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CRITERIA FOR THE DESIGN AND EVALUATION OF MATHEMATICAL MODELS FOR THE ALICE SPRINGS TOWN BASIN

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T. Quinlan

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SUMMARY

The Alice Springs Town Basin is a small alluvial basin, which has provided the town with water. A model of the basin must simulate its geological and hydrological features if it is to be useful. If the available solutions to the equations of groundwater flow are used, the geometry of the model needs to be simple, and to consist of a combination of rectangular and circular elements.

The technique of stepwise regression is proposed as the basis for quantitative methods to analyse and evaluate proposed models. The simplest is that of a homogeneous unconfined aquifer of infinite areal extent with a coefficient of transmissibility between 18,000 and 37,000 Imperial gallons per day per foot, and a storage coefficient between 0.01 and 0.256.

A design for a model to simulate the effects of impermeable boundaries is presented.

LIST OF SYMBOLS USED

Symbol		Dimension
s	= Drawdown in a bore in response to pumping, in feet	L
T	= Coefficient of transmissibility, in Imperial gallons per day per foot	L^2/T
S	= Coefficient of storage	-
t	= Time elapsed since commencement of pumping, in days	T
Q	= Discharge of pumping bore, in Imperial gallons per hour	L^3
r	= Radial distance of the observation bore from the pumping bore, in feet	L
Bo	= Regression constant	-
B_1, B_2, B_3	= Regression coefficients	-

INTRODUCTION

The Alice Springs Town Basin is a small alluvial basin with a maximum depth of about 75 feet and a surface area of roughly 3 square miles. Groundwater was withdrawn from the saturated alluvium for the town supply; until January 1964 this was the town's only source of water. Late in 1964, pumping from the basin virtually ceased when groundwater was obtained from an alternative source, in Palaeozoic sandstone. At that date about 250 million gallons was used annually, of which approximately 200 million was withdrawn by the town supply bores.

From 1953 to 1964 more water was pumped from the basin than was added to it by recharge from the Todd River, and the volume of water in storage fell from 650 to 360 million gallons.

Between 1943 and 1964 information on the occurrence of groundwater and the hydrology of the basin was collected by residents of the town, the Australian Army, the Commonwealth Department of Works, the Water Resources and Animal Industry Branches and the Resident Geologists of the Northern Territory Administration. This information has been used in discussions of the hydrology (Ref. 1, 2, 3) and the geology (Ref. 3) of the basin.

The analysis and interpretation of water level measurements and pumping test data have not yet provided a satisfactory model which could be used to describe and predict the hydraulic behaviour of the basin. Quantitative methods are necessary to determine the efficiency of models, and to provide a basis for comparison between them.

THE DESIGN OF MATHEMATICAL MODELS

A mathematical model consists of one or more equations which can be used to explain and to predict the hydraulic behaviour of a basin.

Many elegant solutions for the equations of groundwater flow have been derived (Ref. 4). These describe the pattern of drawdown about a bore which is withdrawing water from aquifers of simple geometrical shape under one of a number of hydraulic conditions. If these solutions are to be used, the geometry of the model needs to be simple, and to consist of a combination of rectangular and circular elements.

More than one geometrical design could be used to approximate all but the simplest of geological boundaries. Again, some or all of the boundary conditions for more than one of these solutions could be applicable in particular cases. It follows that a choice must be made between more than one design for the model.

An overriding restriction on the design of a model is the level of sophistication at which analysis could be undertaken. In the past this has been set by the availability of type curves for particular solutions of the equations of groundwater flow. The advent of digital computers has replaced this restriction by ones associated with quantitative methods of analysis.

EVALUATION OF MATHEMATICAL MODELS

The differences between possible designs and in the results obtained from them can be small. It is expected that quantitative methods could be useful to evaluate each design and its efficiency. A suitable method for some situations is that of the analysis of regression.

One such case is that of a pumping bore in the ideal aquifer, which is homogeneous and of infinite areal extent. The solution of the equation of groundwater flow in this case uses the Well Function (Ref. 5):

$$s = -\frac{Q}{4\pi T}\int_{u}^{\infty}\frac{e^{-u}}{u}\,du \qquad \text{—— (i)}$$

$$\because\ u = \frac{r^2 S}{4Tt}$$

which, for the units used, can be written in the form:

$$s = \frac{1.91\,Q}{T}\left\{-0.5772 - \log_e u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \cdots\right\} \quad \text{— (ii)}$$

$$\because\ u = \frac{1.56\,r^2 S}{T\,t}$$

Rearrangement of the terms yields the form:

$$\frac{s}{Q} = \frac{1.91}{T}\left\{-\left(0.5772 + \log_e\left(\frac{S}{T}\right)\right) - \log_e\left(\frac{1.56\,r^2}{t}\right) + \left(\frac{S}{T}\right)\frac{1.56\,r^2}{t} - \frac{1}{2.2!}\left(\frac{S}{T}\right)^2\left(\frac{1.56\,r^2}{t}\right)^2 + \frac{1}{3.3!}\left(\frac{S}{T}\right)^3\left(\frac{1.56\,r^2}{t}\right)^3 - \cdots\right\} \quad \text{— (iii)}$$

A regression equation in the observed variables s and t is:

$$\frac{s}{Q} = \left\{B_0 + B_1\log_e\left(\frac{1.56\,r^2}{t}\right) + B_2\left(\frac{1.56\,r^2}{t}\right) - B_3\left(\frac{1.56\,r^2}{t}\right)^2 + B_4\left(\frac{1.56\,r^2}{t}\right)^3 - \cdots\right\} \quad \text{—(iv)}$$

The relations which exist between the aquifer constants, S and T, and the coefficients in the regression equation can be obtained by equating the coefficients of equations (ii) and (iv). These may be written in the form:

$$T = 1.91 \frac{Q}{B_1} \qquad \text{—— (v)}$$

$$\left\{ \left(\frac{S}{T}\right) + \left(\frac{S}{T}\right)^2 + \left(\frac{S}{T}\right)^3 + \cdots \right\} = \frac{1}{2} \left\{ \frac{B_2}{B_1} + e^{-(B_0/B_1 + 0.5772)} + 2.2! \frac{B_3}{B_1} + 3.3! \frac{B_4}{B_1} + \cdots \right\} \qquad \text{—— (vi)}$$

A satisfactory solution to (vi) for $(\frac{S}{T})$ can be obtained from (vi) by iterative methods, which in conjunction with (v) yields a value for S.

In addition, the analysis yields two statistical measures which can be used to evaluate the model. These are confidence limits for the aquifer constants and the coefficient of determination. For a given degree of confidence, the confidence limits are the most probable maximum and minimum values for the aquifer constants. These can be converted into confidence limits for the coefficients S and T by the use of equations (v) and (vi).

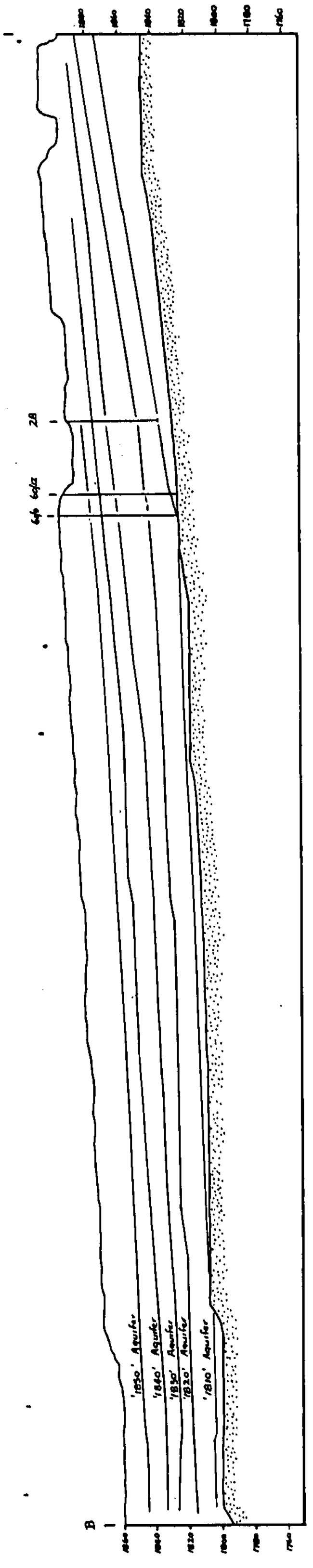
The coefficient of determination, in this context, is the percentage of the observed drawdown which is accounted for by the regression equation.

MODELS OF THE ALICE SPRINGS TOWN BASIN

A summary of the relevant geological and hydrological facts and inferences concerning the basin (Ref. 1, 2, 3) is presented to provide a basis for a discussion of two models which could be used to predict the behaviour of the basin. The design of the first model is that of an ideal unconfined aquifer, and the second is of a strip aquifer consisting of a number of arcuate segments.

Geological Features

1. Alluvium, regarded as Quaternary in age, is preserved in the Town Basin; it rests unconformably on Precambrian sedimentary, igneous and metamorphic rocks. The alluvium consists of a mixture of gravel, sand, silt, and clay, and is considered to be the product of deposition in the channels, and on the levees, deltas and flood plains of the ancestral Todd River.



LONGITUDINAL PROFILE

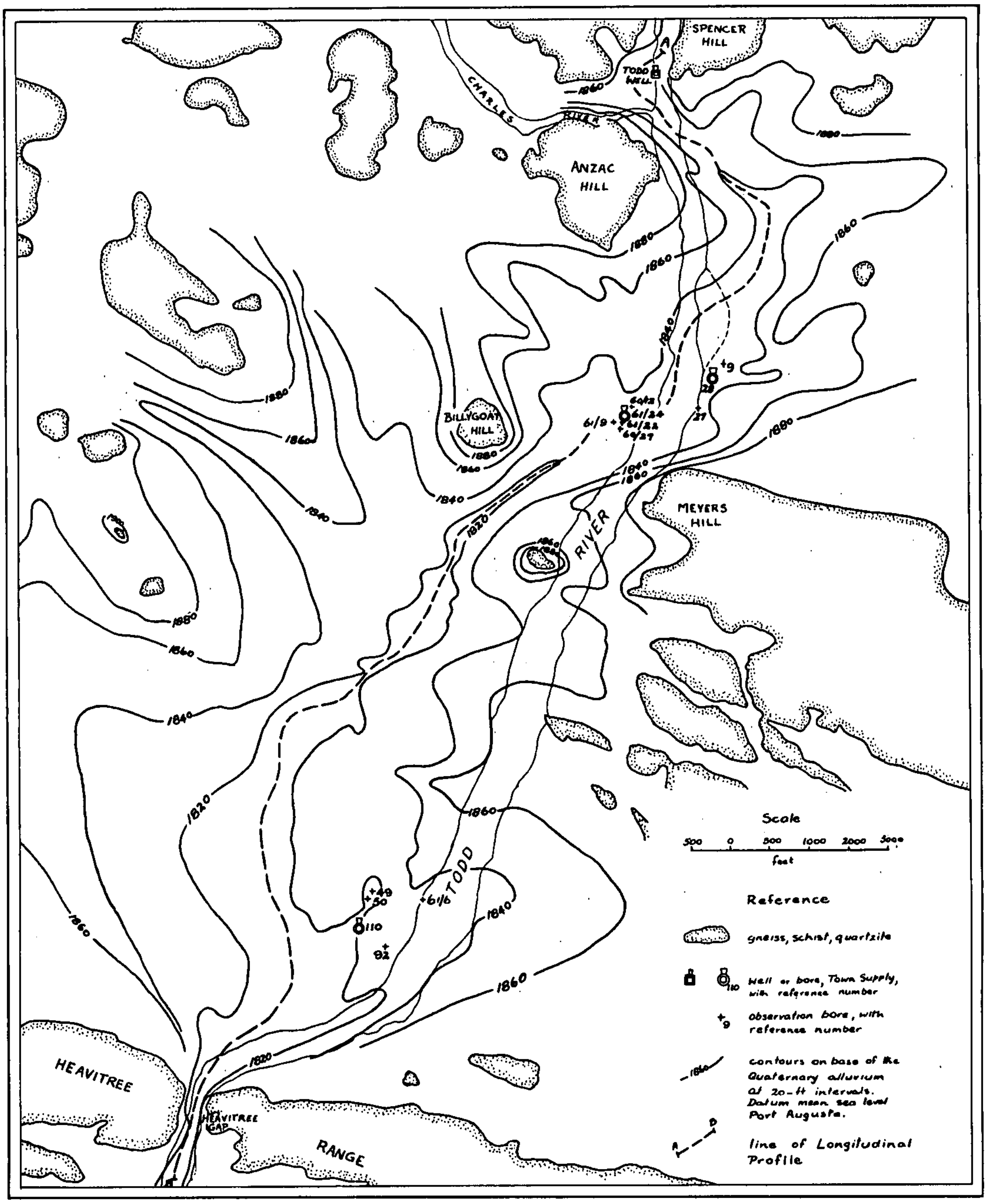


Fig. 1 Geological Structure of the Town Basin, Alice Springs

2. Aquifers of silty sand form about 15 percent of the total volume of saturated alluvium.

3. The aquifers are long narrow bodies, with a lenticular cross section ranging from 2 to 25 feet thick, which anastomose both vertically and horizontally through the alluvium.

4. Five aquifer systems can be recognized, based on subjective correlations between bore logs. Fig. 2 is an example of portion of one of the geological maps which were prepared for each system, using these correlations. Such a synthesis is needed for an understanding of the detailed geology of the basin and the distribution of aquifers within it, and for the development of hypotheses regarding its hydraulic behaviour.

5. The physical processes which controlled the transport and deposition of sediment did not alter materially during the period in which the basin was filled with alluvium.

6. The sediments in the southern part of the basin are more closely packed than those in the northern part, and they presumably have a lower effective porosity and permeability.

Hydrological Features

1. Groundwater is lost from the basin by pumping, evapotranspiration, and by outflow through aquifers within Heavitree Gap. It is added by flow through aquifers at its northern end and, for brief periods following river flow, through recharge areas in the present channel of the river.

2. Groundwater can be extracted from bores and wells which intersect aquifers of silty sand below the piezometric surface. During the period 1954 to 1964 water for the town supply was withdrawn from wells lined with concrete caissons, and from bores which were completed with perforated casing or sand screens.

3. Water levels in bores throughout the basin rise within several days after the Todd River starts to flow, and generally have fallen to their original level within 100 days.

4. Water levels in all bores respond to changes induced by pumping in the vicinity, even if they do not intersect the aquifer which is being pumped.

5. The shape of the piezometric surface can be considered to consist of three components, a mound adjacent to the river, a gradient of 15 feet per mile from north to south, and depressions caused by pumping.

6. The mound in the piezometric surface adjacent to the river encloses permeable alluvium in which recharge water is stored under unconfined conditions until it moves into confined aquifers.

7. The gradient is maintained by inflow to the basin from storage upstream of the Todd Well.

8. Some of the drawdown curves, obtained from aquifer performance tests, have the simple shape of the Theis type curve for the Well Function, which is applicable to an ideal homogeneous aquifer of infinite areal extent. The remainder have drawdown values greater or less than would be expected in the ideal aquifer. The differences may be caused by the presence of hydraulic boundaries, the addition of recharge water to the aquifer during the test, or to dewatering of the aquifer.

A Model of the Ideal Unconfined Aquifer

On a long term basis, the withdrawal of groundwater from the saturated alluvium in the Town Basin can be considered to occur under unconfined conditions (Ref. 3). Provided sufficient time has elapsed since pumping commenced, and the observation bores are sufficiently far from the pumping bore, the Well Function can be applied to values of drawdown to calculate the aquifer constants (Ref. 4, 6).

This is the simplest model, and it ignores most of the geological and hydrological features of the basin. It is used to gain experineoe in the application of quantitative methods, and to provide a basis for comparison between more sophisticated models.

The drawdowns measured during aquifer performance tests of bores 28, 61/24, and 110 (Fig. 1) were used to test the model. The techniques of

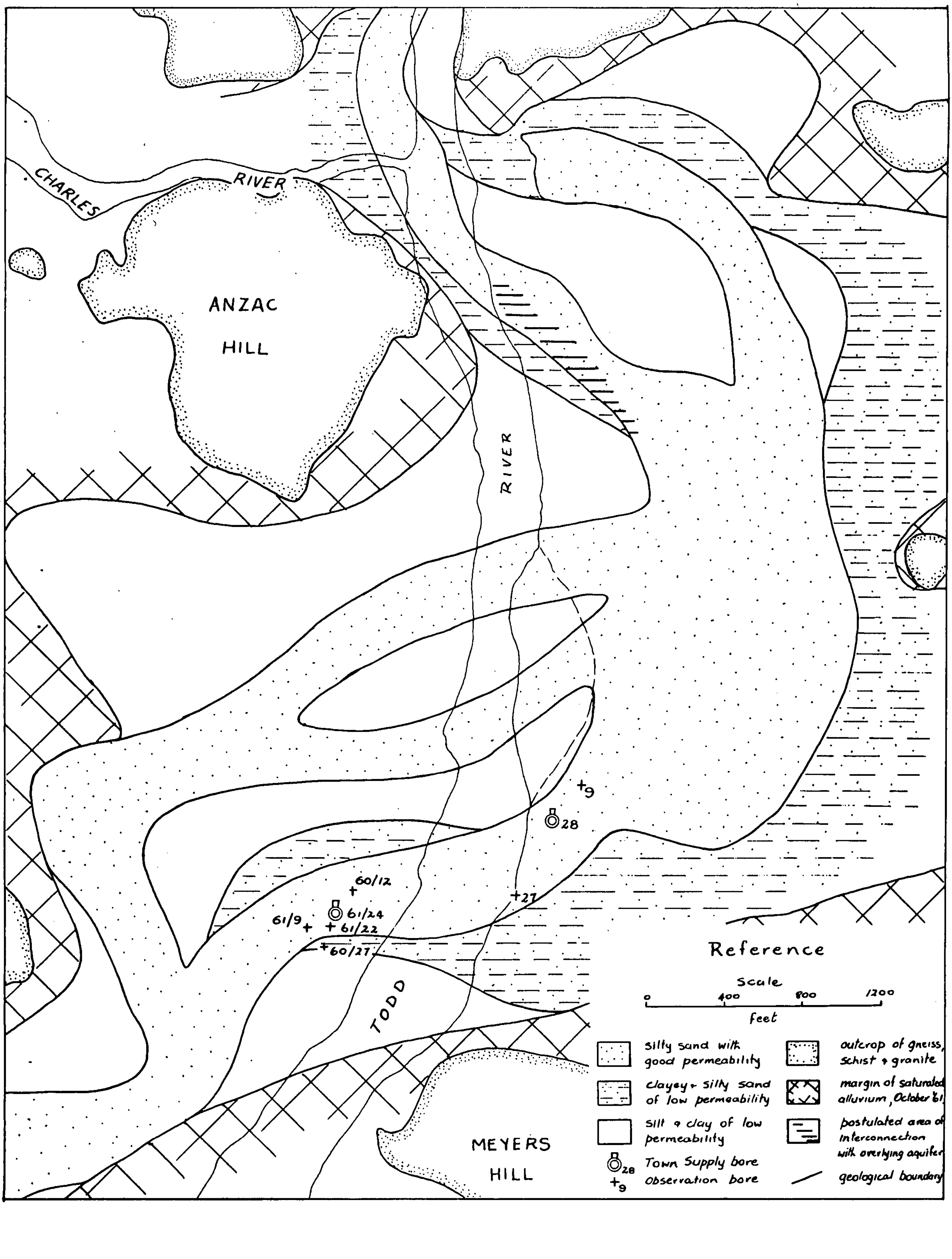


Fig 2 Geological Map for Portion of the '1820' Aquifer

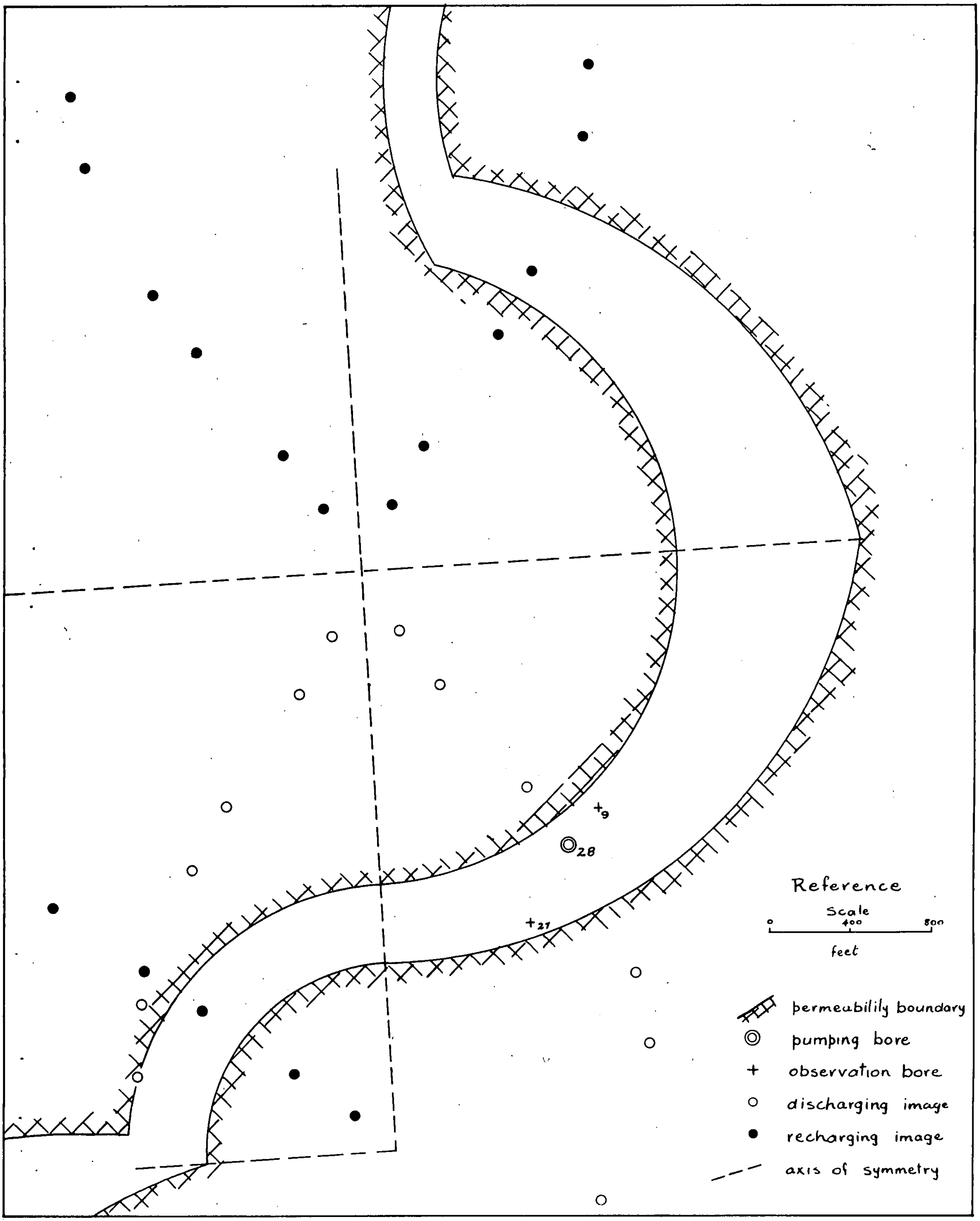


Fig 3. Composite Strip Model for Bore 28

Aquifer Constants for the Model of a Ideal Unconfined Aquifer - TABLE 1

Observation Bore	Coefficient of Transmissibility (Imp. gal./day/ft.)	Coefficient of Storage	Coefficient of Determination (%)	Aquifer Test of Bore	Coefficient of Transmissibility (Imp. gal./day/ft.)	Coefficient of Storage	Coefficient of Determination (%)
9	23,600	0.035	99	28	20,500	0.071	98
27	18,900	0.099	99				
60/12	27,000	0.057	100	61/24	33,000	0.042	86
60/27	26,500	0.257	92				
61/9	37,000	0.048	97				
61/22	33,200	0.056	96				
49	30,000	0.012	93	110	30,000	0.010	88
50	27,200	0.014	99				
92	19,500	0.058	99				
61/6	24,000	0.010	99				

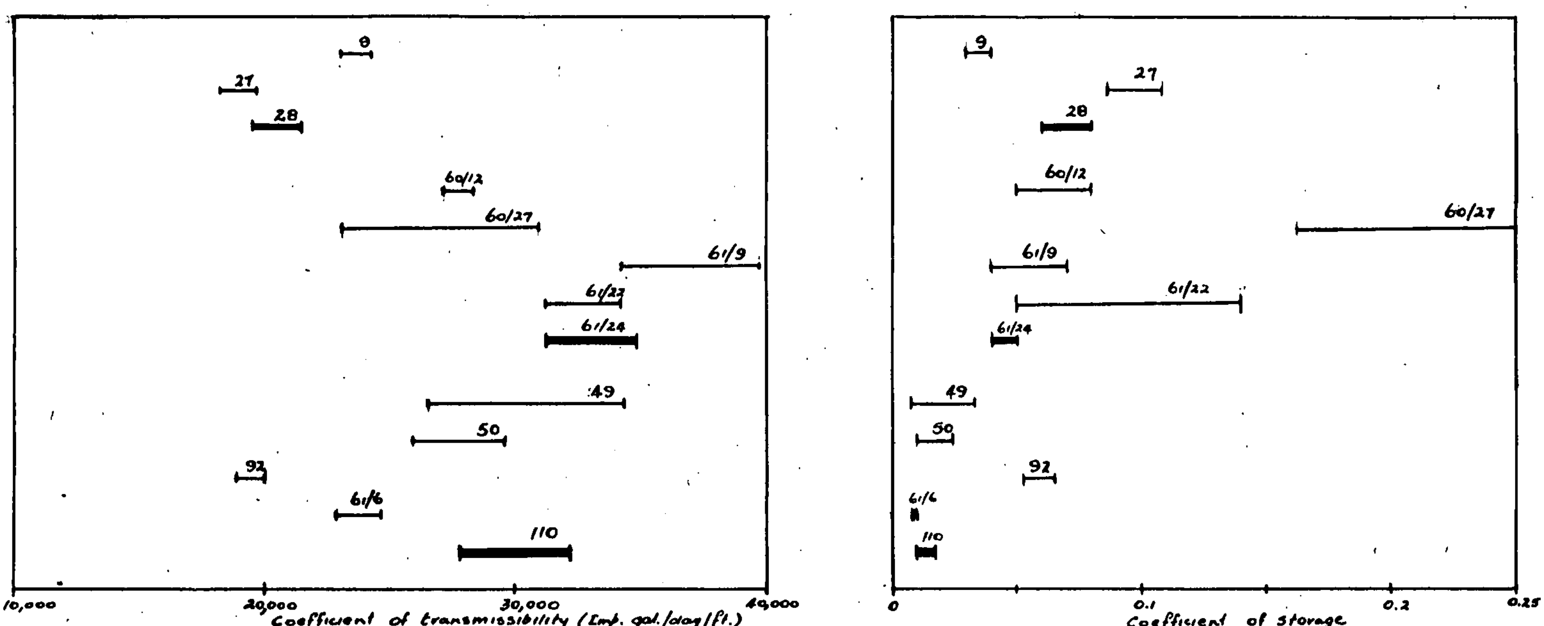


Fig. 4 Confidence intervals for the aquifer constants for bores 9, 27, 28, 60/12, 60/27, 61/9, 61/22, 61/24, 49, 50, 92, 61/6, 110.

regression analysis were used to obtain values for the coefficients of an expansion of the Well Function, as developed on p. 3 . Values for the aquifer constants (Table 1) were then determined from these coefficients and from the solution of a polynomial equation formed from them. In addition, the drawdowns for the observation bores in each test were grouped, and the analysis repeated to obtain a single value for each constant which could be applied to the aquifer. The results are shown on the right hand side of Table 1.

Confidence intervals for the constants were calculated for each of the tests (Fig. 4); the probability that the true values of the aquifer constants lie within these intervals is 95 percent.

The stepwise method of regression (Ref. 7) was used because a number of regression equations are derived during the course of the analysis. At each stage the number of terms in the equation is increased by the addition of that term, of those not previously used, which makes the most significant contribution. In the present application it was found that for a maximum number of four stages, terms were added to the equation in the order in which they appear in the expansion of the integral. For each succeeding stage the next term included was one which was out of sequence, and one for which the sign of the coefficient was opposite to that of the equivalent term in the expansion. The regression equation considered to be most appropriate was the one which contained the largest number of ordered terms.

The values which were obtained for the coefficients of transmissibility are between 18,000 and 37,000 Imperial gallons per day per foot and those for the coefficient of storage between 0.01 and 0.256 (Table 1). The wide range in values is an indication that the proposed model is not particularly suitable for the Alice Springs Town Basin. This is reflected in the absence of significant overlap for the confidence intervals (Fig. 4).

The coefficient of determination is usually taken as a measure of the degree to which the regression equation fits the data. In the present application it is not considered to be sufficiently sensitive for this purpose. The drawdowns measured in bore 27 are shown in Figure 5 in relation to the Theis type curves which mark the confidence limits for the regression. The high value of 99 percent for the coefficient of determination does not indicate that the curvature of the plot is broken at three places (for values of r^2/t of $5.0x10^4$, $1.2x10^4$, and $5.5x10^3$) probably as a result of reflections from impermeable boundaries. Nor does it indicate that the first 8 of the 47 measurements of drawdown plot outside the 95 percent confidence limits.

A Model Consisting of Arcuate Strips

This model, in contrast to the previous one, has been designed to simulate some of the geological and hydrological features of the basin. That portion of it which is proposed for bore 28 is illustrated in figure 4.

Individual aquifers are long narrow bodies of silty sand, and they can be considered to be strips bounded on two sides by impermeable boundaries. That the strips are arcs of circles in plan rather than straight lines is appropriate, as they correspond to meanders of the ancestral Todd River. The effect of variable width of the aquifer can be achieved by the use of eccentric arcs.

The method of images is a convenient tool for the solution of problems with such boundaries, and it is one which can be conveniently used in conjunction with the methods of regression analysis. The system of images which is proposed (Fig. 4) consists of three components. The first is the group of image wells required to satisfy the condition that no flow can occur across the boundaries at the edges of the strip. The position of these images were determined by reflecting each in turn across the arcuate boundaries, in the radial direction appropriate to that boundary. The second group of image wells in the system is used to limit the strip of aquifer to a quadrant of a circle, to allow for changes in its direction which are consistent with the meanders of a river. This restriction is achieved by reflecting the first group of images into the other three quadrants of the circle. As a result an undesirable restriction is introduced into the system, that there can be no flow into or from the ends of the strip. This can be removed by placing an image of the first group of image wells in the quadrants of the adjacent circular systems which are used to represent the adjoining strips of aquifer.

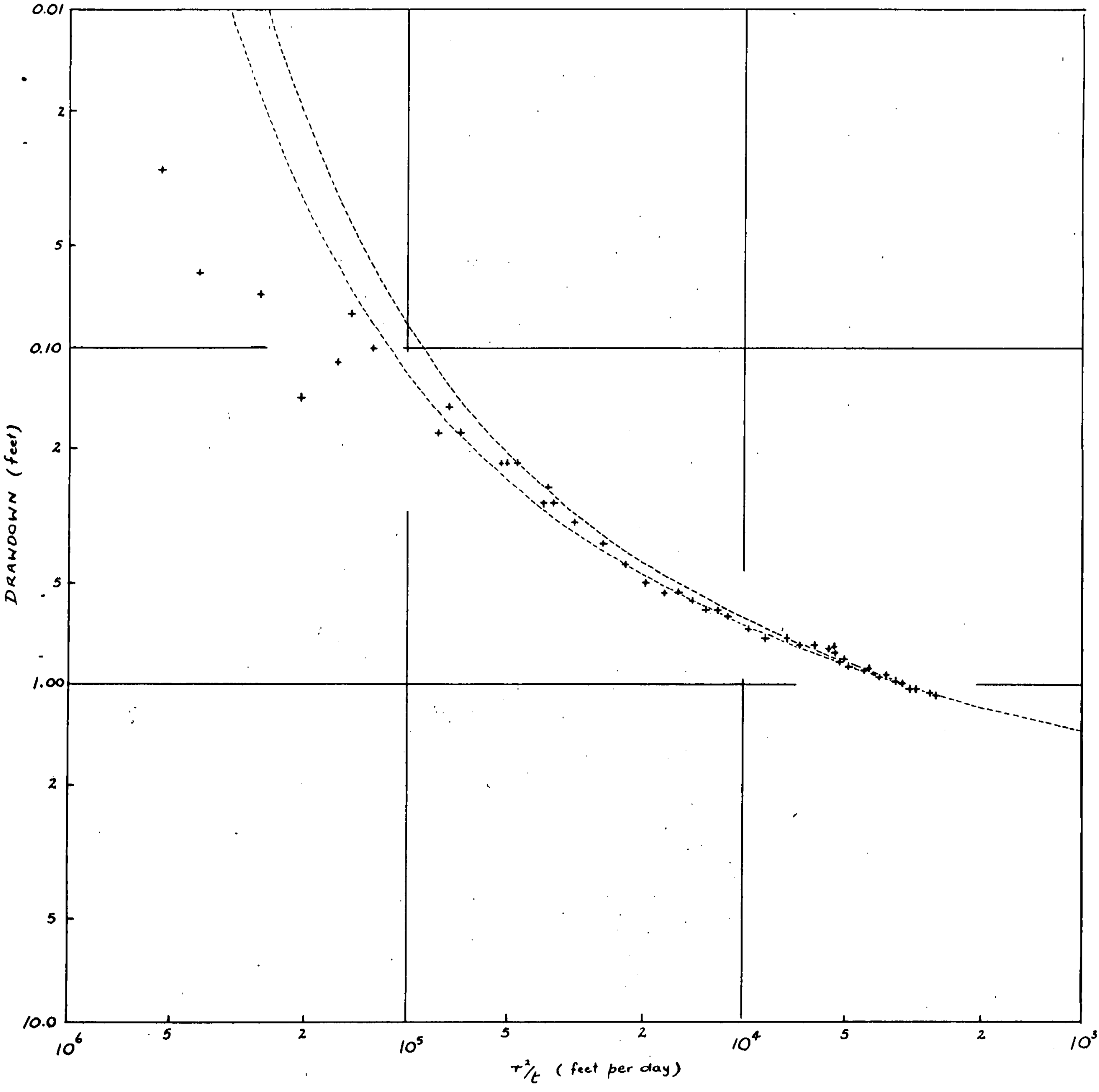


Fig. 5 Confidence interval for the Theis type curve fitted to drawdown measurements in Observation bore 27.

This model is being evaluated by the methods which were applied to the previous model. Final results are not yet available.

CONCLUSIONS

The most appropriate model which could be used to explain and to predict the hydraulic behaviour of the Alice Springs Town Basin, is one in which the geometrical shape of the geological features has been idealized and the hydraulic features have been preserved.

Values of the aquifer constants for a model and the confidence limits for them can be calculated by regression techniques from drawdown observations.

The most appropriate way to evaluate a model is to compare the confidence intervals of the aquifer constants for each observation bore.

The simple model of an homogeneous unconfined aquifer of infinite areal extent is not applicable to the basin.

ACKNOWLEDGEMENTS

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The Director of the Bureau of Mineral Resources has approved of the publication of this paper.

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