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DEPARTMENT OF NATIONAL DEVELOPMENT

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IP Electromagnet Coupling for the Dipole-Dipole Configuration

by

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SUMMARY

Curves of the electromagnetic coupling between collinear dipoles on a uniform half space are presented for various values of dipole length and frequency and for n-values ranging from 2 to 8.

1. INTRODUCTION

Induced polarization (IP) measurements using frequency domain equipment are limited in areas of low resistivity by electromagnetic coupling between transmitting and receiving cables, which introduces errors in the measured values of the frequency effect. The single unknown parameter that affects the electromagnetic coupling is the earth resistivity. It is almost impossible to remove the coupling errors by theoretical calculations, because lateral inhomogeneities in earth resistivity produce situations far too complex to be represented mathematically. However, in some areas the assumption of a uniform earth resistivity will not introduce large errors, and in general this assumption may be used to derive a reliable estimate of the order of magnitude of the coupling.

If a uniform resistivity is assumed, the computation of electromagnetic coupling is not difficult, but is tedious. In some of the early work, the calculations were simplified by dealing only with small coupling effects. In this case the out-of-phase component may be ignored and the coupling is given, with parameters as defined in Chapter 2, by:

percentage coupling $\simeq |kL|^3 \left[n(n^2 - 1)\right] . 100/6\sqrt{2}$ (Madden & Cantwell 1967).

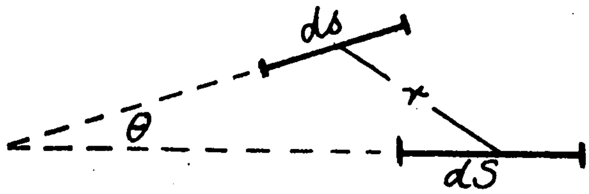
However, with more than a few percent coupling, the out-of-phase component becomes significant and must be taken into account. Millet (1967) has made an accurate computation of electromagnetic coupling, but the method of presentation makes the results difficult to use in routine field work. For practical purposes, it is desirable to have the results directly in terms of frequency, dipole length, dipole separation, and resistivity.

Geoscience Limited (1968) in a technical manual issued to their clients, have presented results in this form, apparently using the approach of Madden and Cantwell (1967). Unfortunately the curves produced give values that are low by about 10 percent; this is not serious, however, as the curves are applicable only to small coupling effects. The error appears to have resulted from making the approximation $6\sqrt{2} \simeq 10$ in calculating the constant $\left[(2\pi\mu)^{3/2} . 100\right] /6\sqrt{2}$

Tha aim of the present work was to make an accurate computation of coupling effects up to 60 percent, and to present the results in a form suitable for routine use in field surveys.

2. MATHEMATICAL DERIVATION

Consider two wire elements dS and ds on a uniform half space. Then, neglecting displacement currents, the coupling between the two wire elements is given by:



$$dZ_{Ss} = \frac{V}{I} = dS\,ds\left[P(r)\cos\theta + \frac{\partial^2 Q(r)}{\partial S\,\partial s}\right]$$

(Sunde, 1949)

Where
$$Q(r) = \frac{\rho}{2\pi r}$$
$$P(r) = \frac{i\mu\omega}{2\pi r}\left[\frac{1-(1+kr)e^{-kr}}{k^2r^2}\right]$$
$$k^2 = \frac{i\mu\omega}{\rho}$$
$$I = I_0 e^{i\omega t}$$

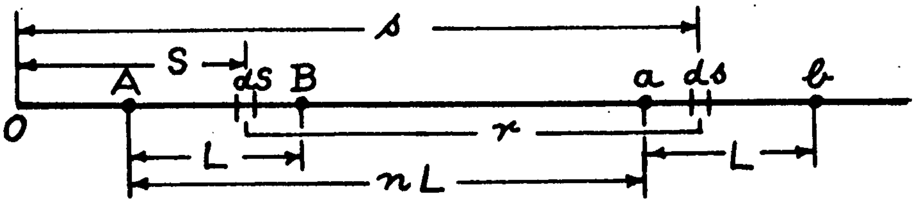
$$\begin{aligned} P(r) &= \frac{\rho}{2\pi}\cdot\frac{1}{r^3}\left[1-(1+kr)e^{-kr}\right] \\ &= \frac{\rho}{2\pi r^3}\left[1-(1+kr)\left(1-kr+\frac{k^2r^2}{2}-\frac{k^3r^3}{6}+\dots\frac{(-kr)^n}{n!}+\dots\right)\right] \\ &= \frac{\rho}{2\pi}\left[\sum_{x=1}^{\infty}\frac{(x-1)(-k)^x r^{x-3}}{x!}\right] \end{aligned}$$

Q(r) is the "conductively coupled" term; i.e the normal signal.
P(r) is the "electromagnetically coupled" term.

Thus, for two insulated wire elements ab and AB the mutual impedance Z_{Ss} is given by:

$$Z_{Ss} = \int_a^b \int_A^B \left[P(r)\cos\theta + \frac{\partial^2 Q(r)}{\partial S\, \partial s} \right] dS\, ds$$

and for collinear dipoles, $\cos\theta = 1$.



$$Z_{Ss} = \int_a^b \int_A^B P(r)\, dS\, ds + \int_a^b \int_A^B \frac{\partial^2 Q(r)}{\partial S\, \partial s}\, dS\, ds$$

$$= \overline{P(r)} + \overline{Q(r)}$$

$$\overline{P(r)} = \int_a^b \int_A^B \frac{\rho}{2\pi} \sum_{x=1}^{\infty} \frac{(x-1)(-k)^x\, r^{x-3}}{x!}\, dS\, ds$$

$$= \frac{\rho}{2\pi} \int_a^b \int_A^B \left[\frac{k^2}{2r} - \frac{k^3}{3} + \sum_{x=4}^{\infty} \frac{(x-1)(-k)^x\, r^{x-3}}{x!} \right] dS\, ds$$

It may be shown (Appendixes 1-4) that:

$$\overline{P(r)} = \frac{\rho}{2\pi L} \left[\frac{k^2 L^2}{2} \left(\ln \frac{n+1}{n-1} + n \ln \frac{n^2-1}{n^2} \right) - \frac{k^3 L^3}{3} + \right.$$
$$\left. + \sum_{x=4}^{\infty} \frac{(-kL)^x}{(x-2)x!} \left\{ (n+1)^{x-1} + (n-1)^{x-1} - 2n^{x-1} \right\} \right]$$

and $\overline{Q(r)} = - \dfrac{\rho}{\pi L n (n^2-1)}$

Define ZX as the complex sum of $\overline{P(r)}$ and $\overline{Q(r)}$
Z as the magnitude of ZX
Q as the magnitude of $Q(r)$

Then the percentage coupling $C = \left(\frac{Q}{Z} - 1 \right) \times 100$

The above formulae were solved using complex arithmethic on a CDC 3600 computer, summing the series in the equation for P(r) for the first 32 terms. The results are presented as curves in Plates 1 to 8.

The computer programme listing and flow diagram are presented in Appendix 5.

3. REFERENCES

GEOSCIENCE INCORPORATED, 1968 - Induced polarisation theoretical electromagnetic coupling curves. Geoscience Inc. Technical Manual NU-68013, April 1968.

MADDEN, T.R., and CANTWELL, T., 1967 - Induced polarisation - a review. In MINING GEOPHYSICS, Vol. 2. Society of Exploration Geophysicists.

MILLET, F.B., 1967 - Electromagnetic coupling of collinear dipoles in a uniform half space. Ibid.

SUNDE, E., 1949 - EARTH CONDUCTION EFFECTS IN TRANSMISSION SYSTEMS. Van Nostrand, New York.

APPENDIX 1

$$\overline{Q(r)} = \iint \frac{\partial^2 Q(r)}{\partial s\, \partial S}\, ds\, dS$$

$$Q(r) = \frac{\rho}{2\pi r}$$

$$\frac{\partial^2 Q(r)}{\partial S\, \partial s} = \frac{\rho}{2\pi}\cdot\frac{\partial^2}{\partial S\, \partial s}\cdot\frac{1}{r} = -\frac{\rho}{2\pi}\cdot\frac{\partial^2}{\partial S\, \partial s}\cdot\frac{1}{S-s} \qquad \text{since } r = s - S$$

$$= -\frac{\rho}{2\pi}\cdot\frac{\partial}{\partial s}\left[-1(S-s)^{-2}\right]$$

$$= 2\,(S-s)^{-3}\cdot\frac{\rho}{2\pi}$$

$$\iint \frac{\partial^2 Q(r)}{\partial S\, \partial s}\, dS\, ds = \frac{2\rho}{2\pi}\int_a^b\int_A^B (S-s)^{-3}\, dS\, ds$$

$$= \frac{\rho}{\pi}\int_a^b \left[\frac{(S-s)^{-2}}{-2}\right]_A^B ds$$

$$= \frac{\rho}{2\pi}\int_a^b \left[(A-s)^{-2} - (B-s)^{-2}\right] ds$$

$$= \frac{\rho}{2\pi}\int_a^b \left[(s-A)^{-2} - (s-B)^{-2}\right] ds$$

$$= \frac{\rho}{2\pi}\left[-\left[(s-A)^{-1}\right]_a^b + \left[(s-B)^{-1}\right]_a^b\right]$$

$$= \frac{\rho}{2\pi}\left[(a-A)^{-1} - (b-A)^{-1} + (b-B)^{-1} + (a-B)^{-1}\right]$$

$$= \frac{\rho}{2\pi}\left[\frac{1}{nL} - \frac{1}{(n+1)L} + \frac{1}{nL} - \frac{1}{(n-1)L}\right]$$

$$= \frac{\rho}{2\pi L}\left[\frac{2}{n} - \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{\rho}{2\pi L}\cdot\frac{2n^2 - 2 - (n^2 - n) - (n^2 + n)}{n\,(n^2-1)}$$

$$= \frac{\rho}{2\pi L}\cdot\frac{-2}{n(n^2-1)}$$

$$= -\frac{\rho}{\pi L n(n^2-1)}$$

$a - A = nL$ $\qquad$ $a - B = (n-1)L$

$b - A = (n+1)L$ $\qquad$ $b - B = nL$

APPENDIX 2

$$\int_A^B\int_a^b \frac{ds\,dS}{(s-S)} = \int_A^B \Big[\ln(s-S)\Big]_a^b dS$$

$$= \int_A^B \Big[\ln|b-S| - \ln|a-S|\Big] dS$$

$$= \Big[-(b-S)(\ln|b-S|-1) + (a-S)(\ln|a-S|-1)\Big]_A^B$$

$$= (B-b)\ln|b-B| + (a-B)\ln|a-B| + (b-A)\ln|b-A| -$$
$$- (a-A)\ln|a-A|$$

$$= -nL\ln|nL| + (n-1)L\ln|(n-1)L| + (n+1)L\ln|(n+1)L| -$$
$$- nL\ln|nL|$$

$$= -2nL\ln|nL| + nL\ln|(n^2-1)L^2| + L\ln\left|\frac{n+1}{n-1}\right|$$

$$= L\ln\left|\frac{n+1}{n-1}\right| + nL\ln\left|\frac{n^2-1}{n^2}\right|$$

$$\text{So} \iint \frac{k^2}{2}\cdot\frac{dS\,ds}{r} = \frac{k^2L}{2}\left\{\ln\left|\frac{n+1}{n-1}\right| + n\ln\left|\frac{n^2-1}{n^2}\right|\right\}$$

since $s - S = r$

$B - b = -nL$ $\qquad$ $b - A = (n+1)L$

$a - B = (n-1)L$ $\qquad$ $a - A = nL$

APPENDIX 3

$$\int_a^b \int_A^B \frac{k^3}{3}\, dS\, ds = \frac{k^3}{3} \int_a^b \Big[S\Big]_A^B ds$$

$$= \frac{k^3}{3} \int_a^b (B-A)\, ds$$

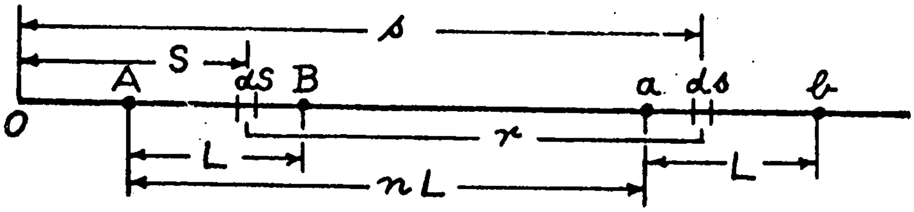
$$= \frac{k^3 (B-A)}{3} \int_a^b ds$$

$$= k^3 \, \frac{(B-A)}{3} \, (b-a)$$

$$= \frac{k^3 L^2}{3}$$

$B - A = L$ $\qquad\qquad$ $b - a = L$

APPENDIX 4



Since $r = s - S$,
For $P > 3$

$$\int_a^b \int_A^B (s-S)^P \, dS \, ds = \int_a^b \left[-\frac{(s-S)^{P+1}}{P+1} \right]_A^B ds$$

$$= \frac{1}{P+1} \int_a^b (s-A)^{P+1} ds - \frac{1}{P+1} \int_a^b (s-B)^{P+1} ds$$

$$= \frac{1}{P+1} \left[\frac{(s-A)^{P+2}}{P+2} \right]_a^b - \frac{1}{P+1} \left[\frac{(s-B)^{P+2}}{P+2} \right]_a^b$$

$$= \frac{1}{(P+1)(P+2)} \left[(b-A)^{P+2} - (a-A)^{P+2} - (b-B)^{P+2} + (a-B)^{P+2} \right]$$

$$= \frac{L^{P+2}}{(P+1)(P+2)} \left[(n+1)^{P+2} + (n-1)^{P+2} - 2n^{P+2} \right]$$

For $P = x - 3$,

$$\int_a^b \int_A^B r^{x-3} \, ds \, dS = \frac{L^{x-1}}{(x-2)(x-1)} \left[(n+1)^{x-1} + (n-1)^{x-1} - 2n^{x-1} \right]$$

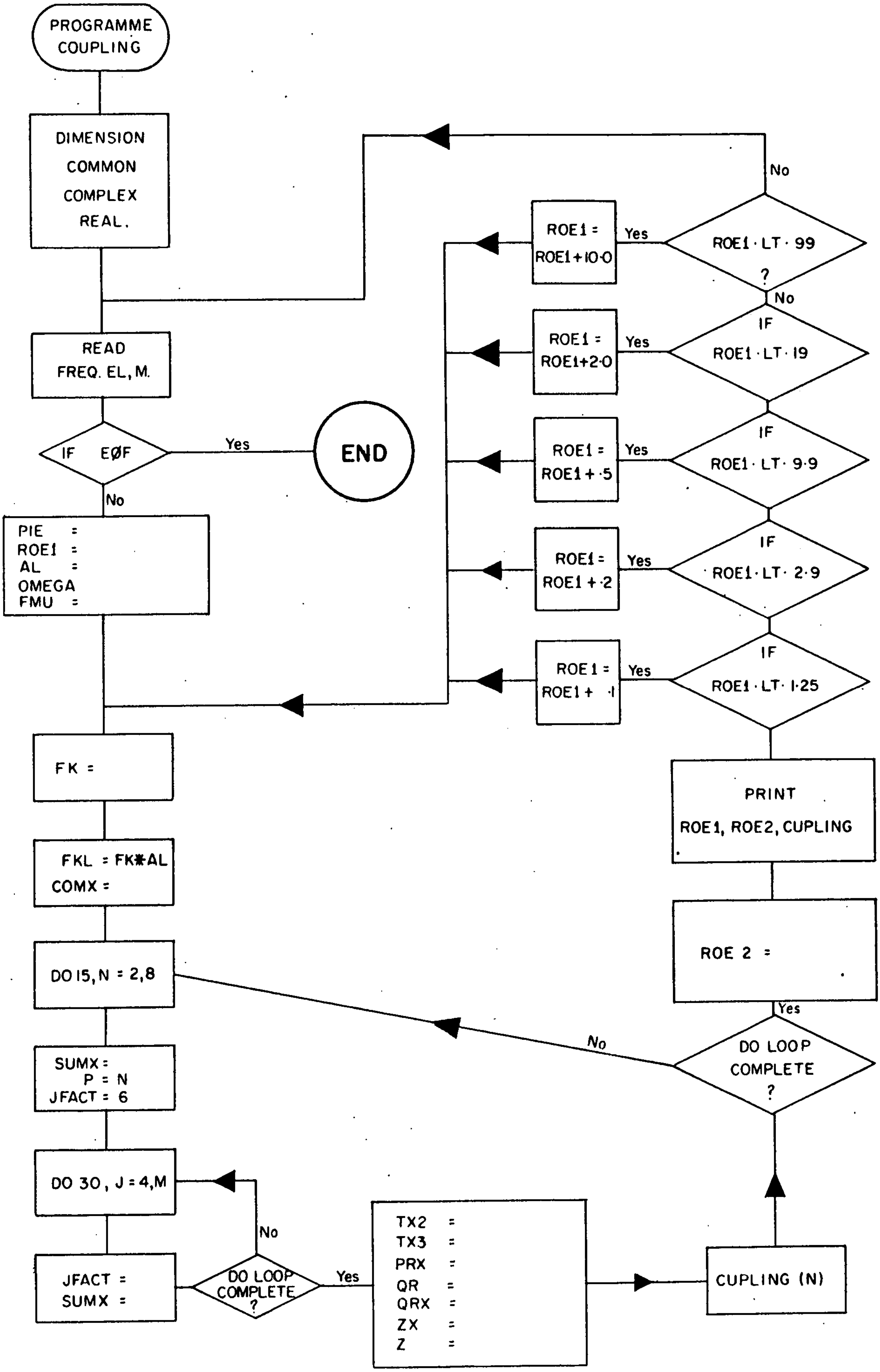
$b - A = (n+1)L$	$a - B = (n-1)L$
$b - B = nL$	$B - A = L$
$a - A = nL$	$b - a = L$

APPENDIX 5

PROGRAMME LISTING

```
      PROGRAM COUPLING
      DIMENSION CUPLING(8)
      COMPLEX COMX,SUMX,TX2,TX3,PRX,QRX,ZX
      REAL JFACT
    2 FORMAT(6X,F5.2,14X,F4,12X,I2)
  111 FORMAT (5X,54HINDUCTIVE COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATI
     1ON/,5X10HFREQUENCY=,F4.1,5H CPS.,4X,7HLENGTH=,F4 ,8H FEET = ,F5.1,
     27H METRES//)
  112 FORMAT(2X,10HROE1  ROE2,6X,3HN=2,4X,3HN=3,4X,3HN=4,4X,3HN=5,4X,3HN
     1=6,4X,3HN=7,4X,3HN=8/)
  116 FORMAT(F6.1,1X,F5.1,2X,7(1X,F6.1))
  119 FORMAT(1H1)
    1 READ 2, FREQ,EL,M
      IF(EOF,60)27,3
    3 PIE =3.14159
      ROE1=0.1
      AL=EL    *12./39.371    OMEGA =2*PIE*FREQ    FMU=4*PIE*10.0**(-7.0)
      PRINT 119
      PRINT 111, FREQ,EL,AL
      PRINT 112
    5 FK=SQRT(( FMU* OMEGA)/ROE1)
      FKL= FK * AL
      COMX = CMPLX(FK,FK)/SQRT(2.0) *AL
      DO 15 N=2,8
      SUMX = CMPLX(0.0,0.0)
      P = N
      JFACT = 6.0
      DO 30, J=4,M
      JFACT = JFACT*J
      SUMX = SUMX + (-COMX)**J*((P+1)**(J-1)+(P-1)**(J-1)-2*P**(J-1))/((
     1 J-2)*JFACT)
   30 CONTINUE
      TX2 =  (COMX)**2 *(ALOG((P+1)/(P-1)) +P*ALOG((P*P-1)/(P*P)))/2.0
      TX3 = COMX**3/3.0
      PRX = SUMX + TX2 - TX3
      QR =-2.0/(N*N*N-N)
      QRX = CMPLX(QR,0.0)
      ZX = PRX + QRX
      Z  = CABS(ZX)
   15 CUPLING(N) = (ABS (QR) - Z) *100.0 / Z
      ROE2 =ROE1 *39.371 /(24 *PIE)
      PRINT 116,ROE1,ROE2,(CUPLING(N),N=2,8)
   33 IF(ROE1.LT.1.95)17,18
   17 ROE1=ROE1+ 0.1
      GO TO 5
   18 IF(ROE1.LT.2.9)19,20
   19 ROE1=ROE1+0.2
      GO TO 5
   20 IF(ROE1.LT.9.9)21,22
   21 ROE1=ROE1 +0.5
      GO TO 5
   22 IF(ROE1.LT.19.0)23,24
   23 ROE1=ROE1+2.0
      GO TO 5
   24 IF(ROE1.LT.99.0)25,1
   25 ROE1=ROE1+10.0
      GO TO 5
   27 END
```

APPENDIX 5 (CONT)

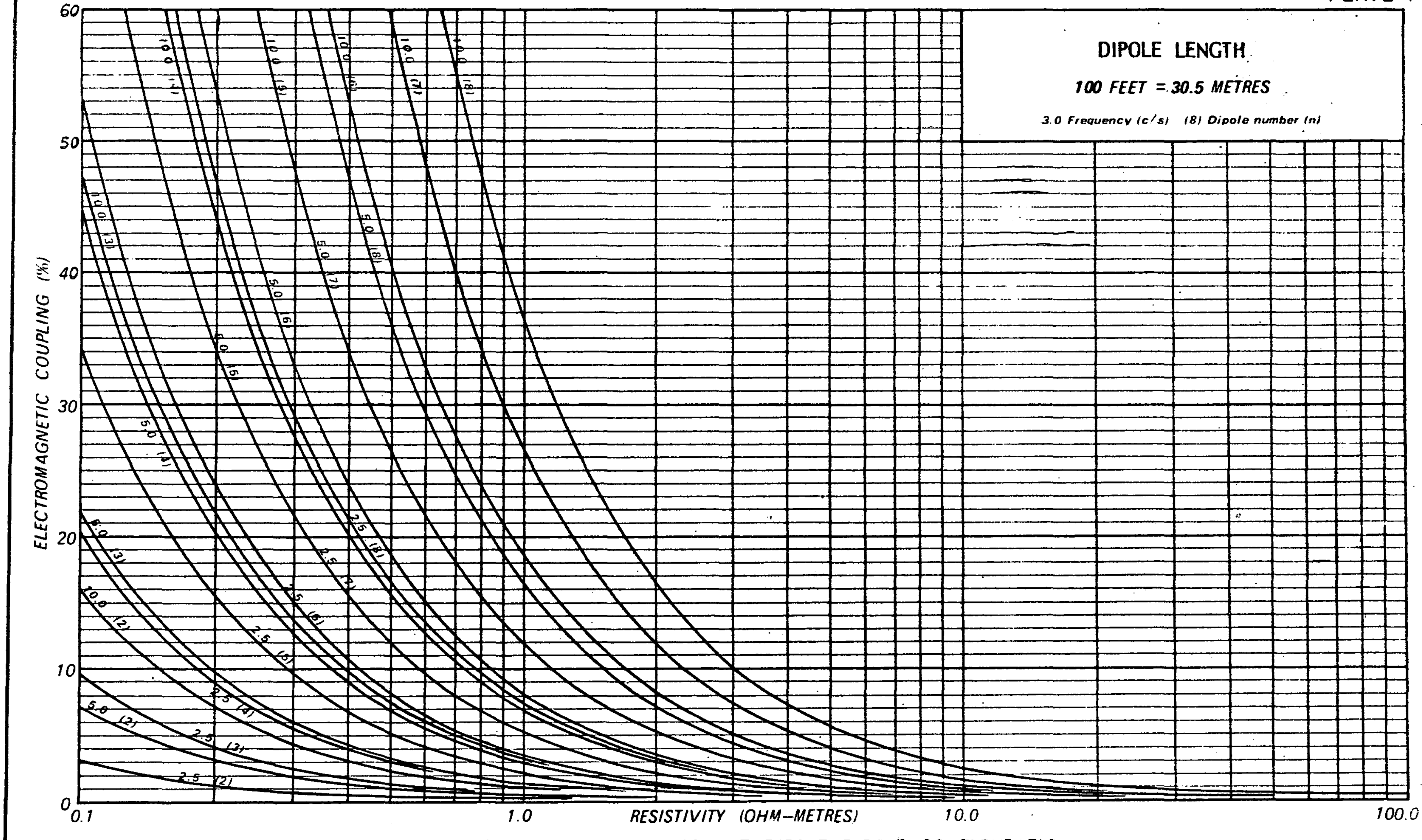


FLOW DIAGRAM – PROGRAMME COUPLING

G29—115A

TO ACCOMPANY RECORD NO 1971/36

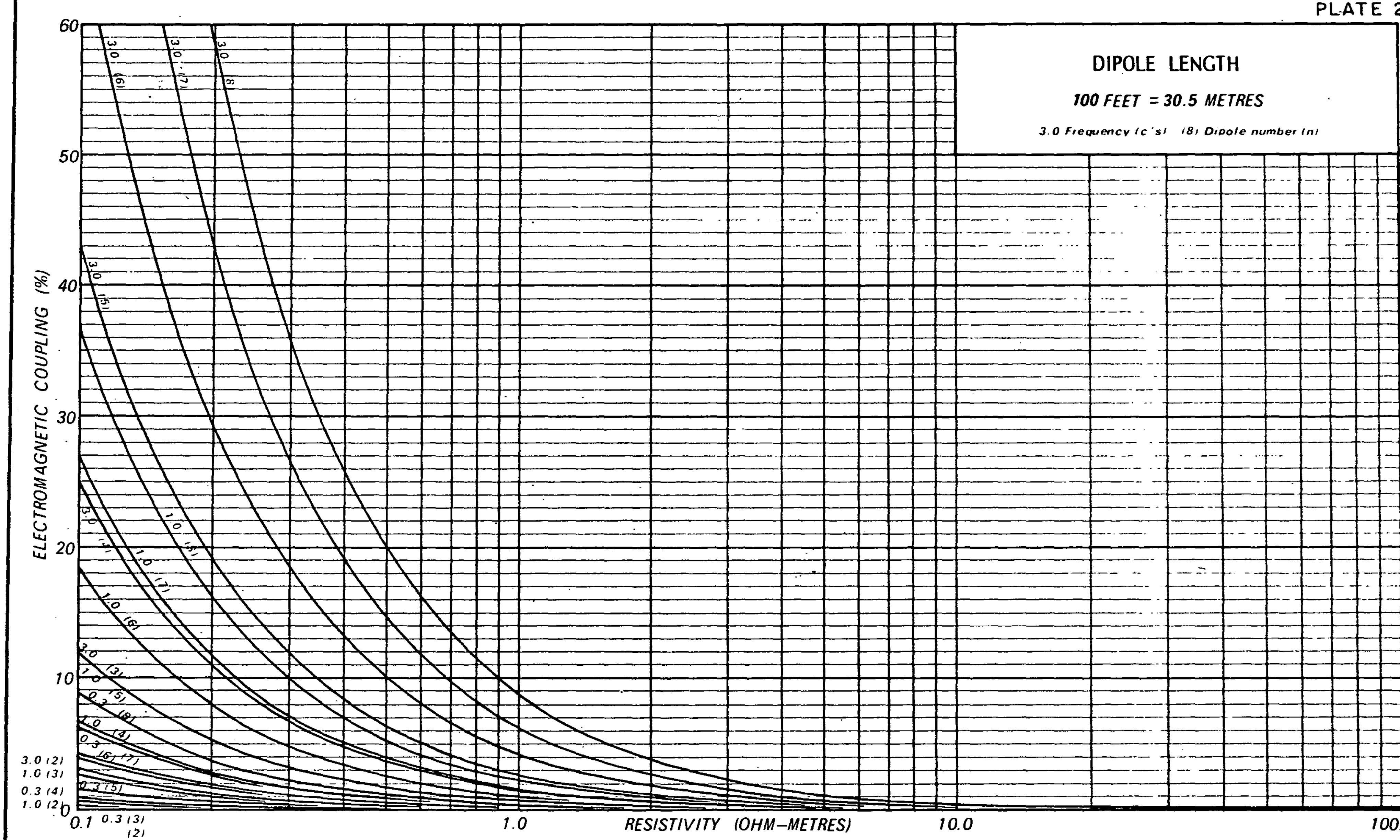
PLATE I



ELECTROMAGNETIC COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATION

(Based on G441-39) RECORD NO 1971/36

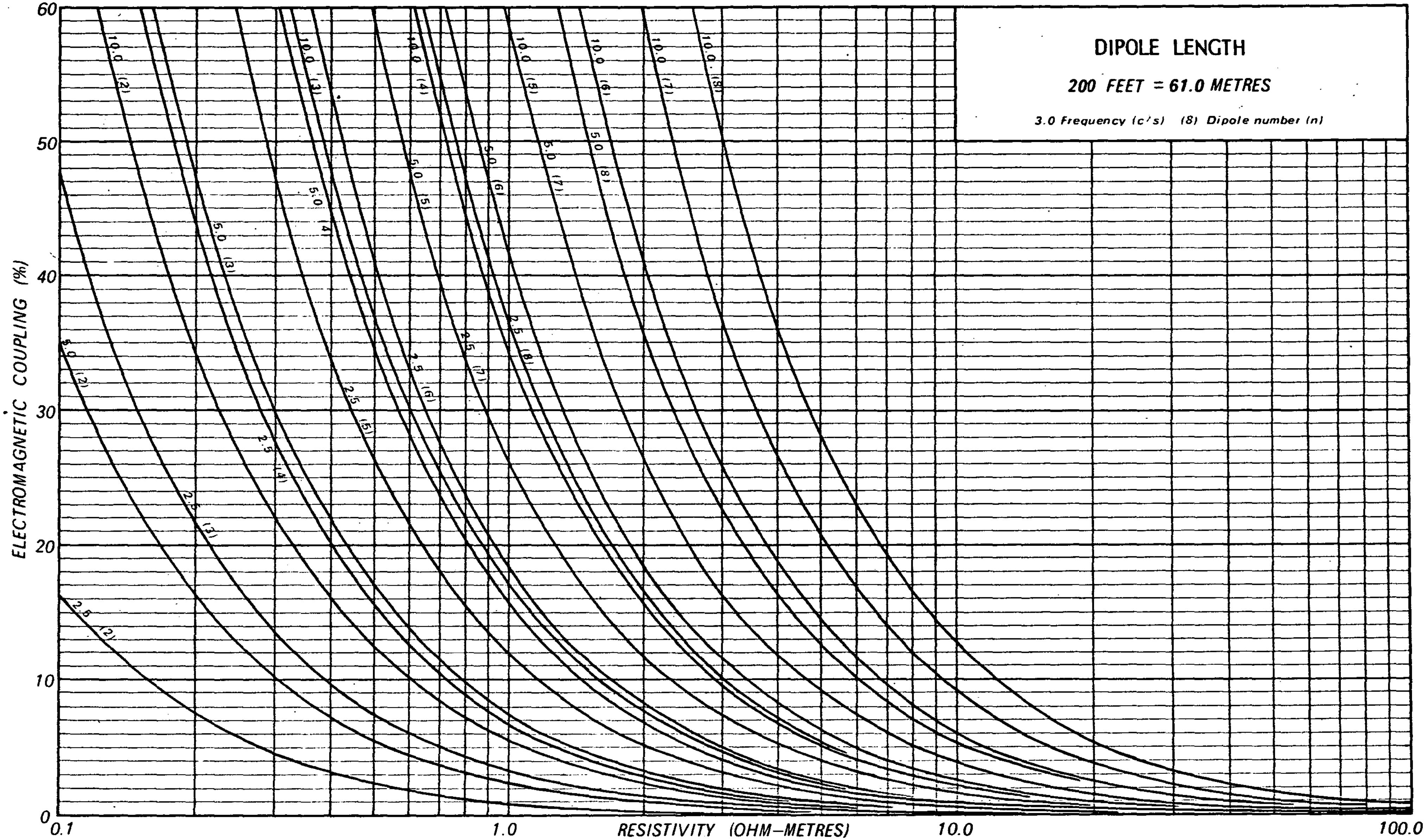
G29-87A



(Based on G441-39) RECORD NO 1971/36

ELECTROMAGNETIC COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATION

G29-85A

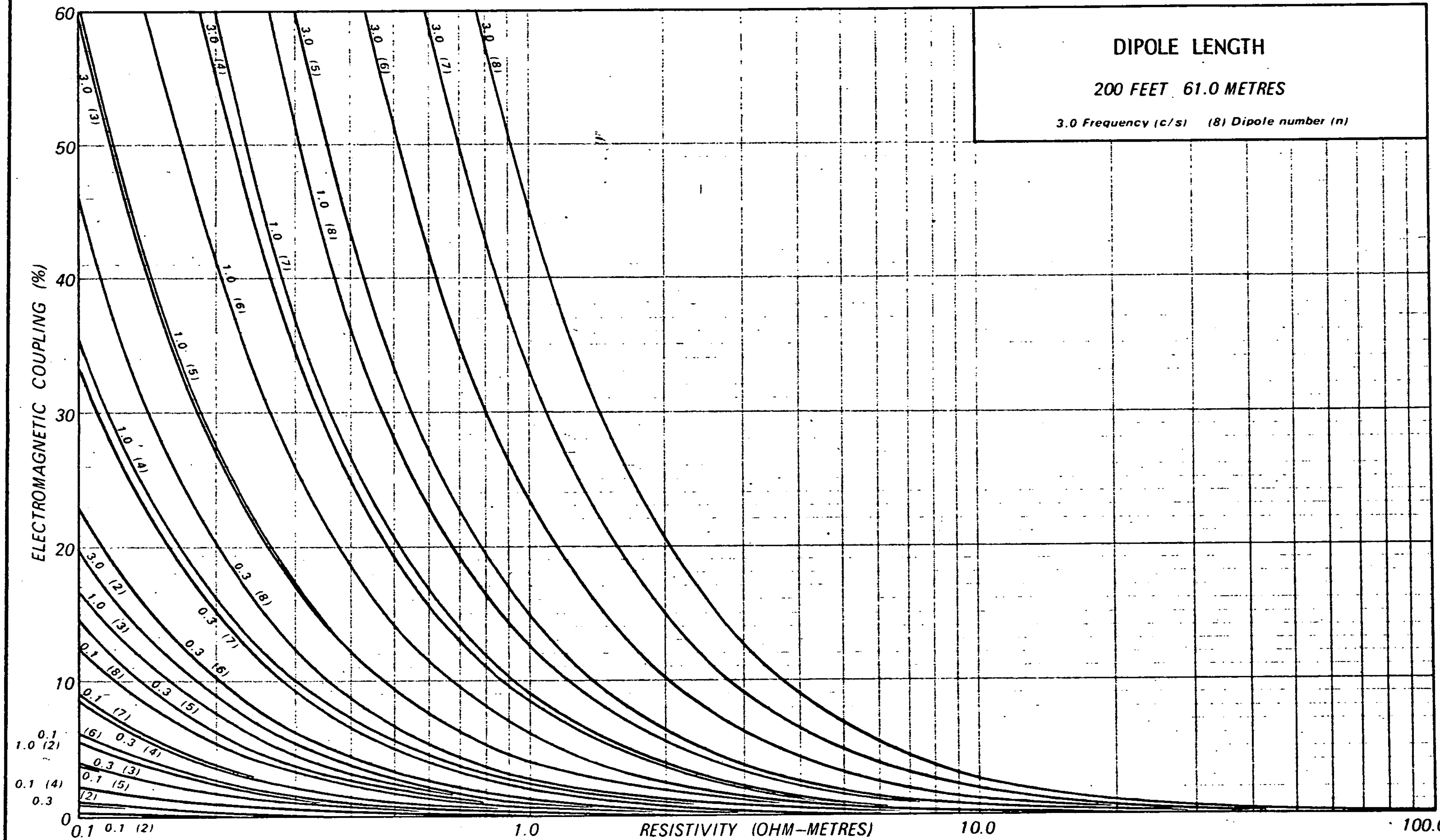


ELECTROMAGNETIC COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATION

(Based on G441-39) RECORD NO 1971/36

G29-84A

PLATE 4

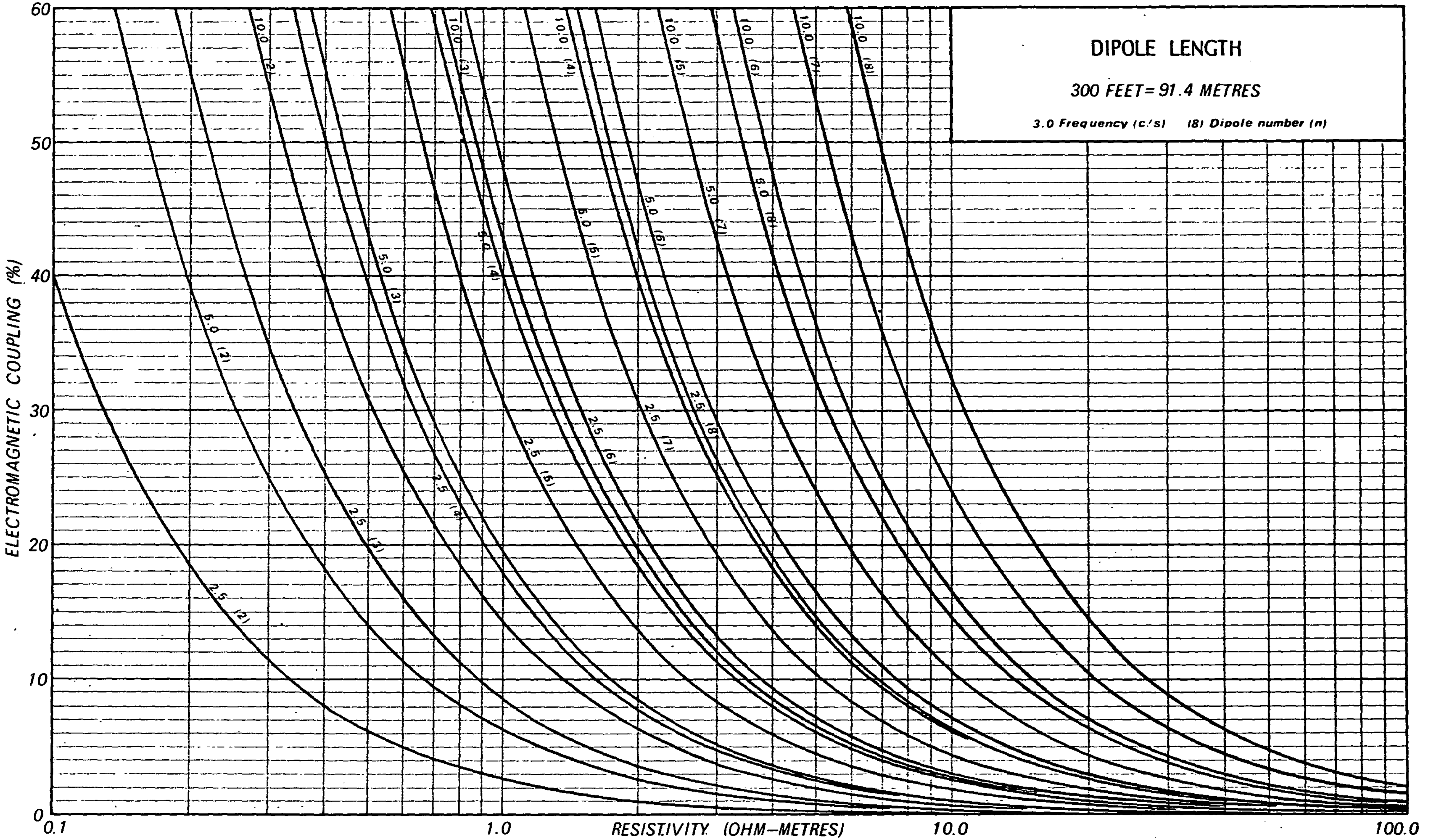


ELECTROMAGNETIC COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATION

(Based on G441-39) RECORD NO 1971/36

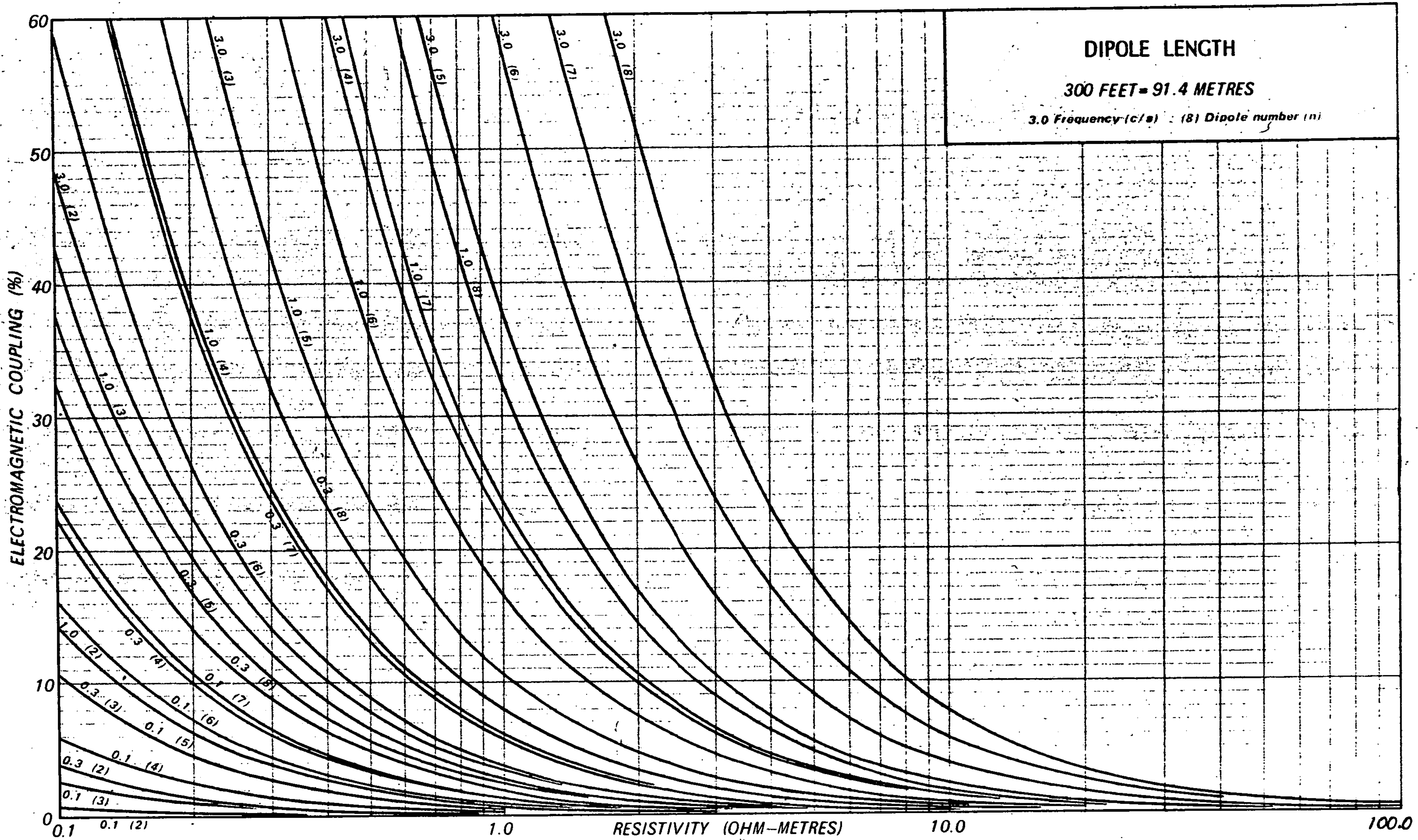
G29-89

PLATE 5



ELECTROMAGNETIC COUPLING FOR THE DIPOLE–DIPOLE CONFIGURATION

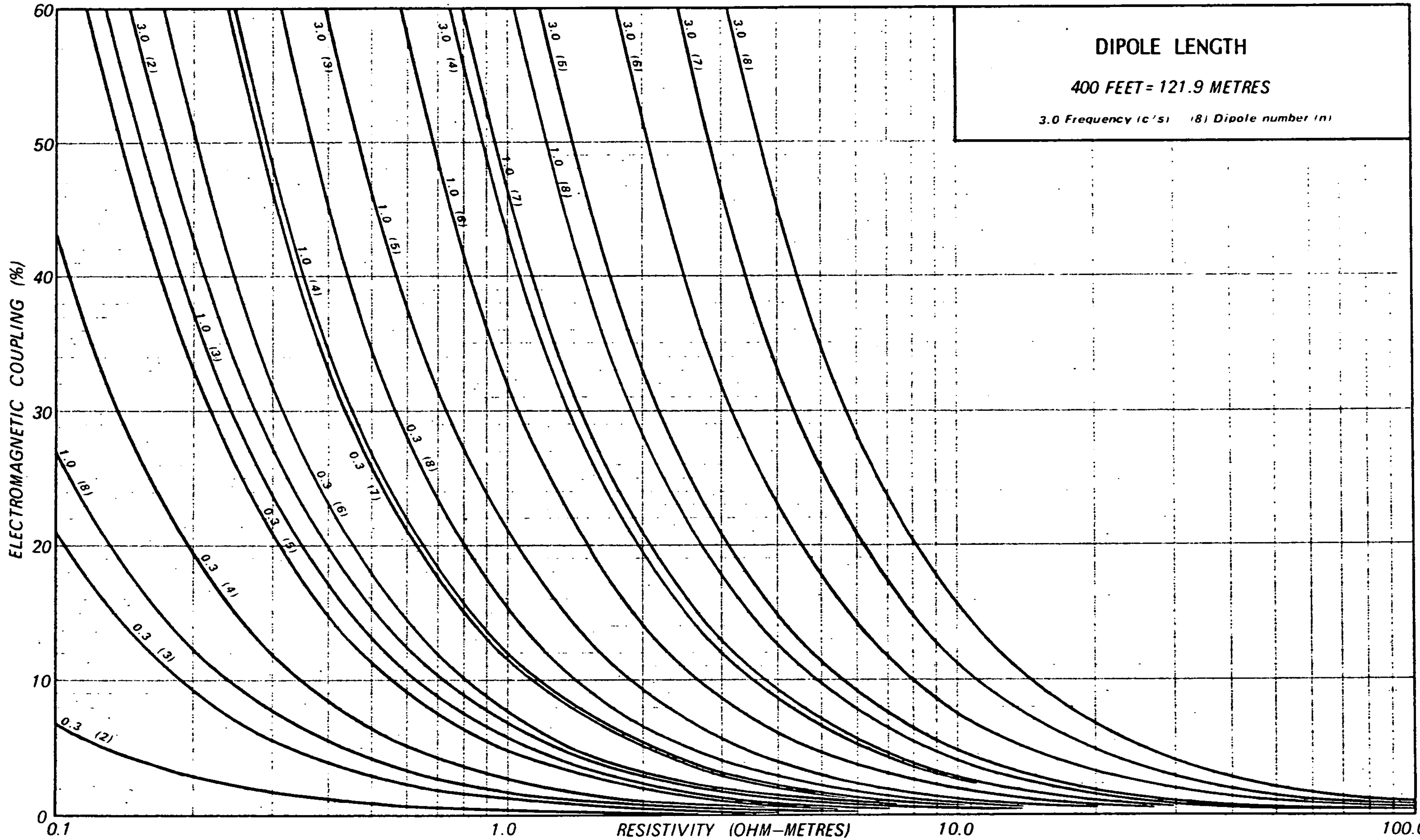
(Based on G441–39) RECORD NO 1971/36 G29–83



ELECTROMAGNETIC COUPLING FOR THE DIPOLE–DIPOLE CONFIGURATION

(Based on G441–39) RECORD NO 1971/36

G29–90

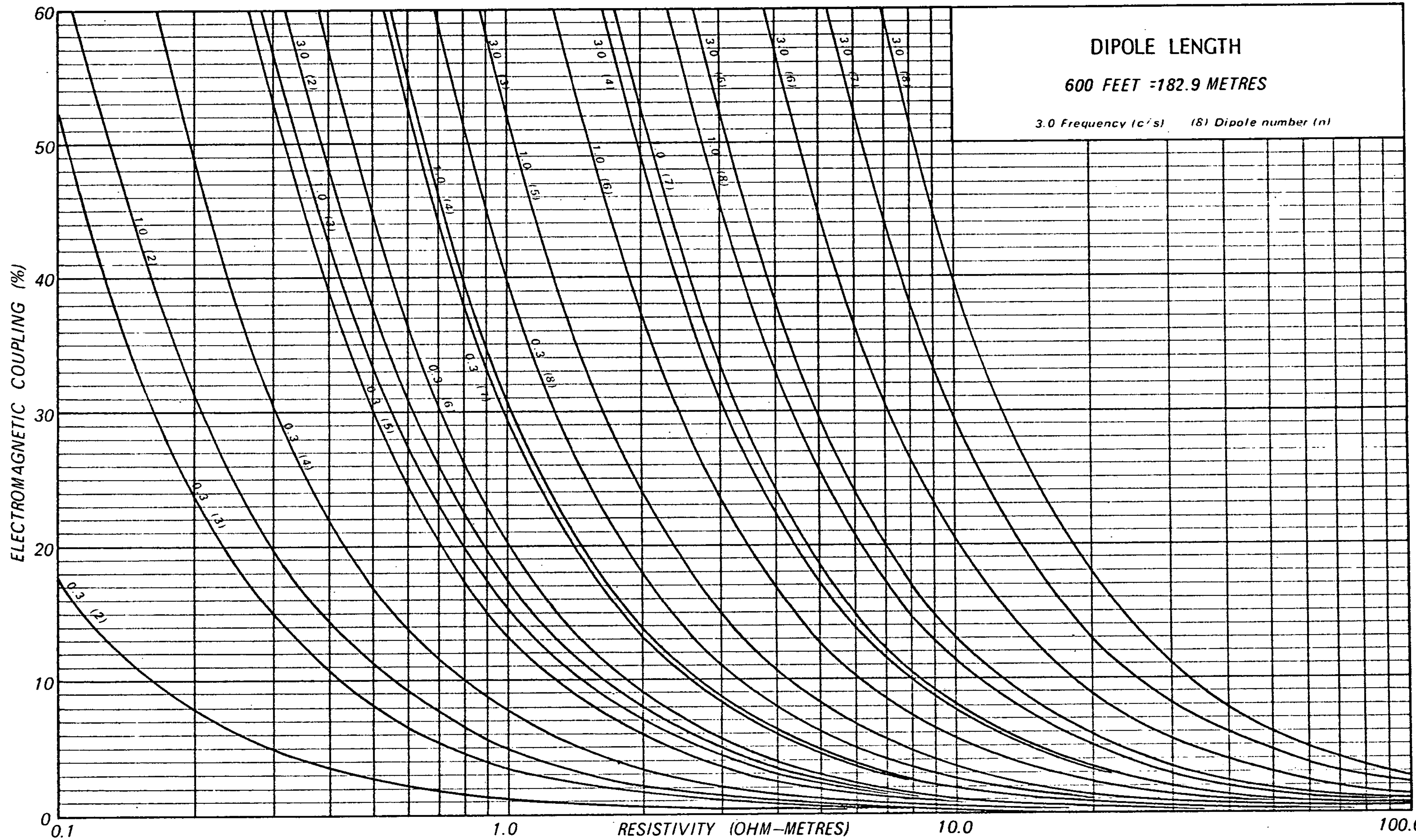


ELECTROMAGNETIC COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATION

(Based on G441-39) RECORD NO 1971/36

G29-86

PLATE 8



ELECTROMAGNETIC COUPLING FOR THE DIPOLE-DIPOLE CONFIGURATION

(Based on G441-39) RECORD NO 1971/36

G29-88A