable departure from this, i.e.

$$\phi = \phi_o + \phi'$$

equation (16) can be written

$$\frac{4\pi^2}{T_m^2 \, m \, g \, l_m} = \frac{\phi_o}{I} + \frac{\phi'}{I} \qquad \ldots 17$$

Now if the left-hand-side is plotted against ϕ' for a number of different tilt angles, the slope of the resulting straight line is 1/I.

K can now be determined from equation (15). Substitution of this value, the measured values of l_c, m, and l_m, and

$$\frac{|E_1|}{R_B + R_C} \quad \text{for} \quad |i_{c1}|$$

in equation (1) gives the ground acceleration corresponding to the measured deflection $|\xi_2|$.

Because the simulated acceleration is sinusoidal, the velocity and displacement sensitivities are determined from the acceleration sensitivities by dividing by w and w^2 respectively.

3. CALIBRATION OF THE COLUMBIA VERTICAL SEISMOGRAPH USING A WILLMORE BRIDGE

If t is the time elapsed since the displacement of the ground was last ζ_o then the displacement of the ground at time t can be represented by the real part of the complex quantity

$$\underline{\zeta} = \zeta_o \, e^{jwt}$$

By letting the real part of the complex quantity

$$\underline{\theta} = \underline{\theta}_o \, e^{jwt}$$

denote the angular displacement of the boom from its rest position at time t, we can write the differential equation of the motion of the boom in complex form as (Bullen, 1947)

$$I\ddot{\underline{\theta}} + \lambda\dot{\underline{\theta}} + I \, w_m^{\,2} \, \underline{\theta} = -l_m \, m \, \ddot{\zeta}$$

where I = the moment of inertia of the seismometer boom about the hinge,

λ = the damping constant,

w_m = the natural angular velocity of the seismometer boom,

l_m = the distance of the centre of mass of the boom from the hinge, and

m = the mass of the boom.

This equation is of the same form as that for the horizontal seismometer and so the theory of stages 1, 2, and 3 hold for the vertical case. Thus we can write the maximum simulated ground acceleration for the vertical case as

$$f = \frac{|E|}{R_c + R_B} \; \frac{K \, l_c}{m \, l_m} \qquad \ldots \; 18$$

where K = the constant of proportionality between the current flowing in the coils and the force acting on them,

$|E_1|$ = the peak bridge drive voltage, and

l_c = the distance of the axis of the coils from the hinge.

To determine $K \, l_c$

Balance the bridge for direct current with the seismometer boom clamped and the seismic galvanometer as the balance detecting galvanometer. Switch off the current. Unclamp the seismometer and place a small known mass m_1 on the seismometer boom at a distance l_1 from the vertical through the hinge. Upon lifting this mass a deflection d_1 will be produced on the recorder. This deflection is related to the mass m_1, by the equation

$$m_1 \, g \, l_1 = kd_1 \qquad \ldots 19$$

where k is a constant.

Now switch on the bridge driving voltage and allow the seismometer to come to rest. Measure the voltage across its terminals using a vacuum tube voltmeter. Calculate the d.c. current i_D flowing through the seismometer from the balance conditions. When the current is switched off, the seismometer will return to its rest position generating a current which will cause the galvanometer to produce a deflection d_2 on the recorder. This deflection is related to the current by the equation

$$l_c \, K \, i_D = kd_2 \qquad \ldots 20$$

From equations (2) and (3)

$$K \, l_c = \frac{m_1 \, g \, l_1 \, d_2}{i_D \, d_1} \qquad \ldots 21$$

Substituting this in equation (18), the simulated acceleration becomes

$$f = \frac{E}{R_B + R_C} \cdot \frac{d_2 \, m_1 \, l_1 \, g}{d_1 \, m \, i_D \, l_m} \qquad \ldots 22$$

Calibration of the vertical seismometer therefore consists simply of determing the quantity

$$\frac{g \, d_2 \, m_1 \, l_2}{i_D \, d_1 \, m \, l_m}$$

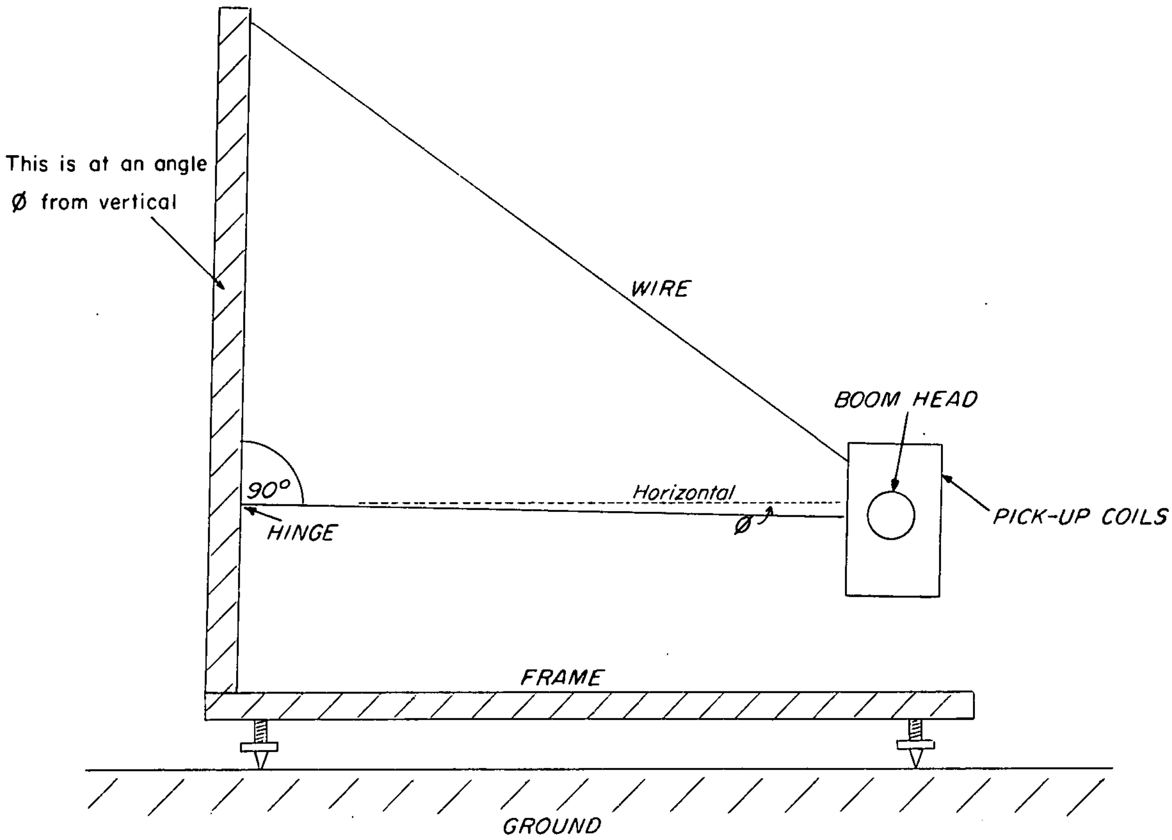
and the galvanometer deflection ξ corresponding to an alternating voltage E. The acceleration sensitivity is then f/ξ, where f is given by equation (22). The velocity and displacement sensitivities are determined by dividing by w and w^2 respectively.

4. REFERENCES.

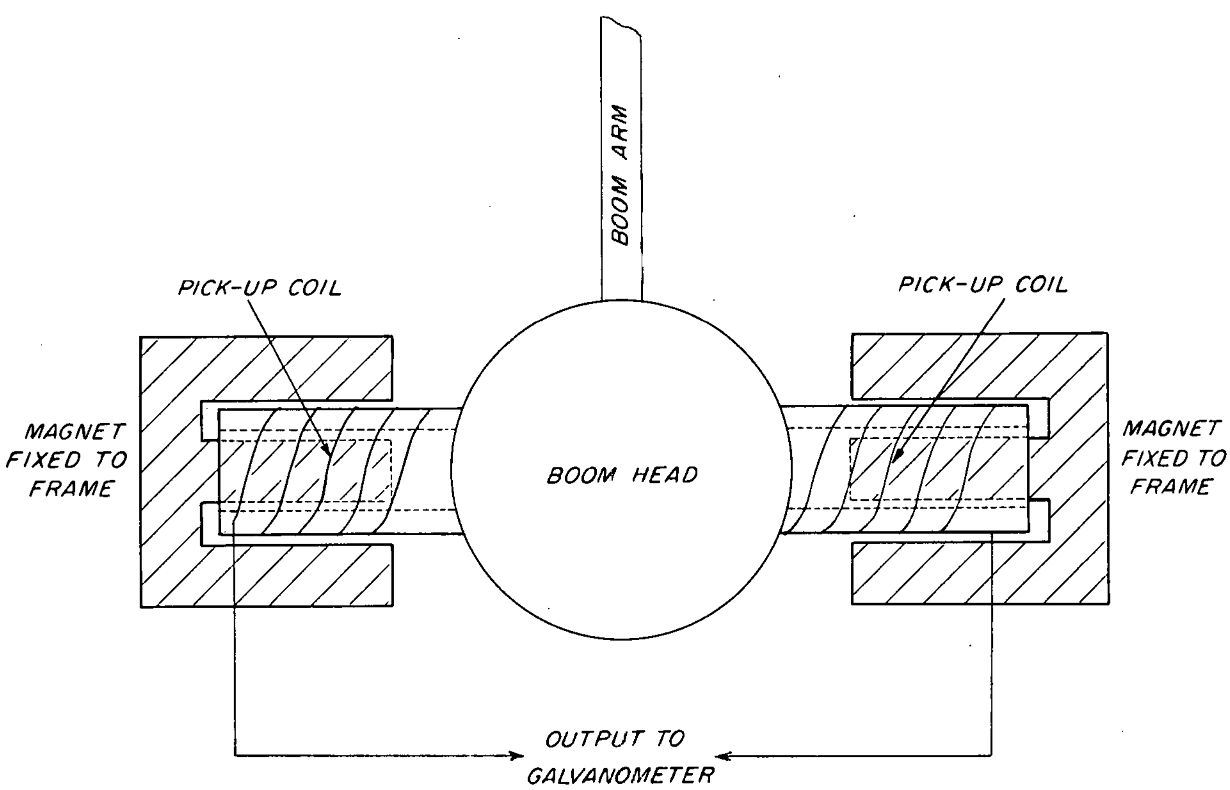
WILLMORE, P. L. 1959 The application of the Maxwell impedance bridge to the calibration of electro-magnetic seismographs. Bull. Seism. Soc. Amer., 49 (1), 99 - 114.

BULLEN, K. E. 1947 INTRODUCTION TO THE THEORY OF SEISMOLOGY. Cambridge University Press.

PLATE I



(a) PRINCIPLES OF THE SPRENGNETHER HORIZONTAL SEISMOMETER



(b) METHOD OF
SIGNAL PICK-UP IN THE SPRENGNETHER HORIZONTAL SEISMOMETER

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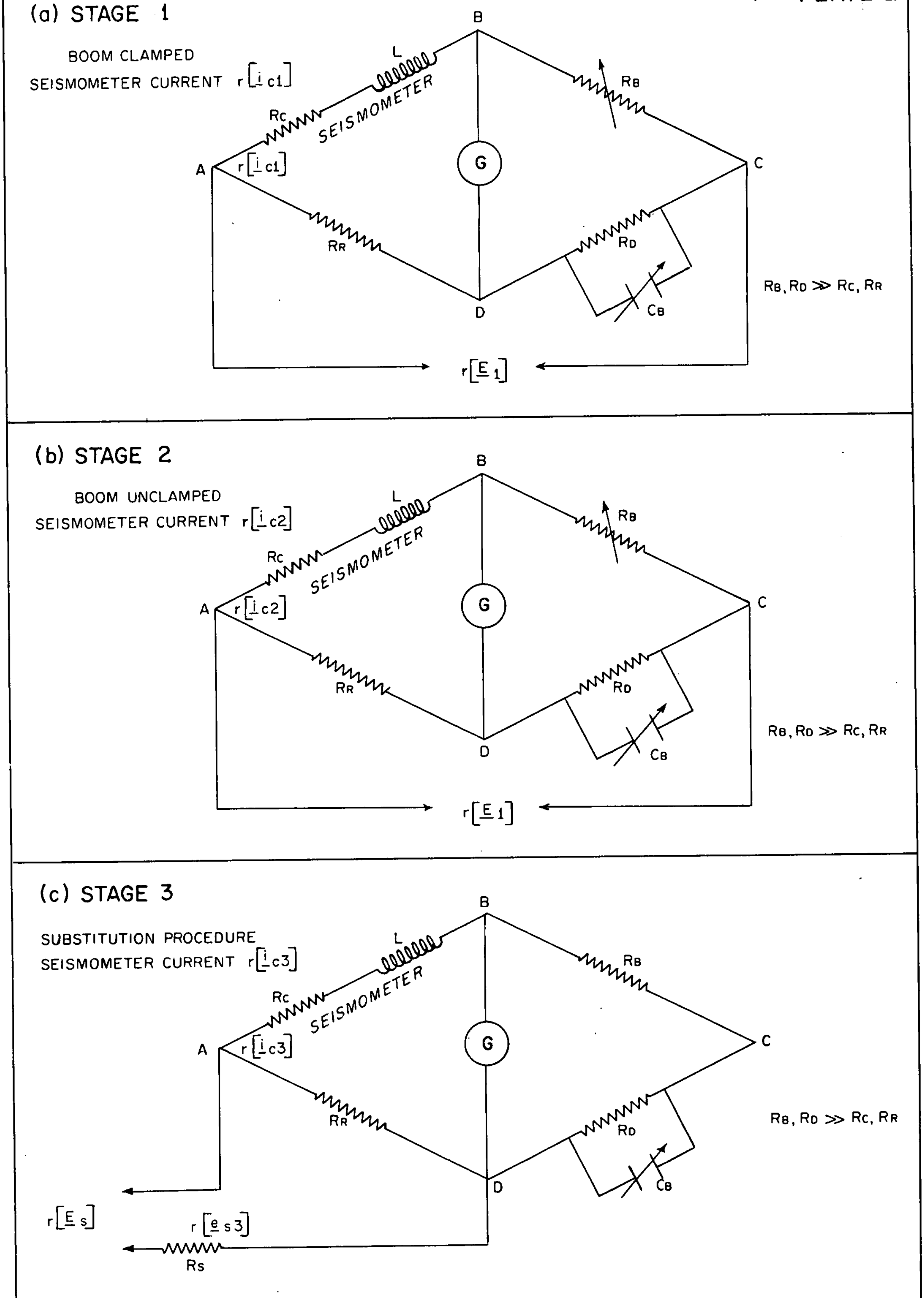
PLATE 2



CIRCUIT DIAGRAMS OF THE WILLMORE BRIDGE

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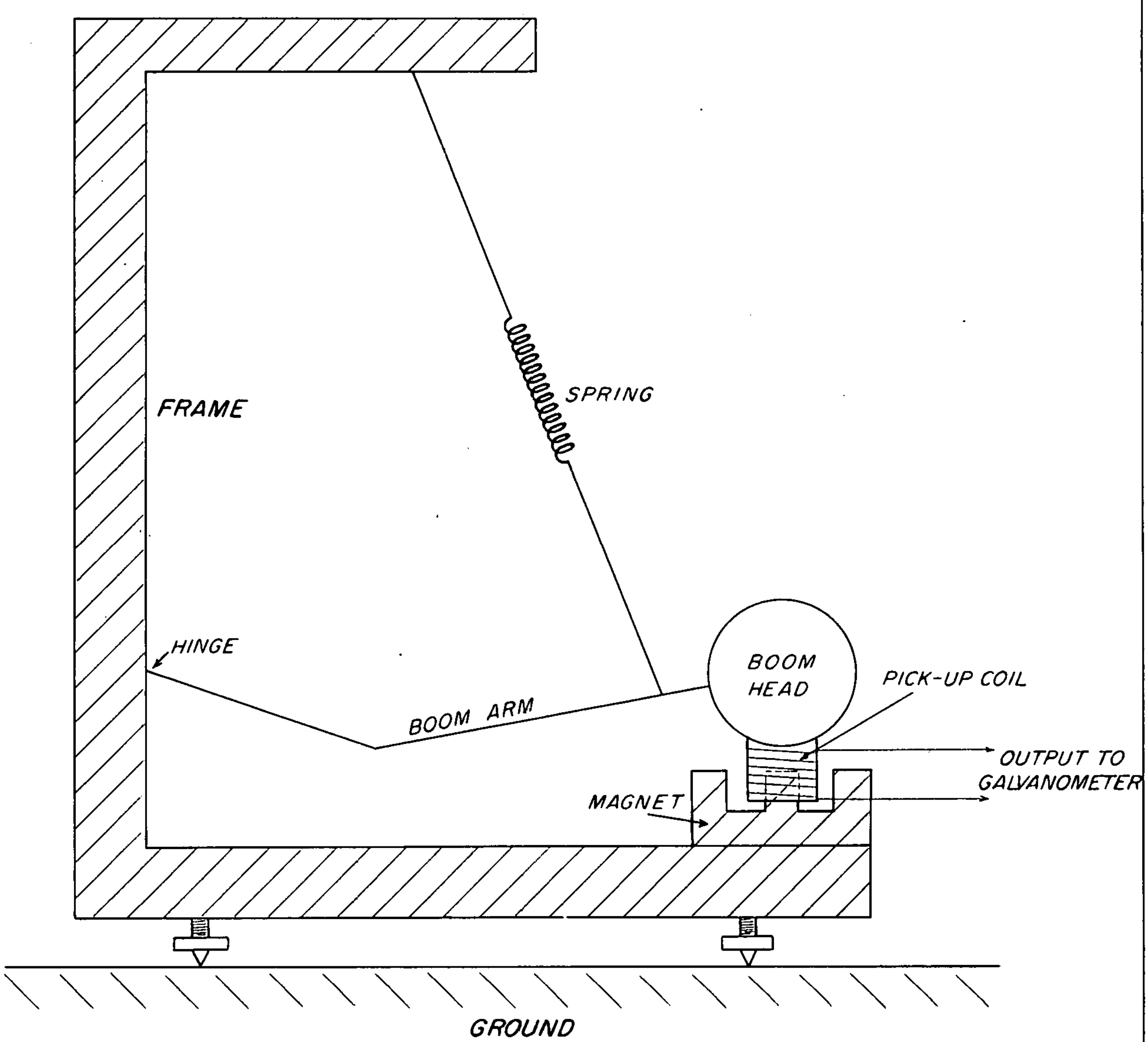

PRINCIPLES OF THE COLUMBIA VERTICAL SEISMOMETER

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