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Computer Programme for Producing Magneto-Telluric Curves for a Horizontally Layered Earth

by

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COMPUTER PROGRAMME FOR PRODUCING MAGNETO-TELLURIC CURVES FOR A HORIZONTALLY LAYERED EARTH

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ILLUSTRATIONS

SUMMARY

Programme MAGTEL was written to produce apparent resistivity and phase curves for magneto-telluric soundings over a horizontally stratified earth. Details of the programme are discussed and a sample of the output is given.

1. INTRODUCTION

In regions where the regional structure can be approximated by horizontal layers, magneto-telluric soundings can be interpreted using master curves such as those of Srivastava (1967) for two-and three-layer models. However, master curves for models consisting of more than three layers are usually required to interpret the field curves, and FORTRAN programme MAGTEL was written to meet this requirement.

Apparent resistivity and phase curves are plotted against period for plane waves, vertically incident on a horizontally stratified earth. Non-vertically incident fields are not included in this programme, as their effects have been shown to be negligible except for extremely long periods and very shallow angles of incidence such as would occur on the continental margins, as shown by Srivastava (1965 and 1967).

2. THEORY

The formula for the impedance of an n-layered earth in response to a vertically incident plane electromagnetic wave is given in MKS units by (e.g. Keller & Frischknecht, 1967):

$$Z = E_x/H_y = E_y/H_x$$

$$= \frac{-i\omega\mu}{\gamma_1} \coth\left(\gamma_1 h_1 + \operatorname{arcoth}\left\{\frac{\gamma_1}{\gamma_2} \coth\left(\gamma_2 h_2 + \right.\right.\right.$$

$$+ \operatorname{arcoth}\left\{\frac{\gamma_2}{\gamma_3} (\gamma_3 h_3 + \operatorname{arcoth}\left\{\frac{\gamma_3}{\gamma_4} \cdots\cdots\cdots\cdots\right.\right.$$

$$\cdots \operatorname{arcoth}\left\{\frac{\gamma_{n-3}}{\gamma_{n-2}} \coth\left(\gamma_{n-2} h_{n-2} + \operatorname{arcoth}\left\{\frac{\gamma_{n-2}}{\gamma_{n-1}} \coth\right.\right.\right.$$

$$\left(\gamma_{n-1} h_{n-1} + \operatorname{arcoth} \frac{\gamma_{n-1}}{\gamma_n}\right)\}\})\})\} \cdots\})$$

where E_x, E_y, H_x and H_y are the electric and magnetic field intensities with respect to an arbitrary set of cartesian co-ordinates in the horizontal plane. The symbols ω, γ, μ represent the angular velocity of the wave, the propagation constant, the magnetic permeability, and the thickness of each layer, respectively. The subscripts refer to the number of each layer, as shown in Figure 1. The symbol i is the complex number $\sqrt{-1}$.

The period of the wave is given by.

$$T = 2\pi/\omega \quad \text{in seconds}$$

The apparent resistivity at a given period is defined by $\rho_a = |Z|^2/(\omega\mu)$

The phase angle of $\underset{\sim}{Z}$ is the difference in phase between the $\underset{\sim}{E}$ and $\underset{\sim}{H}$ fields.

The curves of ρ_a against T, and phase against T give an expression of the thickness and resistivity of the individual layers in an analogous way to resistivity sounding.

3. PROGRAMME MAGTEL

This programme calculates equation (1) for any number of layers n, using the identities

$$\coth \theta = \frac{e^{\theta} + e^{-\theta}}{e^{\theta} - e^{-\theta}}$$

$$\operatorname{arcoth} \theta = \log \sqrt{\frac{\theta + 1}{\theta - 1}}$$

The values of ρ_a and phase are calculated at 50 equal logarithmic intervals of period in the range 1 to 20,000 seconds and placed in arrays in common storage with subroutine PLOTTER.

A listing and flow-chart of programme MAGTEL, along with listings of subroutines PLOTTER, RHOAPLOT, PHASEPLT and MODEL are given in Appendixes 1 and 2.

4. SUBROUTINE PLOTTER

This subroutine plots $\log_{10} \rho_a$ against $\log_{10}$ T and phase against $\log_{10}$ T. An example of the plotted output is shown in Figure 2. This curve is taken from the interpretation of a magneto-telluric sounding at Windorah, Queensland (Pollard & Whiteley, in prep.). The curves are plotted on the same scale as Srivastava's (1965) curves, which can be used for initial interpretation of the field data, and then a more accurate fit obtained using MAGTEL.

All plot routines are designed for the 12-inch CALCOMP plotter attached to the CDC 3600 computer at the C.S.I.R.O. computer centre in Canberra. The range of values of ρ_a acceptable to the plotter is from 0.1 to about 400 ohm-metres.

5. SUBROUTINES RHOAPLOT AND PHASEPLT

Three subroutines plot field data on the same graph as the theoretical curve, as shown in Figure 2.

6. SUBROUTINE MODEL

This subroutine is called from PLOTTER and plots a schematic model of the layers. The resistivity and thickness of each layer are printed.

7. DATA INPUT

Data input is by punched cards, in the following sequence:

(1) The number of layers under format I3.

(2) Resistivity, dielectric constant (generally 10-15 for sedimentary rocks), magnetic permeability (generally 1), and layer thickness in metres, under format (F8.1, 2F5.1, F8.1). Each layer must have a separate data card. The thickness of the bottom layer is assumed infinite, and any value of thickness entered on the last layer card will be ignored.

(3) EOF card.

(4) Indicator IND under format I2.

IND = -1 (negative integer) indicates no field data to be plotted (go to (6)).

IND = 0 (blank card) indicates apparent resistivity field data to follow (go to (5)).

IND = 1 (positive integer) represents the case where data that have been used in a previous case are to be plotted again (go to (6)).

(5) Field values of period, apparent resistivity, and standard deviation in apparent resistivity under format (3F9.1), followed by an EOF card.

(6) Indicator IND under format I2 as in section (4).

IND = -1 or 1 go to (8)

IND = 0 (blank card) go to (7)

(7) Field values of period, phase, and standard deviation in phase under format (3F9.1), followed by an EOF card.

(8) Go back to (1) and repeat for extra cases.

(9) After all cases have been done, the deck is completed by an EOF card and an EOD.

8. REFERENCES

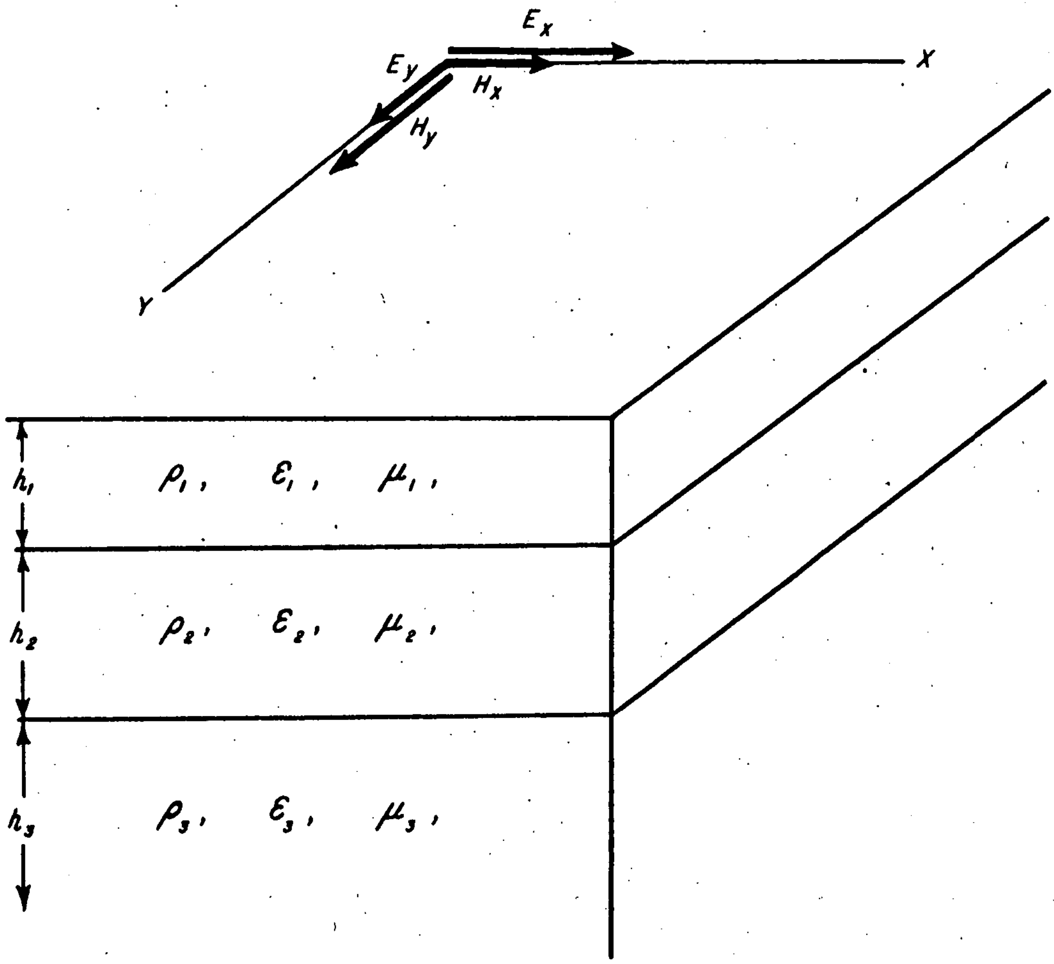
KELLER, G.V., and FRISCHKNECHT, F.C., 1966 - ELECTRICAL METHODS IN GEOPHYSICAL PROSPECTING. Pergamon Press.

POLLARD, P.C., and WHITELEY, R.J., - A combined deep resistivity and magneto-telluric sounding in the Great Artesian Basin, Queensland. Bur. Miner. Resour. Aust. Rec.(in prep.)

SRIVASTAVA, S.P., 1965 - Method of interpretation of magneto-telluric data when the source field is considered. J. geophys. Res. 70, 945.

SRIVASTAVA, S.P., 1967 - Magneto-telluric two and three layer master-curves. Publ. Dom. Obs. Ottawa, Canada.

Fig. 1



SKETCH OF PLANE WAVE INCIDENT ON LAYERED EARTH

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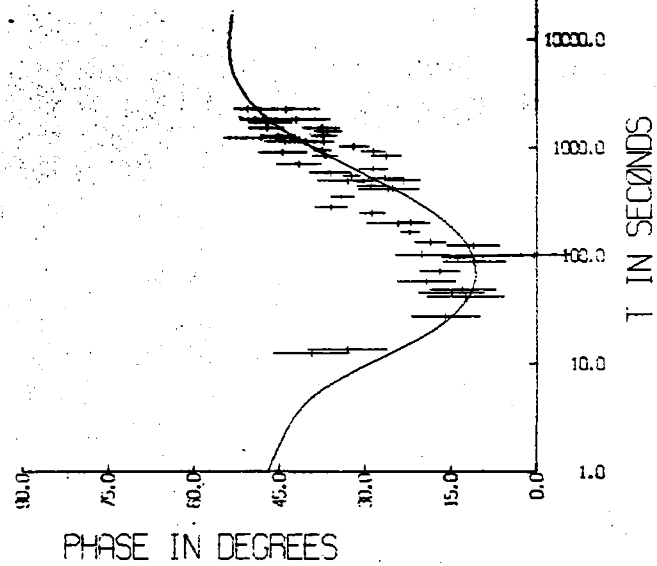
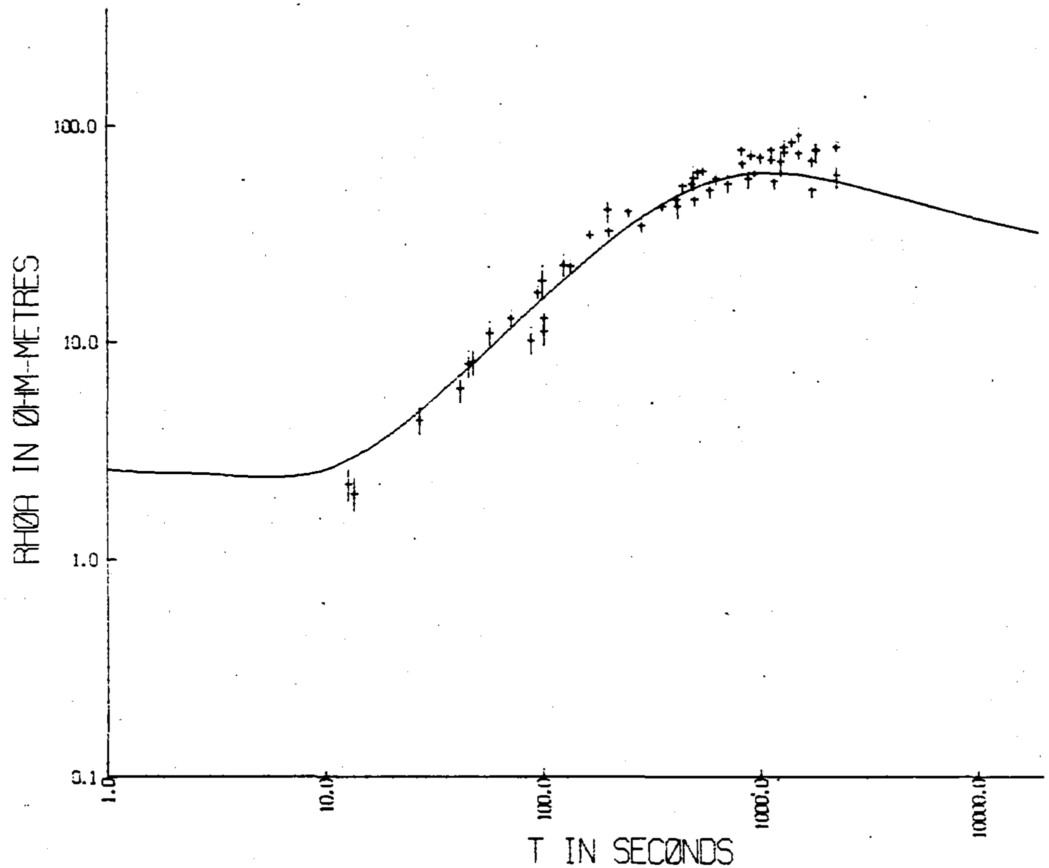


11.4 ØHM-M	5.0 M
55.0 ØHM-M	57.1 M
1.5 ØHM-M	82.3 M
2.6 ØHM-M	1219.2 M
8.0 ØHM-M	3000.0 M
10000.0 ØHM-M	90000.0 M
20.0 ØHM-M	



SAMPLE PLOT OF MAGTEL PROGRAMME

Fig. 2

```
      PROGRAM MAGTEL
C     PROGRAM MAGTEL CALCULATES THE PLANE-WAVE MAGNETO-TELLURIC APPARENT
C     RESISTIVITY AND PHASE CURVES AGAINST PERIOD FOR AN N-LAYERED EARTH
      COMPLEX Z,CI,PK,ARCOTH,PKH,SUM,COTH,PRODUCT,RHOA
      DIMENSION PK(40),AMU(20),EPSILON(20)
      COMMON/SALLY/RHOPARAM(60),PERIOD(60),PHASE(60)
      COMMON/DAN/RHO(20),H(20),NMAX
      CI=CMPLX(0.0,1.0)      $PI=3.1415926536
   17 READ 1,N               $F=5.0*10.0**(-5)              $I=1
C     N IS THE NUMBER OF LAYERS
    1 FORMAT(I3)
      IF(EOF,60)88,3
    3 READ 2,R,AKE,AKM,THICKNES
C     R IS THE RESISTIVITY IN OHM-METERS
C     AKE IS THE DIELECTRIC CONSTANT,AND WILL GENERALLY BE 10.0
C     AKM IS THE MAGNETIC PERMEABILITY,GENERALLY 1.0
C     THICKNES IS THE LAYER THICKNESS IN METRES
C     ALL PARAMETERS ARE IN MKS UNITS
    2 FORMAT(F8.1,2F5.1,F8.1)
      IF(EOF,60)5,4
    4 AMU(I) =AKM*4.0*PI*10.0**(-7)
      EPSILON(I)=AKE*8.854*10.0**(-12)                    $RHO(I)=R
      H(I)=THICKNES                $I=I+1     $GO TO 3
    5 NMAX=N
      DO 15,J=1,51
      IN1=N-1         $IN2=N-2
      DO 77,K=1,N
      BETA=2.0*PI*F*SQRT(0.5*AMU(K)*EPSILON(K)*(SQRT(1.0+1.0/(RHO(K)*
     1RHO(K)*EPSILON(K)*EPSILON(K)*4.0*PI*PI*F*F))-1.0))
      ALPHA=SQRT(BETA*BETA+4.0*PI*PI*F*F*AMU(K)*EPSILON(K))
      PK(K)=CMPLX(ALPHA,BETA)
   77 CONTINUE
      ARCOTH=CLOG(CSQRT((PK(IN1)/PK(N)+1.0)/(PK(IN1)/PK(N)-1.0)))
      DO 6,I=1,NMAX
      IF(I.EQ.1)GO TO 8
      ARCOTH =CLOG(CSQRT((PRODUCT+1.0)/(PRODUCT-1.0)))
    8 PKH=PK(IN1)*H(IN1)
      SUM=ARCOTH + PKH
      COTH=(CEXP(SUM)+CEXP(-SUM))/(CEXP(SUM)-CEXP(-SUM))
      IF(N.EQ.2)GO TO 9
      PRODUCT = PK(IN2)/PK(IN1)*COTH
      N=N-1            $IN1=IN1-1              $IN2 =IN2-1
    6 CONTINUE
    9 Z=  CI*8.0*PI*F*PI*10.0**(-7)/PK(1)*COTH          $ N=NMAX
      PERIOD(J)=1.0/F
      RHOA=CI*Z*Z/(8.0*PI*F*PI*10.0**(-7))
      RHOPARAM(J)=CABS(RHOA)                $PHASE(J)=CANG(Z)*180.0/PI
      F=10.0**(-4.301+J*0.08602)
   15 CONTINUE
      CALL PLOTTER
      GO TO 17
   88 END
```

```
      SUBROUTINE PLOTTER
      COMMON/SALLY/RHOPARAM(60),PERIOD(60),PHASE(60)
      CALL PLOT(0.3951,0.3951,2)
      CALL PLOT(-0.3951,0.0,1)
      CALL PLOT(3.55,4.3010,3)
      CALL PLOT(0.0,4.3010,4)
      CALL PLOT(0.0,0.0,4)                       $K=3
      DO 1,J=1,51
      M=52-J
      CALL PLOT(ALOG10(RHOPARAM(J))+1.0,+ALOG10(PERIOD(M)),K)
      K=4
    1 CONTINUE                    $CALL RHOAPLOT
      CALL PLOT(0.9878,4.6227,3)
      CALL TEXT(16HRHOA IN OHM-METRES,18,4)
      DO 161,J=1,5
      CALL PLOT(0.0395,4.3015-1.0*(J-1),3)
      CALL PLOT(0.0,4.3010-1.0*(J-1),4)
      CALL PLOT(-0.2766,4.2733-1.0*(J-1),3)
      P=1.0*10.0**(J-1)
      ENCODE(7,51,BUF)P
   51 FORMAT(F7.1)
      CALL TEXT(BUF,7,2)
  161 CONTINUE
      CALL PLOTSET(4)
      CALL PLOT(-2.3706,-0.39,3)
      CALL TEXT(12HT IN SECONDS,+12,4)
      DO 162,J=1,4
      CALL PLOT(-4.2615,(J-1)*1.0,3)
      CALL PLOT(-4.3010,(J-1)*1.0,4)
      CALL PLOT(-4.6227,(J-1)*1.0+0.0277,3)
      P=0.1*10.0**(J-1)
      ENCODE(7,51,BUF)P
      CALL TEXT(BUF,+7,2)
  162 CONTINUE
      CALL PLOTSET(1)
      CALL PLOT(-0.3951,6.7160,3)
      CALL PLOT(0.7950,15.0,2)
      CALL PLOT(-0.7950,0.0,1)
      CALL PLOT(0.0,90.0,3)
      CALL PLOT(0.0,0.0,4)
      CALL PLOT(4.7702,0.0,4)                    $K=3
      DO 2,J=1,51
      CALL PLOT(ALOG10(PERIOD(J)),PHASE(J),K)
      K=4
    2 CONTINUE             $CALL PHASEPLT
      CALL PLOT(1.4,-20.0,3)
      CALL TEXT(12HT IN SECONDS,12,4)
      DO 163,J=1,7
      CALL PLOT(0.0795,15.0*(J-1),3)
      CALL PLOT(0.0,15.0*(J-1),4)
      CALL PLOT(-0.3160,15.0*(J-1)-1.05,2)
      P=(J-1)*15.0
      ENCODE(4,53,BUF)P
   53 FORMAT(F4.1)
      CALL TEXT(BUF,4,2)
  163 CONTINUE
```

Appendix 1 (continued)

```
      CALL PLOTSET(4)
      CALL PLOT(-4,3725,-15.0,3)
      CALL TEXT(16HPHASE IN DEGREES,16,4))
      DO 164,J=1,5
      CALL PLOT(-0,0795,18.8685*(J-1),3)
      CALL PLOT(0,0,18.8685*(J-1),4)
      CALL PLOT(0,0795,18.8685*(J-1)-1,05,3)
      P=1,0*10,0**(J-1)
      ENCODE(7,51,BUF)P
      CALL TEXT(BUF,7,2)
  164 CONTINUE
      CALL MODEL
      CALL PLOTSET(1)
      CALL PLOT(-0,7950,169.0,3)
      RETURN                        $END
```

```
      SUBROUTINE RHOAPLOT
      DIMENSION X(200),R1(200),R2(200),R3(200)
      READ 11,IND
   11 FORMAT(I2)
      IF(IND)17,15,6
   15 I=1
    4 READ 1,T,RHOA,ERR
    1 FORMAT(3F9,1)
      IF(EOF,60)6,5
    5 X(I)=4,3010-ALOG10(T)
      R1(I)=ALOG10(RHOA+ERR)+1.0
      R2(I)=ALOG10(RHOA-ERR)+1.0
      R3(I)=ALOG10(RHOA)+1.0                $M=I          $I=I+1
      GO TO 4
    6 DO 16,I=1,M
      CALL PLOT(R1(I),X(I),3)
      CALL PLOT(R2(I),X(I),4)
      CALL PLOT(R3(I),X(I),3)
      CALL TEXT(1H+,0,2)
   16 CONTINUE
   17 RETURN                        $END
```

```
      SUBROUTINE PHASEPLT
      DIMENSION X(200),P1(200),P2(200),P3(200)
      READ 11,IND
   11 FORMAT(I2)
      IF(IND)17,15,6
   15 I=1
    4 READ 1,T,PHASE,ERR
    1 FORMAT(3F9,1)
      IF(EOF,60)6,5
    5 X(I)=ALOG10(T)
      P1(I)=PHASE+ERR
      P2(I)=PHASE-ERR
      P3(I)=PHASE                 $M=I          $I=I+1
      GO TO 4
    6 DO 16,I=1,M
      CALL PLOT(X(I),P1(I),3)
      CALL PLOT(X(I),P2(I),4)
      CALL PLOT(X(I),P3(I),3)
      CALL TEXT(1H+,0,2)
   16 CONTINUE
   17 RETURN                        $END
```

```
      SUBROUTINE MODEL
      COMMON/DAN/RHO(20),H(20),NMAX      $DIMENSION BUF(2)
      DO 701,L=1,NMAX
      X=90.0-(L-1)*7.5
      CALL PLOT(1,59,X,3)
      CALL PLOT(3,58,X,4)
      CALL PLOT(1,4310,X-4.5,3)
      P=RHO(L)                    $ENCODE(14,99,BUF)P
   99 FORMAT(F8,1,6H OHM-M)
      CALL TEXT(BUF,14,2)
      IF(L.EQ.NMAX )GO TO 701
      P=H(L)                      $ENCODE(13,100,BUF)P
  100 FORMAT(3X,F8,1,2H M)
      CALL TEXT(BUF,13,2)
  701 CONTINUE
      RETURN                        $END
```

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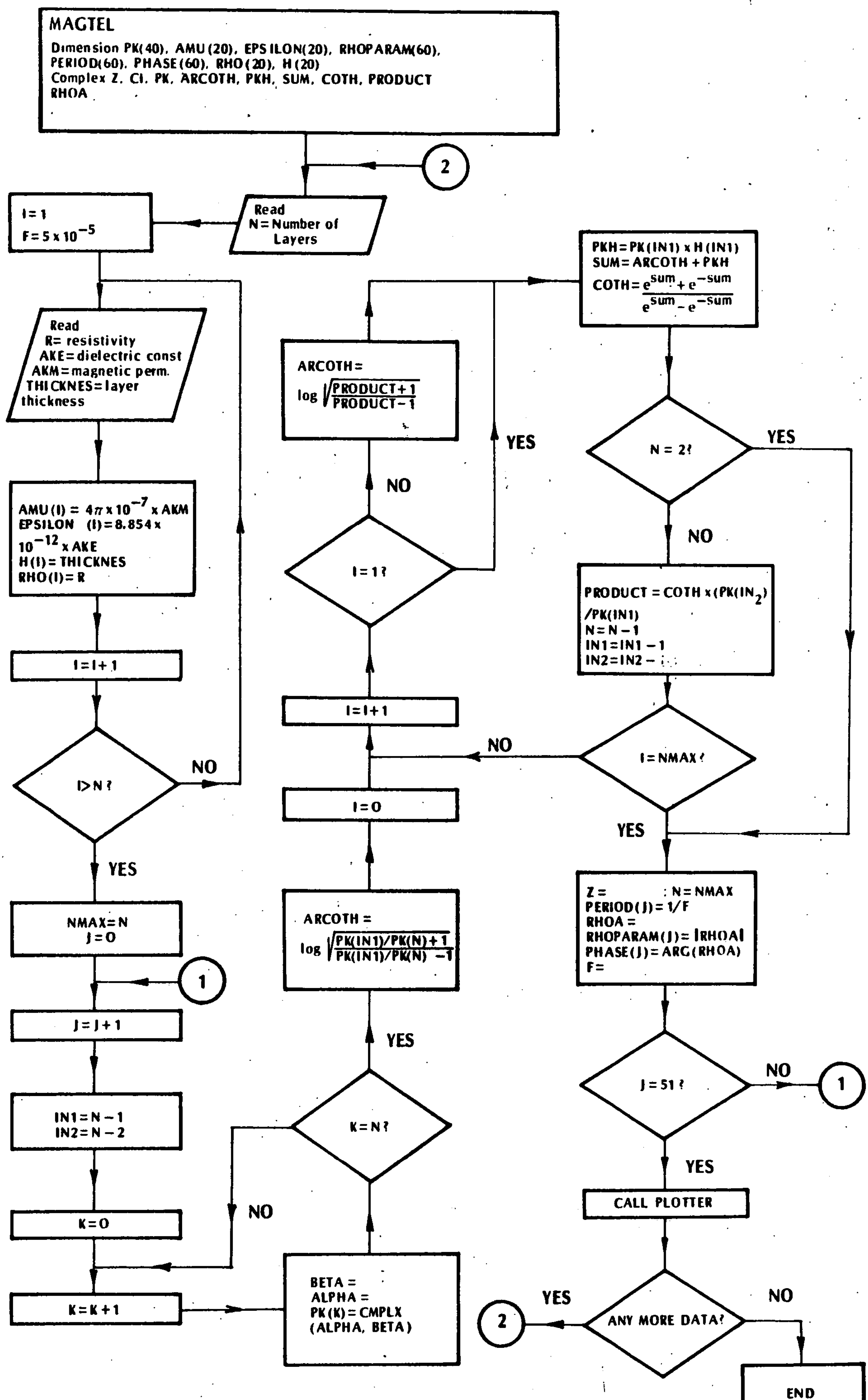
Appendix 2 MAGTEL Program Flow Chart



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