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Record 1971/54

ASTATIC MAGNETOMETER DESIGNED AND BUILT BY THE BUREAU OF MINERAL RESOURCES

by

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SUMMARY

An astatic magnetometer was built by the Bureau of Mineral Resources, and is housed at the Australian National University, Canberra. Three orthogonal pairs of square Helmholtz coils annul the Earth's magnetic field throughout a space of dimensions about 20 cm. The astatic magnet pair has an oscillation period of 8 seconds, and the astaticism (2P/P') achieved was greater than 1000. The measured sensitivity is 1.18×10^{-7} gauss/mm.

The Record gives the theory of astatic magnetometers, and gives details of the design, construction, and operation of this magnetometer.

1. INTRODUCTION

Astatic magnetometers are instruments which measure the directions and intensities of small magnetic moments (usually of the order of 10^{-1} to 10^{-7} e.m.n.). Specifically, they are generally used to measure the magnetic moments of rock specimens.

Basically, an astatic magnetometer consists of a magnet suspended on a torsion fibre. When a magnetic specimen is brought near this magnet, the two magnetic fields interact to cause the magnet to be deflected, that is rotated about the axis of its suspension fibre. The amount of this deflection is a measure of the magnetic moment of the specimen.

However, the actual instrument is, of course, much more complicated than this to enable measurements to be made accurately. The magnetometer consists of the following basic units (see Plates 1-3):

1. The Magnet System: This consists of a pair of magnets of equal moment fastened antiparallel at either end of, and at right angles to, a short glass tube. This system is suspended on a torision fibre inside a housing. Rigidly fastened to the glass tube is a small mirror which reflects a light beam to give a measure of the deflection of the magnet system. This type of magnet system is insensitive to constant magnetic fields (such as the geomagnetic field), but very sensitive to field gradients (such as those produced by magnetic specimens near the magnet system).

2. The De-Gaussing Coils: Three orthogonal pairs of Helmholtz coils are mounted symmetrically around the magnet system. Electric currents are passed through these coils such that they produce a magnetic field equal to, but opposite in direction to, the geomagnetic field at their centre. Thus the magnet system is suspended in magnetic field-free space, and the specimens are raised into this space during measurement to eliminate the effect of components susceptibly induced in the specimens by the geomagnetic field.

3. The Piston: A piston is mounted below the magnet system. On top of this, a specimen can be raised beneath the system, can be rotated, and can be traversed at right angles to the plane of the magnets. These movements are to place the specimen in the desired sequence of positions required for a measurement.

4. The Bench: On the bench at the other end of the room in which the magnetometer is installed, are mounted the controls for the piston movement, together with the measurement scale. The bench is far enough away so that no magnetic interference is created by the operator, and to allow a long light path for the optical measuring system.

5. The Trolley: Running from near the bench to the piston is a trolley-way. The trolley, which can be wound up and down this trolley-way, is to carry the specimens in the specimen holder to and from the piston without the operator needing to approach the magnet system.

6. The Measurement System: The system used to measure the deflections of the magnet system consists of: a galvanometer lamp; the mirror on the magnet system, whith a suitable lens mounted in front of it; and a scale mounted on the bench in front of the operator.

7. The Sensitivity Coil: To calibrate the instrument, a small coil is mounted vertically above the magnet system, and used to apply a known magnetic field gradient across the system.

8. The Zero Coil: Another small coil is mounted above the sensitivity coil. The field of this coil is used to adjust the nil-deflection (i.e. zero) position of the magnet system to bring the light spot back to centre on the measurement scale if it has drifted slightly off-centre.

9. The Control Panel: This panel, mounted on a bench near the operator, is used for distributing and controlling the current to the de-gaussing coils, the sensitivity coil, and the zero coil.

10. The Regulated Power Supply: This supplies voltage-regulated d.c. power to the control panel.

11. The Telescope: A telescope is mounted above the measurement scale. Through this telescope, focussed through a high-mounted mirror near the Helmholtz coils, the rotational position of the top of the piston can be observed.

12. The East-West Indicator: A pointer connected mechanically to the base of the piston shows the east-west traverse position of the piston during measurement.

13. The Lamps: The room needs to be at a low level of illumination to enable the light spot to be seen on the scale. However, several parts of the instrument need to be seen clearly, so several shrouded lamps are used for illumination of the bench, the east-west indicator, the top of the piston, the piston height scale, and the specimen trolley when it is withdrawn to the operator's end of the trolley-way.

The various parts of the magnetometer built by the Bureau of Mineral Resources and housed in the Australian National University's laboratory on Black Mountain, Canberra, are considered in detail in the following chapters, together with the theory and practice of operation of the instrument.

The co-operation of the Department of Geophysics & Geochemistry of the Australian National University, which provides accommodation for the instrument, is gratefully acknowledged.

2. DE-GAUSSING COILS

(i) Purpose

In theory, the magnet system is constructed so that it should be affected only by field differences between the top and bottom magnets (i.e. by field gradients). Thus constant fields (such as the Earth's magnetic field) should not cause any deflection (i.e. rotation) of the system. This is because the magnets of the system are ideally of equal dimensions, of equal magnetic moments, and oriented accurately anti-parallel. A constant field should then produce the same torques on each magnet, but in opposite directions, resulting in no deflection of the system.

In practice, however, it is impossible to obtain two magnets exactly equal and to align them exactly antiparallel. Thus a change in field will produce some deflection. To elimate any errors from this cause, it is necessary to have as little magnetic field as possible in the region of the magnet system.

Also, rock specimens should be in field-free space during measurement, since any field present will produce an induced magnetization in the specimen.

Thus de-gaussing coils are necessary to annul the Earth's magnetic field within a space sufficient to contain the magnet system and the specimen during measurement. This is accomplished by using coils carrying d.c. electric currents to generate a magnetic field equal and opposite to the Earth's field.

(ii) Type of de-gaussing coils

A Helmholtz pair of coils is a set of two identical, circular coils. These are placed with the planes of the two coils parallel and a set distance apart (usually a distance equal to the radius of the coils). When equal currents flow in the same sense in these coils, a region of approximately constant field is produced midway between them. Thus, using this system of coils, a constant field may be annulled over a small region by generating an equal and opposite field.

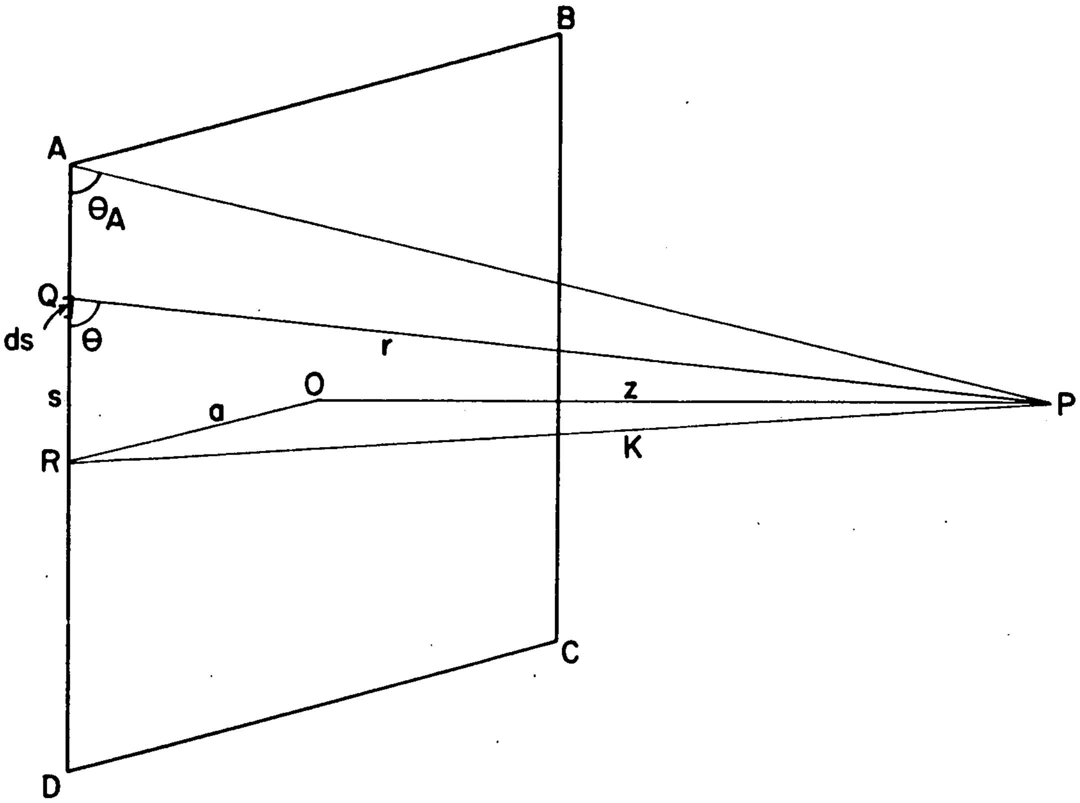


FIGURE 1

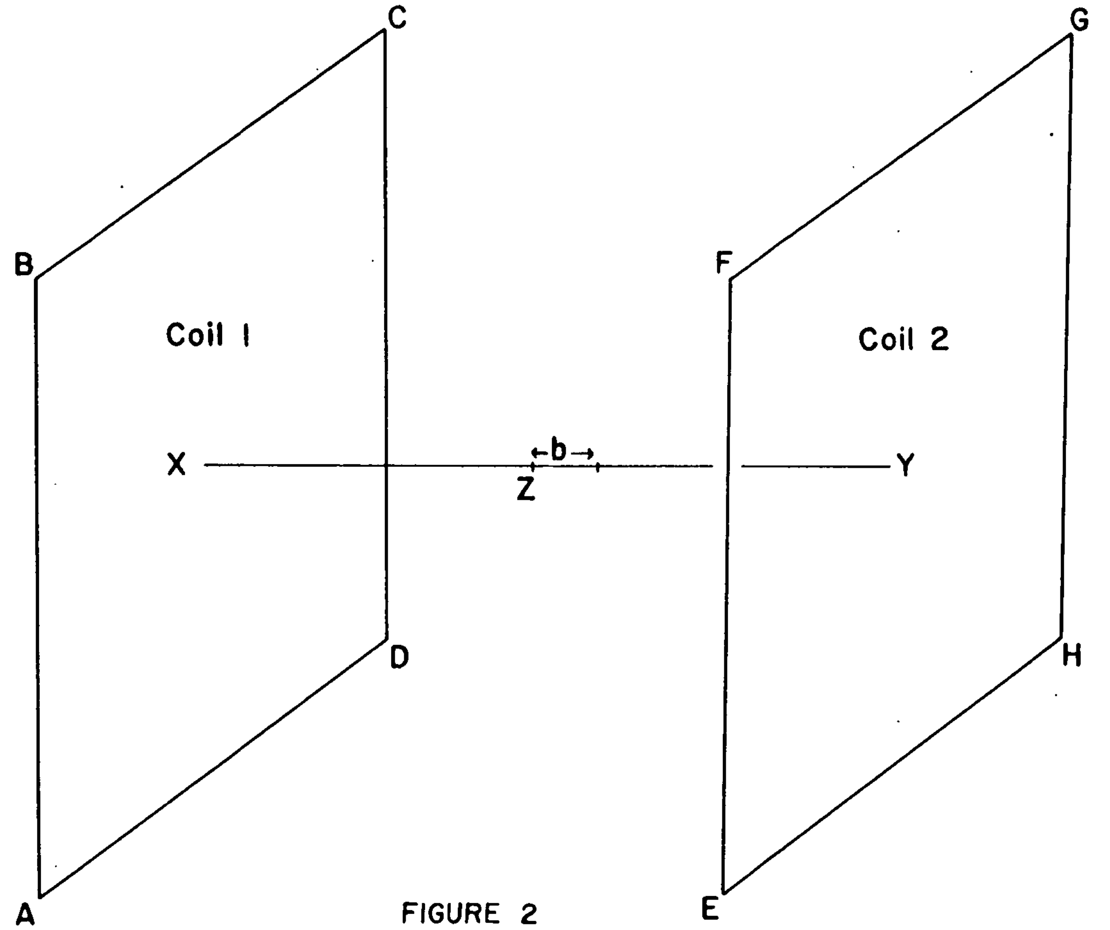


FIGURE 2

In theory it should be possible to create a space free of the Earth's field with a single Helmholtz pair, by aligning the mutual axis of the two coils along the Earth's field. But the difficulties involved in aligning the coils are great, and once the system was installed, changing the orientation of the coils to account for changes due to, say, secular variation, would be a major task.

The more usual arrangement is to use three orthogonal pairs of coils, each pair annulling one of three components of the Earth's field (H_{ns}, H_{ew}, or Z). This system is simple to install, and adjustment involves simply alteration of the coil currents.

Circular Helmholtz coils have the highest symmetry, but square coils are simpler to made and easier to install, and allow better access to the magnet system. Therefore, since square coils have proved themselves to be perfectly satisfactory elsewhere, they have been used here.

(iii) Theory of square Helmholtz coils

Field of a single square coil. In Figure 1, ABCD is a single winding of the coil.

Let AR = RO = a = half the length of a side of the coil.
Let P be a point on the axis of the coil and PO = z.
Let Q be a point on AR such that QR = s.
Let $\angle RQP = \Theta$, RP = k, and QP = r.

The field at P in the direction P to O due to a small length ds of the coil at Q, carrying a current i in the direction R to A, is

$$dH = (i\ ds \sin\Theta)/ r^2 \qquad \text{.........................(1)}$$

where dH is perpendicular to ds

But $\tan\Theta = K / s$, i.e. $s = K \cot\Theta$

Thus $ds / d\Theta = -K \operatorname{cosec}^2\Theta$

$ds = -K \operatorname{cosec}^2\Theta\ d\Theta$

and $1 / r^2 = \sin^2\Theta / K^2$ since $\sin\Theta = K / r$

Thus, from (1)

$$dH = (-K \operatorname{cosec}^2\Theta\ d\Theta \sin\Theta \sin^2\Theta\ i)/K^2$$

$$= (-i \sin\Theta\ d\Theta) / K$$

Integrating from R to A

$$H_{RA} = -\int_{\pi/2}^{\Theta_A} (i \sin\Theta \, d\Theta) / K \quad \text{where } \Theta_A = \angle RAP$$

$$= (i \cos\Theta_A) / K$$

But $\cos\Theta_A = AR / AP = a / AP$

and $AP^2 = AO^2 + OP^2$

$$= AO^2 + z^2$$

$$AO^2 = AR^2 + RO^2$$

$$= a^2 + a^2 = 2a^2$$

Thus $AP = (2a^2 + z^2)^{1/2}$

Therefore $\cos\Theta_A = a(2a^2 + z^2)^{-1/2}$

Therefore the total field at P due to RA (H_{RA}) is

$$H_{RA} = (ia) / K(2a^2 + z^2)^{1/2}$$

$$= (ia) / (a^2 + z^2)^{1/2} . (2a^2 + z^2)^{1/2} \text{ since } K^2 = a^2 + z^2$$

....................(2)

The axial component of this field at P is

$$H_{axial} = H_{RA} . \cos \angle ORP$$

$$= (H_{RA} . a) / K$$

$$= (H_{RA} . a) / (a^2 + z^2)^{1/2}$$

$$= [(ia) / (a^2 + z^2)^{1/2} . (2a^2 + z^2)^{1/2}] . [(a) / (a^2 + z^2)^{1/2}]$$

$$= (ia^2) / (a^2 + z^2) . (2a^2 + z^2)^{1/2}$$

By symmetry over the whole coil, the radial components cancel, so that the total field due to the whole coil at P is the sum of the axial components of the fields generated by the eight segments corresponding to the segment AR. That is, the total field at P due to a single turn (H) is

$$H = (8ia^2) / (a^2 + z^2) . (2a^2 + z^2)^{1/2}$$

....................(3)

or, dividing top and bottom by a^3

$$H = \frac{8ia^2}{a^3} / \left[\left(\frac{a^2 + z^2}{a^2}\right) . \left(\frac{2a^2 + z^2}{a^2}\right)^{1/2}\right]$$

$$= \frac{8i}{a} / [1 + (z/a)^2] [2 + (z/a)^2]^{1/2}$$

$$= \frac{8i}{a} [(1 + x^2)^{-1} . (2 + x^2)^{-1/2}]$$

where $x = z/a$

Separation of coils. For a pair of square Helmholtz coils, it is required that they produce a constant field at a central, axial position between them. Thus, for each of the coils we must find a position along the axis where the field gradient produced is constant (i.e. a point of inflection). The two coils are then placed twice this distance apart, so that the zones of constant gradient coincide, producing a region of approximately constant field by the summation of the fields produced by the two coils.

For the position where the field gradient is constant

$$d^2H / dz^2 = 0$$

or $d^2H / dx^2 = 0$ where $x = z / a$ and a is a constant

$$\text{Now } \frac{dH}{dx} = \frac{-8i}{a}\left[\frac{3x^3 + 5x}{(x^2+2)^{3/2}.(x^2+1)^2}\right] \qquad \text{..........(4)}$$

$$\text{and } \frac{d^2H}{dx^2} = \frac{-8i}{a}\left[\frac{(9x^2+5)(x^2+2)(x^2+1) - 3x(3x^3+5x)(x^2+1) - 4x(3x^3+5x)(x^2+2)}{(x^2+1)^3\,(x^2+2)^{5/2}}\right] \qquad \text{................(5)}$$

Hence for the position of constant gradient, from (5):

$$6x^6 + 18x^4 + 11x^2 - 5 = 0$$

The required solution is $x = 0.5445$

i.e. $z / a = 0.5445$

That is, the gradient of the axial component of the magnetic field due to a single square coil is constant at a distance along the axis of 0.5445 times the half-length of the side of the square from the coil centre. For a square Helmholtz coil pair, the separation of the two coils is twice this distance. Therefore the separation d of the two coils is

$$d = 1.0890\ a$$

$= 0.5445$ L where L is the length of a side of one of the coils.

Field of a square Helmholtz coil pair. From (3) the field of a single coil at the point of constant gradient on the axis would then be

$$\frac{8\, i\, a^2 N}{(a^2 + z^2)\,(2a^2 + z^2)^{\frac{1}{2}}}$$ where N = number of turns of wire

For a coil pair, the total field at the central point between them, if they are the correct distance apart, would be twice this since the fields are additive.

But the distance z from each coil to the centre of the pair is 0.5445a. Thus the total field (H) is

$$H = \frac{16\, i\, a^2\, N}{(a^2 + (0.5445a)^2).(2a^2 + (0.5445a)^2)^{\frac{1}{2}}}$$

$$= \frac{16\, i\, a^2\, N}{1.964686\, a^3} = \frac{8.144\, i\, N}{a} \quad \text{e.m.u.} \qquad \text{..........(6)}$$

If the field is measured in oersteds or gauss, then

$$H = \frac{0.8144\, i\, N}{a}$$ oersteds (gauss) where i is in amps
a is in cm .

or $H = 1.6288 \times 10^{-3} \cdot i\, N/L$ oersteds ....(7)

where i = current in milliamps
N = number of turns in each coil
L = length of side of coil in cm .

Zone of zero field. In Figure 2, ABCD is coil 1 of the square coil pair, and EFGH is coil 2. XY is the axis of the pair, and Z is the central position between them.

The total field (H) at the central point Z is due to two equal contributions (H_1 and H_2) from coils 1 and 2 respectively. From equation (6) the field at Z is

$$H = H_1 + H_2 = 8.144\, i\, N/a$$

For there to be a zone of approximately constant field, the total field H generated by the coils must remain almost constant over a region on both sides of Z.

i.e. $H_1 + H_2 = 8.144\ i\ N/a$ for a region $Z \pm b$

where b is a small distance along XY

From equation (3)

$$H_1 = \frac{8\ i\ a^2\ N}{(a^2 + z_1^2)\ (2a^2 + z_1^2)^{1/2}}$$

$$H_2 = \frac{8\ i\ a^2\ N}{(a^2 + z_2^2)\ (2a^2 + z_2^2)^{1/2}}$$

where z_1 is the distance of Z from coil 1 , and z_2 the distance from coil 2.

Consider a point displaced a small distance b along the axis XY for Z such that $z_1 = Z + b$ and $z_2 = Z - b$.

Then

$$H = H_1 + H_2$$

$$= \frac{8\ i\ a^2\ N}{(a^2 + (Z + b)^2)\ (2a^2 + (Z + b)^2)^{1/2}} + \frac{8\ i\ a^2\ N}{(a^2 + (Z - b)^2)\ (2a^2 + (Z - b)^2)^{1/2}}$$

$= 8.144\ i\ N/a$ for a zone of constant field.

Therefore

$$\frac{8.144}{8} = \frac{1}{(1 + (\frac{Z + b}{a})^2)(2 + (\frac{Z + b}{a})^2)^{1/2}} + \frac{1}{(1 + (\frac{Z - b}{a})^2)(2 + (\frac{Z - b}{a})^2)^{1/2}}$$

But $Z = 0.5445\ a$

Thus

$$1.0180 = \frac{1}{(1.2965 + 1.0890\ b/a + b^2/a^2)\ (\ 2.2965 + 1.0890\ b/a + b^2/a^2)^{1/2}}$$

$$+ \frac{1}{(1.2965 - 1.0890\ b/a + b^2/a^2)\ (2.2965 - 1.0890\ b/a + b^2/a^2)^{1/2}}$$

The right hand side of this expression is found, on substitution of values for b/a, to give the following values:

b/a	Expression
0.00	1.0180
0.01	1.0180
0.02	1.0180
0.03	1.0179
0.04	1.0179
....	
0.10	1.0179
0.11	1.0179
0.12	1.0178

Thus to an accuracy of 1 part in 10^4, there is a zone of constant field extending a distance b either side of the centre point, such that

$$b = 0.11\ a$$

i.e. the total length of the zone of constant field along the axis is given by

$$2b = 0.22\ a$$
$$= 0.11\ L$$

That is, a pair of square coils, if separated by 0.5445 times the length of a side, will give a zone of constant field (to within 1 part in 10^4) of length 0.11 times the side length (or approximately one-tenth) along the axis of the coils.

(iv) Details of Square coils constructed

The de-gaussing coils of the BMR astatic magnetometer are three mutually perpendicular pairs of square Helmholtz coils. The largest pair of coils is the pair which annuls the vertical (Z) component of the Earth's field. These are mounted on duralumin and wooden supports, with their axis vertical (the plane of each coil horizontal).

The next largest coils are those which annul the horizontal component in the north-south direction (H_{ns}). These are mounted inside, and fastened to, the Z coils, with their axis along the north-south magnetic meridian.

The smallest are those which annul the horizontal east-west component (H_{ew}). These are mounted inside, and fastened to, the H_{ns} coils, with their axis east-west. If the axis of the H_{ns} coils was aligned accurately along the magnetic meridian, there would be no residual east-west component. In practice, however, perfect alignment of the H_{ns} coils would take a great deal of unnecessary effort, and would not allow easy correction for secular variation.

The formers for each of the individual coils are of U-channel duralumin (see Fig. 3). The sides of each coil are held together by being bolted to square duralumin blocks situated one inside each corner. The two coils of each pair are held apart at the correct distance by four duralumin rods, one inch in diameter bolted to the centres of the corner blocks, except for the vertical coils, which are separated by wooden beams. Specifications of the coils are given in Table 1.

(v) Installation of the coils

In the following sections, all magnetic field measurements were made with a milli-oersted meter. This is similar to a single-component fluxgate magnetometer, except that it is calibrated from 1 gamma to 10^5 gammas (1 oersted). The detecting probe can be placed in various orientations to measure each of the components required.

With the coils assembled, the three components of the Earth's magnetic field parallel to the axes of the three coils were measured with the milli-oersted meter. These were:

Z = 52,000 gammas = 0.52 oersteds

H_{ns} = 24,500 gammas = 0.245 oersteds

H_{ew} = 70 gammas = 0.0007 oersteds

From equation (7), the theoretical current (in milliamps) required to annul each of these components can be calculated as follows:

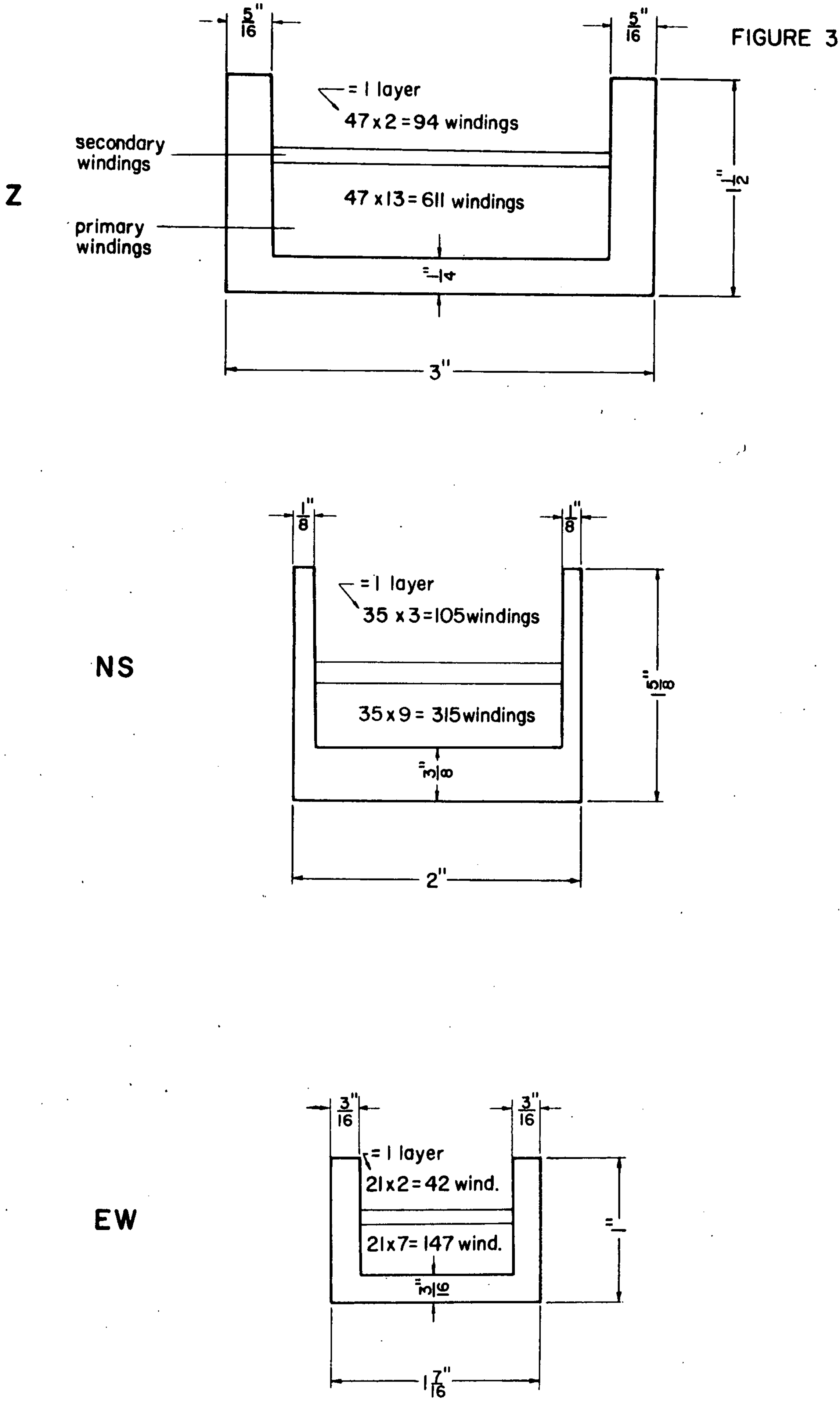
$$H = 1.6288 \times 10^{-3} \frac{i\,N}{L} \text{ oersteds}$$

$$i = \frac{H\,L.10^3}{1.6288N} \text{ milliamps}$$

TABLE 1: Specifications of Square Helmholtz Coils

	Primary Coils			Secondary Coils			Number of turns of wire		Separation of Coils		Size zone of zero field	
	Length side of former cm	Length outside windings cm	Mean side length cm	Length inside windings cm	Length outside windings cm	Mean side length cm	Primary	Secondary	Theoretical cm	Actual cm	Theoretical cm	Actual cm
H_{EW}	190	191.8	190.9	191.8	192.3	192.2	147 (7 layers of 21 turns)	42 (2 layers of 21 turns)	103.9	104.9	21	20
H_{NS}	196	198.3	197.2	198.3	199.1	198.7	315 (9 x 35)	105 (3 x 35)	107.4	108.7	21.7	19
Z	206	209.4	207.7	209.4	209.9	209.6	611 (13 x 47)	94 (2 x 47)	113.1	114.2	22.8	22

FIGURE 3



Details of duralumin formers and windings of the square Helmholtz degaussing coils

For the Z component, i = 108.5 milliamps; the measured current necessary is 117 mA.

For the H_{ns} component, i = 94.2 mA; the measured current necessary is 94 mA.

For the H_{ew} component, i = 0.56 mA; the measured current necessary is 1.1 mA.

The differences between the calculated and actual currents required are due to short-circuiting within the windings of the coils (see section vii).

(vi) Electrical circuits for the Helmholtz coils

Power supply. This is a twin-output, voltage-regulated power supply operating off the normal mains supply (240-V, 50-Hz). Each of its two outputs can supply from 0 to 30 volts and from 0 to 500 milliamps d.c. It was manufactured by the Electronics Workshop of the Research School of Physical Sciences, Australian National University, and is used in lieu of the commercial power supply originally purchased, whose maximum voltage, 18 volts, was found to be insufficient.

Control Panel 1. This contains the following circuits:

(a) The H_{ew} Helmholtz primary coil circuit, to supply the correct current for annulling the EW component.

(b) The H_{ns} primary coil circuit.

(c) The Z primary coil circuit.

(d) A potentiometer 'balance' circuit incorporating three constant-voltage (3V each) Mallory cells in series with a micro-ammeter. This circuit is to ensure the repeatability of the set currents in each of the primary coil circuits. These set currents are obtained by balancing the constant voltage of the Mallory cells against the voltage drop across fixed resistors in each of the circuits. A selector switch allows the potentiometer circuit to be switched into each primary coil circuit in turn.

(e) The sensitivity coil circuit (see chapter 3).

(f) The zero coil circuit (see chapter 3).

In the above circuits, current adjustments are made with one or more helical potentiometers. No milliammeters were included in the primary coil circuits. Once the set currents are obtained (with the aid of the milli-oersted meter to measure the residual field inside the Helmholtz coils) it is not necessary to know the exact values of the currents. The potentiometer circuit ensures that the set currents are repeated. However, to enable the currents to be measured as required, in each circuit sockets and a switch are included so that a milliammeter may be inserted into the circuit.

Control Panel 2. This is a separate control panel for the secondary windings on the Helmholtz coils. Power (20V) is supplied from one side of the power pack, and provision is made for a milliammeter to be plugged into the circuits. A selector switch ensures that only one secondary coil can be used at a time, and a 250-mA fuse is included so that the total current drain on the power supply will be kept below its maximum of 500 mA.

(vii) Determination of currents required to give null field

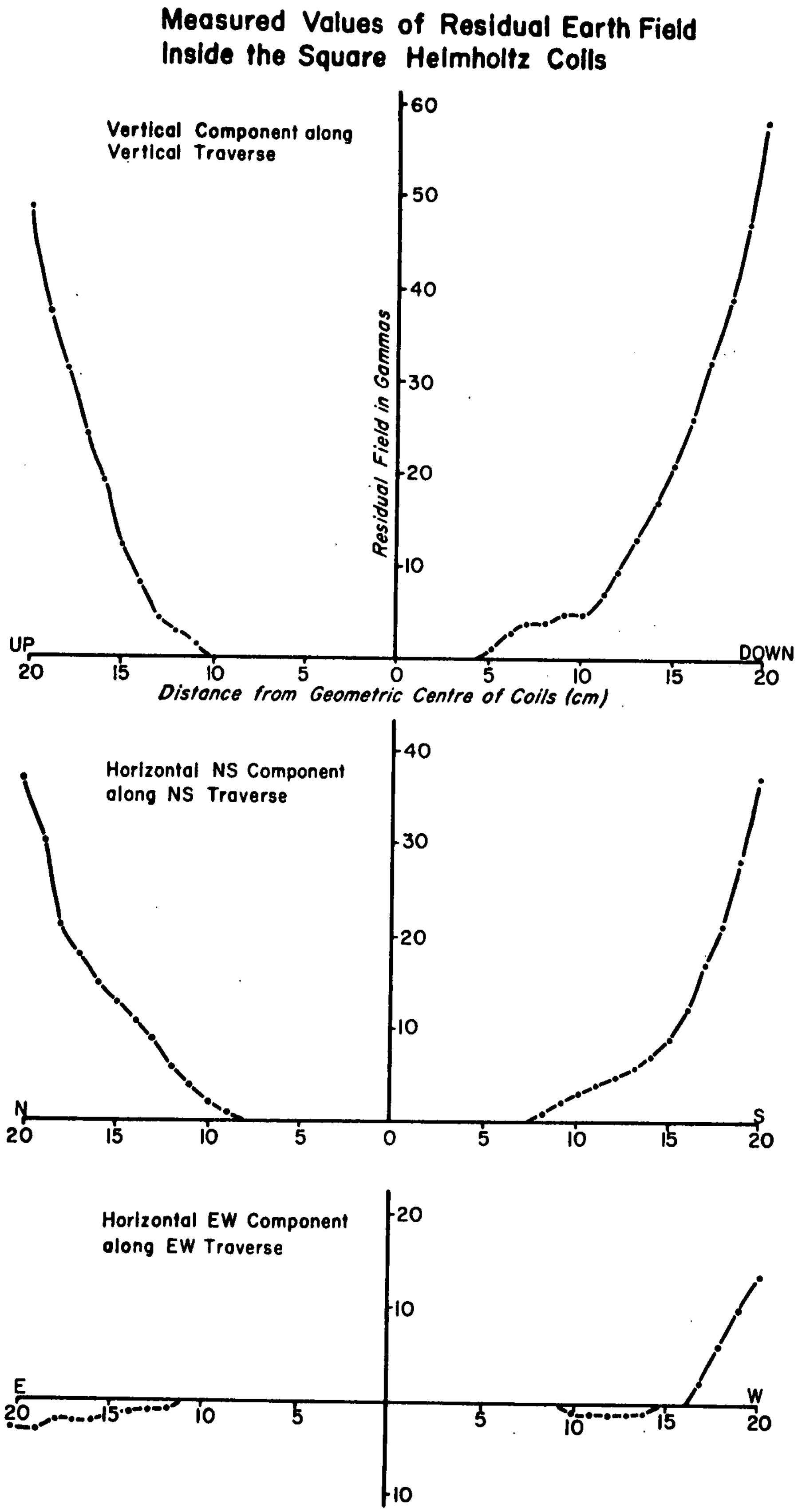
The milli-oersted meter was used to measure the residual part of the Earth's field at the centre of the Helmholtz coil system when the coils were generating an opposing field. The currents were then adjusted until the residual field was zero.

The field-free space was then checked by measuring the residual field along traverses in the NS, EW, and vertical (Z) directions. Ideally, the residual field should be approximately zero over the range calculated in Table 1, and should increase gradually on either side. The actual residual field gradients measured showed no zone of zero field.

These field gradients with no null zone are due to there being, effectively, different numbers of turns of wire in the two coils of each Helmholtz coil pair. This is probably due to short-circuiting of some of the turns during transport of the coils.

To compensate for this effect in any pair, the current flowing through the coil that had effectively the greater number of windings was decreased. This was done by shunting with fixed resistors (in parallel with the coil windings), the correct values of the shunts being found by trial and error. With suitable shunts, the residual fields were found to be quite satisfactory (see Figure 4).

FIGURE 4


Measured Values of Residual Earth Field
Inside the Square Helmholtz Coils
Vertical Component along
Vertical Traverse
Residual Field in Gammas
60
50
40
30
20
10
UP
DOWN
20 15 10 5 0 5 10 15 20
Distance from Geometric Centre of Coils (cm)
Horizontal NS Component
along NS Traverse
40
30
20
10
N
S
20 15 10 5 0 5 10 15 20
Horizontal EW Component
along EW Traverse
20
10
10
E
W
20 15 10 5 5 10 15 20

The shunts and currents actually used are:

Shunts : (a) Top coil of Z pair shunted 1465 ohms

(b) South coil of H_{ns} pair shunted 10,000 ohms

(c) East coil of H_{ew} pair shunted 20 ohms.

Currents : (a) Z coils 117 mA

(b) H_{ns} coils 94 mA

(c) H_{ew} coils 1.1 mA.

The necessity to shunt the Helmholtz coil windings explains the differences between the theoretical and actual currents required, as outlined in (v).

(viii) Calibration of the secondary coils

The secondary coils are separate windings on the Helmholtz coils. They contain fewer turns than the primary windings, and are used to generate known fields in known directions, which may be used during the adjustment of the magnet system.

To calibrate these coils, it is necessary to determine the field strengths produced at the centre of the coils for known currents through each of the coils in turn. This was found by measuring the fields produced, with the milli-oersted meter. The results are plotted in Figure 5, and these graphs fall very near to the following linear equations:-

For H_{ew} secondary $H = 35.8\ I$

H_{ns} secondary $H = 86.2\ I$

Z secondary $H \doteq 73.9\ I$

where H = field in gammas

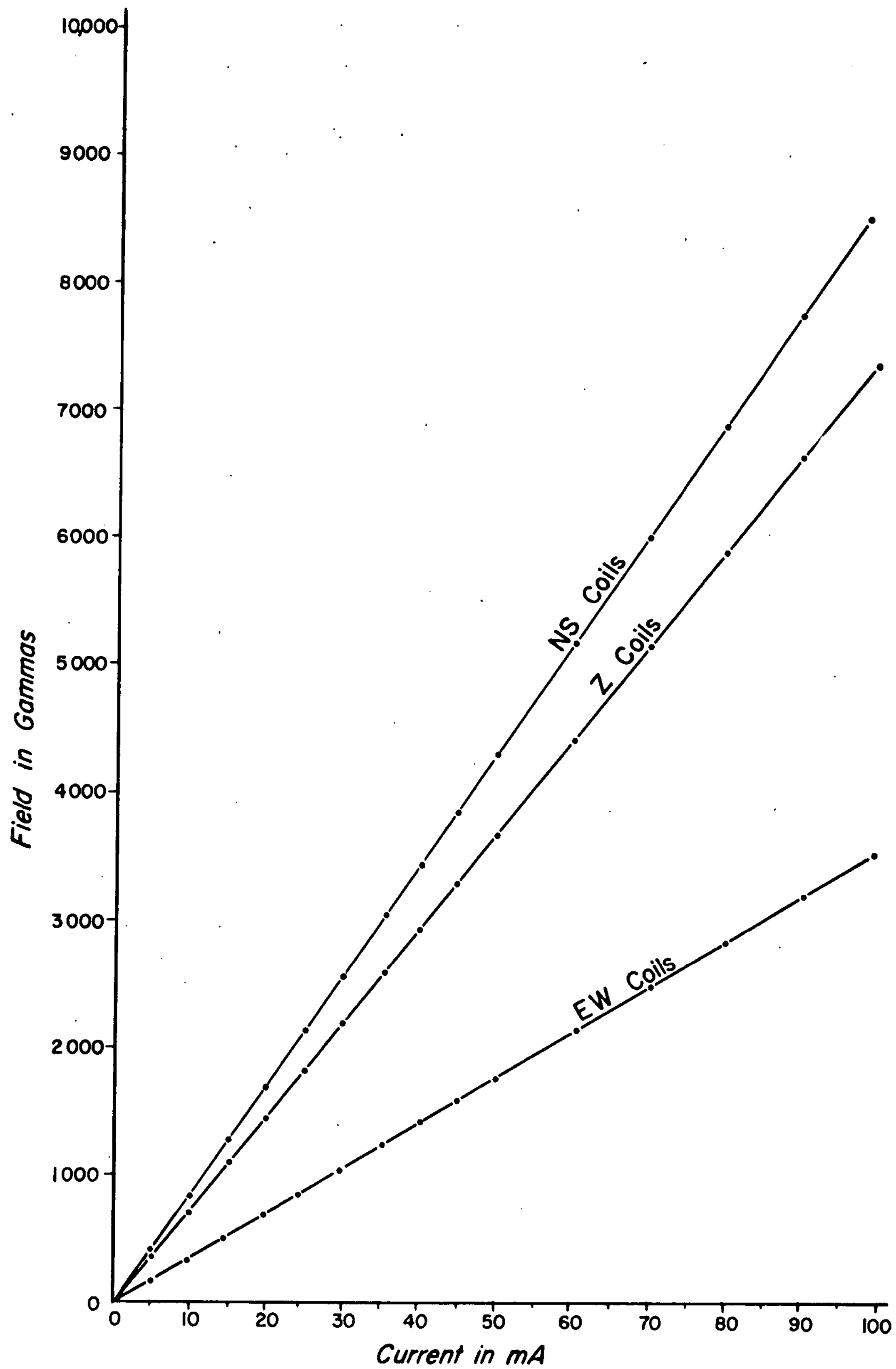
I = current in milliamps

To calculate the fields produced theoretically:

$$H = 1.6288 \times 10^{-3}\ N\ i/L$$

FIGURE 5

MEASURED FIELDS PRODUCED BY SECONDARY COILS



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Thus for H_{ew} $H = 35.59\ I$

for H_{ns} $H = 86.07\ I$

for Z $H = 73.05\ I$

The agreement with the experimental values is satisfactory.

As a further check on whether these coils also were short-circuited, field gradients produced by them were measured, and are plotted in Figure 6. The gradients are quite symmetrical, showing that there is no short-circuiting.

3. SENSITIVITY AND ZERO COILS

The sensitivity and zero coils are two small circular coils mounted vertically above and some distance from the magnetic system. Their purposes are:

The sensitivity coil produces a known field gradient across the magnet system to enable it to be calibrated in terms of angular deflection per unit field difference between the top and bottom magnets.
The zero coil supplies a variable field gradient across the magnet system. This allows one to return the magnet system to its set zero position.

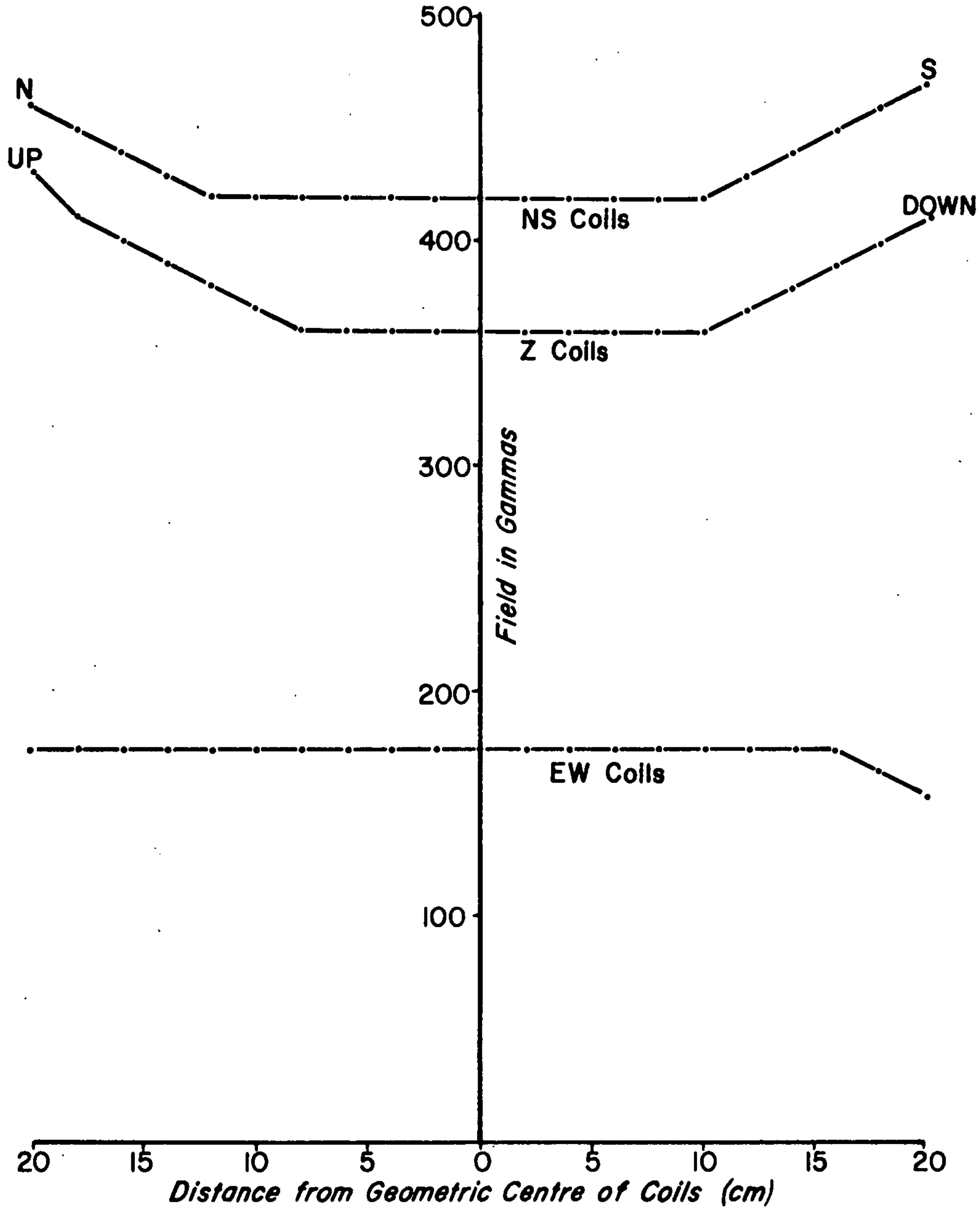
(i) Sensitivity coil

This coil is mounted from the ceiling, with its centre vertically above the magnet system. The plane of the coil is in the NS vertical plane, parallel to the magnetic axis of the system. A field generated by the coil will then be at right angles to the direction of magnetization of the magnets. It can then be used to generate a known field gradient, the field difference between the top and bottom magnets causing the system to be deflected about its axis.

The magnet system is calibrated by producing a known field difference, and by measuring the deflection of the system by the movement of the galvanometer light spot on its scale. The sensitivity of the instrument is given in terms of gauss per millimetre of deflection, or oersteds per millimetre of deflection.

FIGURE 6

Fields Produced by 5mA Current in Secondary Coils



Design of coil. To determine approximately the number of turns of wire required, the following estimates were made:

Sensitivity 2×10^{-7} gauss/ mm

Deflection of light spot 20 cm

Current in coil I = 50 mA

Distance from centre of coil to centre of magnet system y = 1 metre

Separation of magnets x = 8 cm

From the above assumptions, the required field difference

$$H - H' = 2 \times 10^{-5} \text{ gauss.}$$

The field H produced by a circular coil of radius a, with N turns, at a point distant y from the centre of the coil, and in the plane of the coil, and carrying a current I, is

$$H = 10^{-3} \pi a^2 I N / y^3 \text{ gauss}$$

where a, y are in metres

I is in amps.

The field difference between the magnets is then given by:

$$H - H' = 10^{-3} \pi I a^2 N \left[\frac{1}{(y - \frac{1}{2}x)^3} - \frac{1}{(y + \frac{1}{2}x)^3} \right] \text{ gauss}$$

A coil of convenient radius would be 5 cm.

Thus N = 422

Specifications of sensitivity coil. The coil former was machined from a disk of Tufnol, 1 inch thick and five inches in diameter. A groove 5/8 of an inch wide and 7/8 of an inch deep was machined around the edge to take the windings. The coil was wound with 23 SWG wire, rayon-covered.

Internal diameter of coil = 82 mm

External diameter = 104.8 mm

MEAN DIAMETER = 93.4 mm

Thus

Radius a = 46.7 mm = 0.0467 m

Turns N = 396 (18 layers of 22 turns each)

Distance y = 1.0227 m

Separation x = 8.47 cm = 0.0847 m

It was found that a current of 50 mA produced too great a deflection, so 10 mA was used instead. Therefore the field difference produced by a current of 10 mA is

$$638.9 \times 10^{-8} \text{ gauss.}$$

The usual system for determining the sensitivity is to reverse the 10 mA current, and read the total deflection between normal 10 mA and reversed 10mA. Thus the total field difference will be twice that quoted above. Therefore the sensitivity of the instrument is given by dividing 1277.8×10^{-8} by the total deflection in mm.

(ii) Zero Coil

Because of magnetic disturbances, it is found that the zero position of the light spot tends to change with time. Thus it is necessary to have some system whereby the light spot can be brought back near to its correct zero position. This is done by applying a field difference with a second coil, called the 'zero coil'. This coil is mounted vertically above the magnet system, just below the sensitivity coil. A variable current can be passed through this coil in either direction. The specifications of the coil are:

Former and wire	: as for sensitivity coil.
Internal diameter	= 82 mm
External diameter	= 106.2 mm
Mean diameter	= 94.1 mm
Turns	N = 440 (20 layers of 22 turns each)

(iii) Mounting for coils

The two coils are mounted on two aluminium strips suspended vertically from the ceiling. The strips are arranged so that the coils may be moved small distances in the EW and NS directions, to allow for centralizing the coils above the magnet system. They also allow the coils to be rotated about a vertical axis, to enable them to be aligned parallel to the magnets of the magnet system.

4. THE MAGNET SYSTEM

The design of magnet systems for astatic magnetometers has been discussed by Blackett (1952), Collinson, Creer, Irving and Runcorn (1957), Collinson and Creer (1960), and Roy (1963). The details of design will not be discussed here, as reference can be made to these authors.

A magnet system consists basically of two small magnets fastened some distance apart at the two ends of a rod. The magnets are of equal magnetic moment and aligned antiparallel to one another. The system is suspended on a fine torque fibre or ribbon. A mirror is also fastened to the rod, and produces an image of an illuminated cross-hair on a scale some large distance from the system. This allows angular deflections of the magnet system about its vertical axis to be read.

Because the two magnets are equal in moment and antiparallel, the system is not sensitive to constant magnetic fields, which would produce equal torques at the magnets but in opposite directions.However, the system is very sensitive to field gradients, which produce field differences between the two magnets.

During measurement, a rock specimen is brought from a large distance below to a position close below the lower magnet. The magnetic moment of the specimen produces a larger field at the lower magnet than at the upper, thus producing a twist of the magnet system on its fibre and a deflection of the image of the cross-hair on its scale. By placing the specimen in a series of set positions, and measuring the deflections, the orientation and strength of the magnetic moment of the specimen may be calculated.

(i) Magnets

The magnets for this system are two Magnadure cylinders, 2.83 mm in diameter and approximately 6 mm long. They were magnetized in a steady field of 11,000 gauss along their axes, and mounted on the two ends of a thin-walled glass tube (O.D. =1 mm) with their axes at right angles to the length of the tube. The weights of the magnets are approximately 180 mg, and their moments approximately 6 e.m.u. (measured on the A.N.U. igneous magnetometer).

(ii) Theoretical sensitivity of magnet system

For each magnet, the following approximations are made:

Mass	m = 0.18 g
Length	L = 6 mm = 0.6 cm
Diameter	d = 3 mm = 0.3 cm
Moment	P = 6 e.m.u.

Then (from Roy, 1963), the moment of inertia I_1 of each magnet is given by

$$I_1 = m/6 \,.\, (L^2 + d^2)/2$$
$$= 0.00675$$

Moment of inerta I_2 of system less magnets
estimated $I_2 = 0.001$

Moment of inertia of total system I

$$I = 2I_1 + I_2$$
$$= 0.0145$$

Then ratio P/I = 414

Sensitivity $\frac{\Theta}{h} = \frac{T^2}{4\pi^2}\frac{P}{I}$ Where Θ = deflection in radians
h = field difference
T = period in seconds

For a period T = 10 seconds

$$\frac{\Theta}{h} = 1048 \text{ radians/gauss}$$

Thus

sensitivity = 1048 radians/gauss
$= 6.004 \times 10^4$ degrees/gauss
$= 0.1665 \times 10^{-4}$ gauss/degree
$= 1.85 \times 10^{-7}$ gauss/mm deflection
for a scale distance of 5 metres.

For a period T = 8 seconds,

sensitivity = 1.18×10^{-7} gauss/mm deflection.

(iii) Astaticism

Ideally, the two magnets of the system should have exactly equal moments, and should be aligned accurately antiparallel, so that there is no residual moment remaining after the vector addition of the two moments. However, this is rarely possible, and the astaticism is a measure of how closely the system approaches this ideal. It is defined as the ratio of the total moment of the two magnets (i.e. 2P) to the residual moment (called P').

i.e. Astaticism $A = 2P/P'$

The larger the astaticism, the closer is the approach of the system to the ideal.

The astaticism of the system is determined by the following procedure:

1. The magnet system is suspended in the Earth's field on a torsionless fibre (in this case, a filament of terylene, which is a close approximation).

2. The magnets are placed parallel to one another (i.e. both north-seeking poles pointing north).

3. The period of free oscillation of the system (T_2) about its axis is determined in the Earth's field.

4. The magnets are placed antiparallel.

5. The period of free oscillation (T_1) in the Earth's field is again determined.

Then with the magnets parallel,

$$T_2 = 2\pi(I/H.2P)^{\frac{1}{2}}$$

where I = moment of inertia

H = horizontal component of Earth's field

2P = total moment of two magnets

With the magnets antiparallel,

$$T_1 = 2\pi(I/H.P')^{1/2}$$

where P' is the residual magnetic moment.

Thus $A = 2P/P' = T_1^2/T_2^2$

(iv) Assembly of magnet system

The magnet system was assembled with the stronger of the two magnets at the bottom. Loops of fine wire were placed around the magnets and the wire ends glued into the glass tubing. A fine wire hook was also fastened to the top magnet. The gluing medium was shellac. This was used because the application of heat to the dried shellac melts it, so allowing the rotation of the lower magnet with respect to the top one during astaticizing. On cooling, the shellac returns to normal.

Period T_2. The lower magnet was rotated until it was approximately parallel to the top one. The system was suspended by a terylene filament in the centre of the Helmholtz coil system, but with the coils switched off. Its period of free oscillation in the Earth's field was found to be 0.38 seconds.

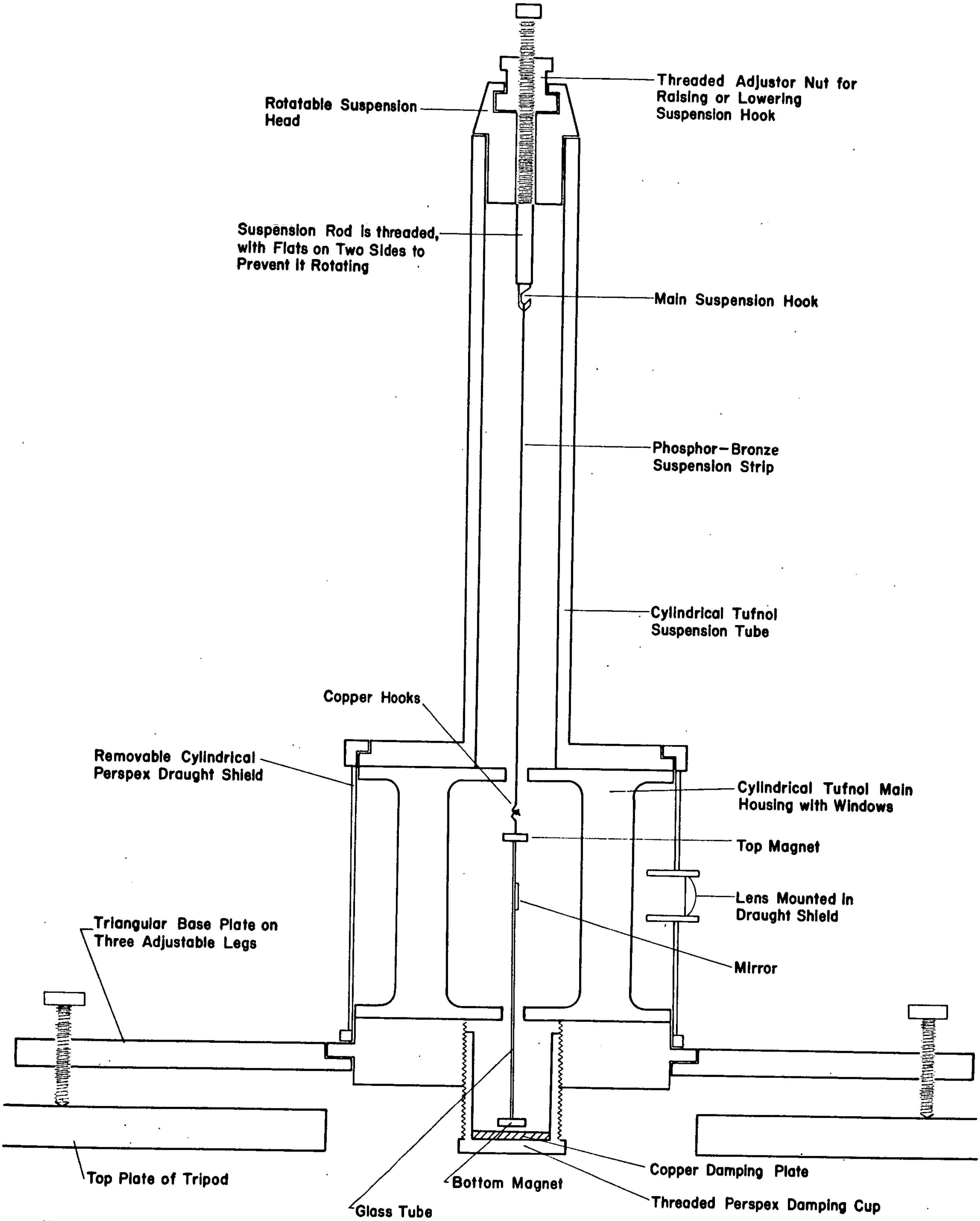
Astaticizing. The magnets were then placed approximately antiparallel by rotating the lower magnet with the shellac melted by the heat of a small soldering iron. The free period in the Earth's field was again measured.

The magnets were placed as closely to antiparallel as possible by rotating the lower magnet small amounts until the longest possible period was obtained.

Since the stronger of the two magnets was put at the bottom of the system, the astaticizing procedure was continued by grinding small amounts from this magnet. This would cause the strength of this magnet to approach that of the top magnet, the closeness of approach being shown by the increase in the free period.

HOUSING FOR MAGNETIC SYSTEM

FIGURE 7



Note: scale and details are not exact

Often it is necessary to use small lengths of magnetized steel wire as 'trimmer' magnets to obtain the final degree of astaticism required. However, this proved unnecessary in the present case.

Period T_1. The final period of free oscillation with the magnets antiparallel obtained was 13.5 seconds.

Thus the astaticism of this system is

$$A = 2P/P' = T_1{}^2/T_2{}^2 = (13.5)^2/(0.38)^2$$
$$= 1260$$

That is, the out of balance moment P' is 0.08% of the total moment 2P. This degree of astaticism is quite sufficient for a magnet system of the sensitivity required.

Mirror. A square mirror, 2 mm x 2 mm and 0.25 mm thick, was glued to the glass rod just below the top magnet, with a minute drop of shellac. The plane of the mirror is at right angles to the axes of the magnets, and it faces towards the same direction as the north-seeking pole of the top magnet.

Torsion fibre. The torsion fibre from which the system is suspended is a phosphor bronze ribbon of dimensions 0.003 inches by 0.0003 inches. It has a fine copper wire hook glued to each end, and its total length from hook to hook is approximately 20 cm.

Orientation of magnets. The system was suspended in its case in field-free space with the south-seeking pole of the top magnet towards magnetic north.

Dimensions of magnet system:

Magnets	length = approx. 0.6 cm each
	diameter = 0.283 cm
	strength = approx. 6 e.m.u.
	weight = approx. 180 mg each

Glass tube	length = approx. 8 cm
	diameter = 0.1 cm
Mirror	Height = 2 mm
	width = 2 mm
	thickness = 0.25 mm
Torsion fibre	width = 0.003 inches
	thickness = 0.0003 inches
	length = approx. 20 cm hook to hook

Separation of magnets = 8.47 cm

Length of system from bottom magnet to hook = 9.34 cm

Sensitivity. A current of 10 mA reversed through the sensitivity coil gives a total deflection of 120 mm with the scale 5 metres from the magnet system. The free period of oscillation of the system in field-free space and suspended on its torsion fibre is approximately 8 seconds. The sensitivity is then

$$\text{sensitivity} = \frac{2 \times 638.9 \times 10^{-8}}{120} \text{ gauss/mm}$$

$$= 1.065 \times 10^{-7} \text{ gauss/mm}$$

or approximately 1.1×10^{-7} gauss/mm.

This sensitivity is quite close to the theoretical sensitivity of 1.18×10^{-7} gauss/mm calculated in section (ii) of this chapter for a free period of approximately 8 seconds.

5. MAGNET SYSTEM HOUSING AND TRIPOD

The tripod that supports the magnet system above the piston is mounted on a concrete block which goes down to bedrock. This block has an air gap around it to isolate the system from vibrations transmitted through the floor of the building. No other parts of the magnetometer are mounted on this block; they are all mounted to the floor or ceiling. Thus vibrations from the piston raising and traversing gears are not transmitted to the magnet system. The base of the piston itself is suspended within a central recess in the concrete block.

The housing for the magnet system itself was designed and constructed with the following requirements in mind:

1. The magnet system must be completely enclosed to prevent it being disturbed by air currents (i.e. draughts).

2. The hook from which the system is suspended must be rotatable in a horizontal plane to allow the plane of the magnets to be oriented north-south. As well, it must be adjustable in a vertical direction to alter the position of the magnet system, without rotating the system.

3. The magnet system must be visible during installation, for ease of adjustment, and also so that it can easily be seen whether the suspension fibre is intact.

4. A mounting for a lens is required on the front of the housing, level with the mirror of the magnet system. This lens is necessary for the optics of the galvanometer lamp system.

5. A cup, adjustable in a vertical direction, is required around the lower magnet. This cup holds a copper plate close beneath the lower magnet to give electromagnetic eddy-current damping of the system.

6. The whole housing must be tiltable in any direction to allow centring of the lower magnet in the damping cup.

7. There must be no structure beneath the damping cup to prevent the close approach of a specimen from below.

With these points in mind, the magnet system housing was constructed as shown in Figure 7. This figure, however, is not exact in either scale or detail.

Access to the magnet system is either by lifting off the draught shield or by screwing down the damping cup. If the magnet system has to be handled at any stage, note that THE MAGNET SYSTEM IS EXTREMELY FRAGILE. When removing, or adjusting, the damping cup, take care that the cup is turned in very small steps, each of only a few degrees, allowing the magnet system to settle between steps. The electromagnetic eddy-currents produced by moving the copper plate in the damping cup deflect the magnet system. If the cup is turned too vigorously, the suspension fibre will be twisted and broken.

After the magnet system is unhooked from the suspension fibre, the fibre itself can be removed by carefully raising the suspension head vertically until the lower hook of the fibre has cleared the suspension tube. If the lower hook fouls the inside of the tube while it is being raised, the fibre will undoubtedly be broken.

Upon replacement of the magnet system, the legs of the triangular base plate are adjusted so that the lower magnet hangs centrally within the damping cup. The cup is then adjusted carefully upwards until critical damping of the system is obtained.

If the heights of the base plate legs are altered; or if the magnet system is raised or lowered by the adjustment in the suspension head; or if the suspension fibre is replaced, the height constant (Chapter 9, section iv) has to be remeasured with a cathetometer, and the correction factors (Chapter 7, section v) re-computed.

To obtain the correct north-south orientation of the plane of the magnets, the following procedure is used:

1. Place a weak magnet (e.g. a magnetized 2-cm length of heavy piano wire) in the specimen holder, with its south-seeking pole towards magnetic north.

2. Raise this magnet on the piston until it is close beneath the damping cup, with the piston in the central position of its east-west traverse. This will rotate the magnet system until it is parallel (within a few degrees) to the magnet (i.e. oriented north-south) owing to the attraction of the lower magnet.

3. Adjust the position of the galvanometer lamp and/or the scale until the light spot is central on the scale.

4. Remove the weak magnet from the piston.

5. Orient the magnet system by carefully rotating the suspension head in small steps until the light spot is back central on the scale.

The magnet system is now oriented with the plane of its magnets in the north-south magnetic meridian.

Note that the north-seeking pole of the bottom magnet of the magnet system (marked with a spot of white paint on the end of the magnet) must point towards magnetic north.

6. PISTON AND HEIGHT CONTROL

The piston is the mechanical unit beneath the magnet system which manipulates the specimen into the various positions required during a measurement. It was designed with the following functions in mind:

1. It must be able to raise a specimen from an effectively infinite distance below the magnet system to a position close below it.

2. The final position to which the piston is raised must be controlled by a stop which is easily variable, and whose positions are accurately known and easily repeatable.

3. The top of the piston must be able to pick up the specimen holder from the specimen trolley, and retain the holder in exactly the same position each time.

4. The top of the piston, with the specimen holder, must be rotatable through 360° in both directions.

5. The piston, with holder, must be able to be traversed a small distance at right angles to the magnet system so that the specimen can be placed a short known distance off-centre each side of centrally below the magnet system.

The piston thus consists of the following parts:

(a) The topmost part of the piston is a truncated cone of nylon with a locating peg in the side. This fits into the corresponding conical recess and tapered locating slot in the base of the specimen holder. When the piston is raised beneath the specimen holder in the trolley, the cone-in-cone construction automatically centres the holder on top of the piston, and the peg-in-slot locates it radially and prevents it from turning with respect to the top of the piston.

(b) The top cone is mounted on the end of a long vertical aluminium rod which runs down the centre of the piston. This rod is a sliding fit so that it is free to move vertically.

(c) Concentric about the lower half of the central rod is an aluminium tube. This is held by a bearing at the base of the piston housing which prevents it from moving vertically, but allows it to rotate. A gear wheel operates in gear teeth cut on the outside of the tube to rotate it. The cetntral rod is splined to this tube so that the two rotate together. Thus when the gear wheel is turned, the rotation tube together with the central rod, top cone, and specimen holder are all rotated about a vertical axis.

(d) Concentric about the rotation tube is another long aluminium tube, equal in length to the central rod. This is a sliding fit between the rotation tube and the outer casing so that it is free to move vertically. It has a locating peg running in a slot in the fixed outer casing which prevents it from rotating. A gear wheel operates in a row of gear teeth cut vertically down the side of the tube to raise and lower it. A nylon insert in the top of the tube abuts underneath the base cone. Thus when the tube is raised or lowered, the top cone and central rod, together with the specimen holder, move with it. The insert in the top of the raising tube is marked at 0°, 90°, 180° and 270° positions so that the rotation position of the top cone can be seen.

(e) Concentric about this raising tube is another aluminium tube of the same length which forms the outer piston casing. This casing is rigidly fastened to a mounting plate about half way down its length. The nylon insert in the top of the raising tube rests on top of another insert in the top of the casing when the piston is fully lowered. The lower part of the casing, which contains the rotation tube with its lower bearing, and the lower halves of the raising tube and the central rod when the piston is down, hangs inside a recess in the concrete mounting block for the tripod support of the magnet system. The gear wheels for the rotation and raising of the piston are mounted on the side of the casing tube near the mounting plate.

(f) The mounting plate of the piston is a sliding fit in a horizontal direction in another plate, the support plate. Another gear wheel mounted on this plate working in a row of gear teeth cut in the mounting plate moves the whole piston sideways to give the east-west traverse movement.

The ancillary gear to the piston is as follows:

1. The support plate is rigidly fastened to two aluminium U-channel beams which span the central concrete block. The mounts which fasten the ends of these beams to the floor on either side of the block are adjustable so that the piston can be set vertical, and can be centralized beneath the magnet system.

2. Long aluminium rods run the operator's bench to give remote control of the raising, traversing, and rotation gears. Turning of the handles on the ends of these rods moves the piston and specimen as required.

3. On the bench end of the rod that controls the raising gear is the piston height scale unit. This consists of a gear on the rod, which is driven by another gear with the handle attached. This second gear, as well as driving the gear which rotates the rod, rotates a third gear which has a peg attached. This peg abuts against an adjustable stop when the piston is raised, to prevent it going any higher. The adjustable stop is on the back of a calibrated scale disc which can be rotated and clamped in position where required. This disc is marked in centimetre divisions to give the height which the piston has reached when it is raised until the peg hits the stop. With this system, the height required can be set on the scale disc, and the piston raised to the same preset height each time during a measurement.

An adjustable friction device built into the height scale unit prevents the piston from moving down under its own weight after it has been raised.

With this unit, the desired height can be set easily, and the height read off the scale, without the operator having to refer to the piston itself at all.

The total movement of the piston from fully lowered to fully raised is approximately 38 cm. In its fully raised position (height scale set on -1 cm), the top of the piston cone is 7.67 cm from the lower magnet of the magnet system.

7. THEORY OF MEASUREMENT

(i) Introduction

When the magnetization of a specimen is measured, three separate components are required. These are:

1. The direction of magnetization in the horizontal plane, in degrees clockwise from the horizontal reference mark ("strike" line) obtained from the field orientation marks. This is called the DECLINATION (D).

2. The direction of magnetization in the vertical plane. This is measured as positive degrees (degrees up from the horizontal plane) or negative degrees (degrees down from the horizontal plane). This is called the INCLINATION (I).

3. The total INTENSITY of magnetization (M) in e.m.u. cm^{-3}.

The simplest method of measuring D is to place the specimen beneath the magnet system with its reference mark to the east (i.e. perpendicular to the plane of the magnet system), noting the deflection produced (indicated on the scale by the galvanometer light spot). This gives a measure of the intensity of the horizontal component of magnetization in the strike direction (0°). The specimen is then rotated through 90° and the deflection noted. This then gives a measure of the intensity of the horizontal component of magnetization at right angles to the strike direction (i.e. at 90°). From these two components, the direction of magnetization in the horizontal plane (D) may be calculated.

Also, knowing the sensitivity (Sm) of the instrument from the deflection produced by the calibrated sensitivity coil, the intensity of the horizontal magnetic moment may be calculated.

This procedure, however, must be modified to obtain a measure of the vertical component of magnetization. To do this, the above procedure is performed twice: once with the specimen displaced a small distance (y = 0.272 cm) to the west of the magnet system's vertical axis; and again with the specimen displaced the same distance to the east. The differences in the deflections in the west and east positions then give a measure of the vertical component of magnetization.

Now, knowing the horizontal and vertical components, the intensity (M) and direction (D and I) of the total moment may be calculated.

When a specimen of low intensity is being measured, a greater degree of accuracy is obtained by measuring four components (0°, 90°, 180°, 270°) instead of two. The four-component routine is called a "long" measurement (instrument code +05), and the two-component one is called a "short" measurement (instrument code -05).

To minimize the effects of any inhomogeneities of magnetization in the specimen, and to eliminate any stray induced components of magnetization, the measurement routine is repeated with the specimen inverted, and the means of the deflections are used in the calculation.

(ii) Specimen positions during measurement

The basic positions of a specimen during measurement are as follows:

R1. Upright specimen in the west position, strike line at 0°.

R2. Upright specimen in the west position, strike line at 90°.

R3. Upright specimen in the west position, strike line at 180°.

R4. Upright specimen in the west position, strike line at 270°.

R5. Upright specimen in the east position, strike line at 270°.

R6. Upright specimen in the east position, strike line at 180°.

R7. Upright specimen in the east position, strike line at 90°.

R8. Upright specimen in the east position, strike line at 0°.

R9. Inverted specimen in the east position, strike line at 0°.

R10. Inverted specimen in the east position, strike line at 270°.

R11. Inverted specimen in the east position, strike line at 180°.

R12. Inverted specimen in the east position, strike line at 90°.

R13. Inverted specimen in the west position, strike line at 90°.

R14. Inverted specimen in the west position, strike line at 180°.

R15. Inverted specimen in the west position, strike line at 270°.

R16. Inverted specimen in the west position, strike line at 0°.

For a short measurement, steps R3, 4, 5, 6, 11, 12, 13, 14 may be omitted.

When the specimen is inverted, the direction of rotation is reversed (from 0°, 90°, 180°..... to 0°, 270°, 180°.....) to maintain the same sense of rotation of the magnetic moment of the specimen with respect to the magnet system.

The sixteen positions listed above are in the order in which they are done during a routine measurement.

(iii) Data required for each measurement

Piston height. To allow for specimens of differing intensities to be measured without sending the light spot off scale, the specimen may be raised on top of the piston to various heights below the magnet system. The handle that raises the piston has a scale attached which may be preset to the height required.

The height scale is set such that the maximum deflection produced by the specimen during measurement does not exceed 10 cm.

Corrected specimen height (Zc). To obtain the specimen height (Zc in cm), the piston height must be corrected for each specimen holder used. For the 3.5-cm cylinder holder, the correction is zero. For the 2.2-cm cylinder holder, the correction is-1.2 cm.

Sensitivity (Sm). The sensitivity of the instrument is obtained by measuring the deflection in mm produced by reversing 10-mA current through the sensitivity coil (see Chapter 3).

Diameter (d) and thickness (h) of specimens. The diameter and thickness (or height) of the cylindrical specimens is required in cm. The standard specimen size used is diameter = 2.2 cm, thickness = 2.2 cm.

Deflections. The 16 (or 8) deflections are then measured. Note that a zero reading of the position of the light spot is taken between each pair of deflection measurements. The differences (positive or negative) between the deflection positions and the zero reading are the deflections required.

(iv) Calculation of measurement

Mathematical theory. If a specimen of magnetic moment P is placed at a distance Z below the lower magnet of a magnet system, and displaced a distance y (where y is much less than Z) from the vertical axis of the magnet system at right angles to the plane of the magnets, the horizontal fields Hy (Figure 8a) and Hz (Figure 8b) produced at the magnets by the horizontal (Py) and vertical (Pz) components of magnetization of the specimen deflect the magnet system through an angle

$$\Theta = \Theta y + \Theta z$$

where

$$\Theta y = \frac{P}{\sigma}\left[Hy1 - \left(1 - \frac{1}{A^2}\right)^{\frac{1}{2}} Hy2\right]$$

$$\Theta z = \frac{P}{\sigma}\left[Hz1 - \left(1 - \frac{1}{A^2}\right)^{\frac{1}{2}} Hz2\right]$$

where A = astaticism of magnet system (see Chapter 4).

σ = torsion constant of suspension fibre.

When a specimen has dimensions much smaller than Z, the moment of the specimen can be taken, as a first approximation, to be a magnetic dipole. This then gives:

$$\Theta = \frac{P}{\sigma} Py \left[\frac{(\cos^2\chi 1 + 2\sin^2\chi 2)\cos^3\chi 1}{Z^3} - \frac{(1 - 1/A^2)^{\frac{1}{2}}(\cos^2\chi 2 + 2\sin^2\chi 2)\cos^3\chi 2}{(Z + \ell)^3}\right]$$

where ℓ = separation of magnets and χ_1, χ_2 are angles as show n in Figure 8.

$$\Theta z = \frac{P}{\sigma} Pz \left[\frac{3\cos^4\chi 1 \sin\chi 1}{Z^3} - \frac{(1 - 1/A^2)^{\frac{1}{2}} 3\cos^4\chi 2 \sin\chi 2}{(Z + \ell)^3}\right]$$

For small values of χ (when $y \ll Z$), we can make the following approximations:

$$\cos\chi 1 = \cos\chi 2 = 1$$

$$\sin\chi 1 = y/Z$$

$$\sin\chi 2 = y/(Z+\ell)$$

and $\sin^2\chi 1$ and $\sin^2\chi 2$ may be neglected.

Thus

$$\Theta y = \frac{P}{\sigma} \; Py \left[\frac{1}{Z^3} - \frac{(1 - 1/A^2)^{\frac{1}{2}}}{(Z + \ell)^3} \right] \qquad \text{..........(1)}$$

$$\Theta z = \frac{P}{\sigma} \; Pz \; 3y \left[\frac{1}{Z^4} - \frac{(1 - 1/A^2)^{\frac{1}{2}}}{(Z + \ell)^4} \right] \qquad \text{..........(2)}$$

However, the moment of the specimen may not always approximate a dipole. Papapetrou (Blackett, 1952) has calculated the horizontal fields produced outside but near the vertical axis by a circular cylinder of radius a and height h, and uniformly magnetized.

Placing this cylinder of magnetic moment P with its geometrical centre at a distance $(Z^2 + y^2)^{\frac{1}{2}}$ below the lower magnet (Figure 8), the horizontal fields produced at the lower magnet by the horizontal component Py and the vertical component Pz are given by:

$$Hy = \frac{Py \; Fy \; (u, v)}{Z^3} \qquad \text{..........(3)}$$

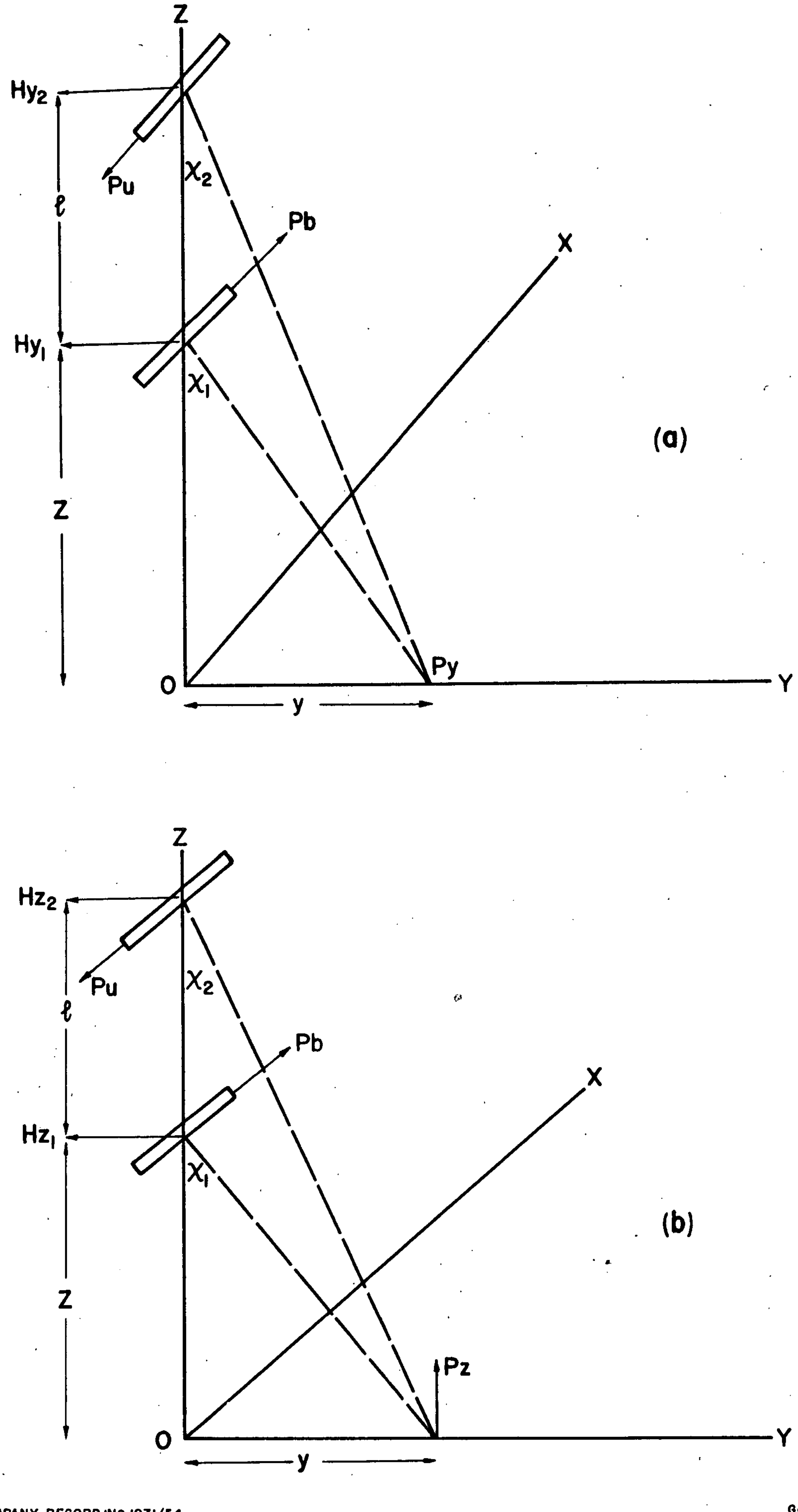
$$Hz = \frac{3y \; Pz \; Fz(u, v)}{Z^4} \qquad \text{..........(4)}$$

where

$$Fy(u,v) = \frac{1}{2uv^2} \left[\frac{1 - u}{[(1 - u)^2 + v^2]^{\frac{1}{2}}} - \frac{1 + u}{[(1 + u)^2 + v^2]^{\frac{1}{2}}} \right]$$

$$Fz(u,v) = \frac{1}{6u} \left[\frac{1}{[(1 - u)^2 + v^2]^{3/2}} - \frac{1}{[(1 + u)^2 + v^2]^{3/2}} \right]$$

FIGURE 8



and $u = h/2Z$ $v = a/Z$

Comparing equations (3) and (4) with equations (1) and (2), we can see that (1) and (2) can be corrected for a non-dipole specimen by applying Papapetrou's factors Fy and Fz. Thus these equations become:

$$\Theta y = \frac{P\,Py}{\sigma}\left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{(1 - 1/A^2)^{1/2}\,Fy(u_2,v_2)}{(Z + \ell)^3}\right]$$

$$\Theta z = \frac{P\;3y\;Pz}{\sigma}\left[\frac{Fz(u_1,v_1)}{Z^4} - \frac{(1 - 1/A^2)^{1/2}\,Fz(u_2,v_2)}{(Z + \ell)^4}\right]$$

where $u_1 = h/2Z$, $u_2 = h/2(Z + \ell)$

$v_1 = a/Z$, $v_2 = a/(Z + \ell)$

The angles Θy and Θz are small, so that the deflections R corresponding to Θy and Θz can be taken as:

$$Ry = 2L\Theta y$$

$$Rz = 2L\Theta z$$

where L is the distance from the magnet system to the scale.

Also, $A > 1000$, so that the term $(1 - 1/A^2)^{1/2}$ can be taken as 1.

Thus:

$$\frac{Ry}{2L} = \frac{P\,Py}{\sigma}\left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{Fy(u_2,v_2)}{(Z + \ell)^3}\right] \quad \text{..........(5)}$$

$$\frac{Rz}{2L} = \frac{P\;3y\;Pz}{\sigma}\left[\frac{Fz(u_1,v_1)}{Z^4} - \frac{Fz(u_2,v_2)}{(Z + \ell)^4}\right] \quad \text{..........(6)}$$

The sensitivity coil produces a field difference of $Sc = 1277.8 \times 10^{-8}$ gauss between the two magnets (see Chapter 3). This produces a deflection of Sm mm.

Thus

$$Sm = \frac{Sc\ P\ 2L}{\sigma}$$

That is

$$\frac{P\ 2L}{\sigma} = \frac{Sm}{Sc}$$

Therefore, from equations (5) and (6)

$$Ry = \frac{Py\ Sm}{Sc}\left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{Fy(u_2,v_2)}{(Z+\ell)^3}\right]$$

$$Rz = \frac{Sm\ 3y\ Pz}{Sc}\left[\frac{Fz(u_1,v_1)}{Z^4} - \frac{Fz(u_2,v_2)}{(Z+\ell)^4}\right]$$

Therefore:

$$Py = \frac{Sc\ Ry}{Sm}\left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{Fy(u_2,v_2)}{(Z+\ell)^3}\right]^{-1}$$

$$Pz = \frac{Sc\ Rz}{Sm\ 3y}\left[\frac{Fz(u_1,v_1)}{Z^4} - \frac{Fz(u_2,v_2)}{(Z+\ell)^4}\right]^{-1}$$

The horizontal component Px can be found by rotating the specimen through 90°, so that:

$$Px = \frac{Sc\ Rx}{Sm}\left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{Fy(u_2,v_2)}{(Z+\ell)^3}\right]^{-1}$$

The deflections Rx, Ry and Rz correspond to the mean horizontal deflections at 90° (Y_{90}), and at 0° (Y_0), and to the mean vertical deflection (V).

Thus:

$$Px = 8Y_{90} \frac{Sc}{8Sm} \left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{Fy(u_2,v_2)}{(Z + \ell)^3} \right]^{-1}$$

$$Py = 8Y_0 \frac{Sc}{8Sm} \left[\frac{Fy(u_1,v_1)}{Z^3} - \frac{Fy(u_2,v_2)}{(Z + \ell)^3} \right]^{-1}$$

$$Pz = 16V \frac{Sc}{16\ Sm\ 3y} \left[\frac{Fz(u_1,v_1)}{Z^4} - \frac{Fz(u_2,v_2)}{(Z + \ell)^4} \right]^{-1}$$

Mean horizontal deflection at 0°. From the measured deflections R1......R16, the mean horizontal deflection with the specimen strike line at 0° (Y_0) is calculated as follows:

$$8Y_0 = (R8 + R1) - (R6 + R3) + (R16 + R9) - (R14 + R11)$$

For short measurements, the calculations is:

$$8Y_0 = 2[(R8 + R1) + (R16 + R9)]$$

Mean horizontal deflection at 90°. The mean horizontal deflection with the specimen strike line at 90° (Y_{90}) is calculated as follows:

$$8Y_{90} = (R7 + R2) - (R5 + R4) + (R15 + R10) - (R13 + R12)$$

For a short measurements, the calculation is :

$$8Y_{90} = 2[(R7 + R2) + (R15 + R10)]$$

Mean vertical deflection. The mean deflection (V) due to the vertical component is calculated as follows:

$$16V = (R8 - R1) + (R7 - R2) + (R6 - R3) + (R5 - R4) + (R16 - R9) + (R15 - R10) + (R14 - R11) + (R13 - R12)$$

For short measurements, the calculation is:

$$16V = 2[(R8 - R1) + (R7 - R2) + (R16 - R9) + (R15 - R10)]$$

Calculation of D. The direction of magnetization in a horizontal plane with respect to the strike reference line on the specimen is calculated as follows:

$$\tan D' = \frac{Px}{Py} = \frac{8Y_{90}}{8Y_0}$$

The quadrant in which D lies is determined by the signs of Y_{90} and Y_0. The convention is as follows:

Y_{90}	Y_0	Declination
+	+	$D = 360° - D'$
+	-	$D = 180° + D'$
-	-	$D = 180° - D'$
-	+	$D = D'$

Calculation of I. The total horizontal component of magnetization of the specimen is given by:

$$Ph = (Px^2 + Py^2)^{1/2}$$

The inclination is then given by:

$$\tan I = Pz/Ph = Pz/(Px^2 + Py^2)^{1/2}$$

Calculation of M. The intensity of magnetization is given by:

$$M = (Px^2 + Py^2 + Pz^2)^{1/2}$$

M is then divided by the volume of the specimen in cm^3 (8.36 cm^3 for a standard specimen) to give the intensity in e.m.u. cm^{-3}.

Specimen distance Z and constants. The specimen distance Z used in the calculations is the actual distance from the centre of the lower magnet to the geometrical centre of the specimen. This is given by:

$$Z = 6.37 + \text{piston height} - \text{holder correction} - \tfrac{1}{2}h$$

$$= 6.37 + Zc - \tfrac{1}{2}h \text{ cm.}$$

The separation of the magnets ℓ = 8.47 cm.

The sensitivity constant Sc = 1277.8 x 10^{-8} gauss/mm.

The off-centre distance y = 0.272 cm.

(v) Simplified calculation of components

Since standard specimens (2.20 x 2.20 cm) used on the magnetometer, at set heights (usually 1 cm apart), Papapetrou's corrections for the specimen shape and the effect of the upper magnet, together with the various constants in the formulae, may be simplified into a table of correction factors as follows:

Corrected Height Zc	Correction Factor for vertical component Cz	Correction Factor for horiz. components Ch
-2.2	0.087	0.046
-1.2	0.264	0.109
-0.2	0.643	0.216
0.8	1.342	0.379
1.8	2.512	0.616
2.8	4.340	0.939
3.8	7.051	1.364
4.8	10.91	1.947
5.8	16.26	2.660
6.8	23.41	3.547
7.8	32.84	4.561
8.8	45.10	5.912
9.8	60.41	7.601
10.8	80.22	9.390
11.8	104.1	11.40
12.8	132.3	13.30
13.8	168.7	15.96
14.8	208.2	22.80

Using these correction factors, the three orthogonal components of magnetization of a specimen (Px, Py, Pz) may be calculated as follows:

$$Px = \frac{8Y_{90} \cdot Ch}{Sm} \times 10^{-3} \text{ e.m.u.}$$

$$Py = \frac{8Y_{0} \cdot Ch}{Sm} \times 10^{-3} \text{ e.m.u.}$$

$$Pz = \frac{16V \cdot Cz}{Sm} \times 10^{-3} \text{ e.m.u.}$$

From these, the inclination I and intensity of magnetization M may be calculated as follows:

$$\tan I = \frac{Pz}{(Px^2 + Py^2)^{1/2}}$$

$$= \frac{16V \cdot Cz}{[(8Y_{90} \cdot Ch)^2 + (8Y_0 \cdot Ch)^2]^{1/2}}$$

$$M = (Px^2 + Py^2 + Pz^2)^{1/2} / \text{Volume}$$

$$= \frac{[(8Y_{90} \cdot Ch)^2 + (8Y_0 \cdot Ch)^2 + (16V \cdot Cz)^2]^{1/2} \times 10^{-3}}{8.36 \text{ Sm}}$$

$$\text{e.m.u. cm}^{-3}$$

The declination D may be calculated as shown in section (iv) above.

8. ROUTINE OF MEASUREMENT

The sequence of steps in making a measurement on the astatic magnetometer are as follows:

1. Switch on the galvanometer lamp, and the lamps which illuminate the east-west position indicator and rotation scale, the piston height scale, the desk, and the specimen trolley.

2. With the potentiometer circuit, check that the currents in the de-gaussing coils are correct. Adjust as necessary.

3. Centre the galvanometer light spot on the measurement scale, using the zero coil. Note that the current in this coil can be reversed if necessary.

4. Determine the sensitivity of the instrument (Sm) by measuring the total deflection in millimetres produced by reversing 10mA current through the sensitivity coil. Note down Sm, and switch off the sensitivity coil.

5. Place the specimen upright in the specimen holder, with the top reference mark (the strike direction from the field orientation procedure) opposite the arrow on the holder.

6. Place the specimen holder in the trolley, with the arrow pointing to the right (east).

7. Wind the trolley down to the end stop beneath the magnet system; raise the piston to pick up the specimen holder; wind the trolley back a few feet out of the way of the piston; and lower the piston with holder to the bottom of its travel.

8. Check that the piston rotation scale is at zero (through telescope) and that the piston is in the west position (shown by the arrow on the scale next to the piston).

9. Raise the specimen on the piston to a height such that on rotation the light spot is not deflected more than 10 cm from its zero position. Set the piston height scale stop to the nearest centimetre division. Note down this height and work out the corrected height (Zc).

10. Lower the piston to the bottom of its travel. Read the zero position of the light spot and note it in the appropriate column of the measurement sheet (upright, rotation 0°, position west).

11. Raise the piston to the preset height (up against the stop which was set in step 9); read the deflection produced, and note it next to the zero reading.

12. Lower the piston to the bottom, and rotate it to the 90° mark. Note the zero reading in the appropriate column (upright, rotation 90°, position west).

13. Raise the piston to the stop, and note down the deflection produced.

14. Repeat steps 12 and 13 for rotation positions 180° and 270°.

15. Traverse the piston to the east position, still with the rotation at 270° from step 14, and measure the zero readings and deflections for rotation positions 270°, 180°, 90° and 0°.

16. Wind the trolley up to the end stop beneath the specimen holder, lower the piston to place the holder on the trolley, and wind out the trolley. Invert the specimen, still with the top reference mark opposite the arrow and the arrow pointing to the right. Place the specimen holder back on the piston, remembering to move the trolley back out of the way afterwards.

17. Still in the east position, measure the zeros and deflections for rotation positions 0°, 270°, 180°, and 90° in that order.

18. Traverse the piston to the west position, and measure the zeros and deflections for rotation positions 90°, 180°, 270°, and 0°.

19. Bring out the specimen holder and remove the specimen from the holder.

20. Determine the 16 deflections by subtracting the zero readings from the deflection readings. Note that deflections to the right on the scale are positive, and to the left are negative (that is, the subtraction of the readings is an algebraic subtraction).

The deflection readings thus produced correspond to the deflections R1...R16 in chapter 7 as follows:

Specimen	East-west Position	Rotation Position	Deflection
Upright	west	0°	R1
..	..	90°	R2
..	..	180°	R3
..	..	270°	R4
..	East	270°	R5
..	..	180°	R6
..	..	90°	R7
..	..	0°	R8
Inverted	East	0°	R9
..	..	270°	R10
..	..	180°	R11
..	..	90°	R12
..	West	90°	R13
..	..	180°	R14
..	..	270°	R15
..	..	0°	R16

The mean deflections produced by the magnetic components of the specimen in the direction of the reference mark (Y_0), in the horizontal plane at right angles to the reference mark (Y_{90}), and along the vertical axis of the specimen (V) are then calculated as shown in chapter 7. The direction of magnetization of the specimen (declination D and inclination I) and its intensity of magnetization (M) are calculated as shown in the same chapter.

As noted in chapter 7, for strong specimens which are homogeneously magnetized, a short measurement may be used. This omits all readings in the 180° and 270° rotation positions with the specimen upright, and in the 180° and 90° rotation psotions with the specimen inverted (that is, deflections R3, 4, 5, 6, 11, 12, 13, 14).

9. MAGNETOMETER CONSTANTS AND DIMENSIONS

(i) De-gaussing coils

Side length of coils	EW = 190.9 cm
	NS = 197.2 cm
	Z = 207.7 cm
Number of turns of wire	EW = 147
	NS = 315
	Z = 611
Separation of coils	EW = 104.9 cm
	NS = 108.7 cm
	Z = 114.2 cm
Resistance shunts	EW = 20 ohms on E coil
	NS = 10,000 ohms on S coil
	Z = 1465 ohms on top coil
Fields to be cancelled (approx.)	EW = 0.0007 gauss
	NS = 0.245 gauss
	Z = 0.52 gauss
Coil currents (approx.)	EW = 1.1 mA
	NS = 94 mA
	Z = 117 mA
Potentiometer circuit sensitivity	
	EW = 0.2 gammas per microamp
	NS = 30 gammas per microamp
	Z = 60 gammas per microamp

(ii) <u>Sensitivity coil</u>

Radius = 4.67 cm

Number of turns = 396

Distance from centre of coil to centre of magnet system = 102.27 cm

Current = 10 mA reversed

Field difference produced by coil between magnets of magnet system = 1277.8×10^{-8} gauss

(iii) <u>Magnet system</u>

Material of magnets = Magnadure

Dimensions of magnets = 6 mm long by 3 mm diameter cylinders (approx.)

Mass of magnets = 0.18 gram each

Magnetic moment of magnets = 6 e.m.u. each (approx.) along the axes of the cylinders

Glass mounting tube = 8 cm long by 1 mm diameter (approx.)

Mirror = 2 mm by 2 mm by 1/4 thiok (approx.)

Overall length of magnet system (including hook at top) = 9.34 cm

Separation of magnets = 8.47 cm

Moment of inertia of magnet system = 0.0145 g cm^2

Astaticism of magnet system = 1260 when assembled
= 1015 after two years (11.2.69)

Orientation of magnet system = north seeking pole of bottom magnet points towards magnetic north

Suspension fibre = 0.003 inch by 0.0003 inch phosphor bronze strip approximately 20 cm long (with hooks)

Trimmer magnets = nil

Sensitivity of magnet system (approx.) = 1.1×10^{-7} gauss per mm deflection

(iv) Piston

Distance from the piston top to the centre of the lower magnet with the height scale at zero = 8.67 cm

Height constant (distance from the centre of the lower magnet to the base of the recess in the 3.5 cm holder with height scale at zero) = 6.37 cm

Correction factor for 3.5 cm holder = 0

Correction factor for 2.2 cm holder = -1.2 cm

EW off-centre traverse distance = 0.272 cm either side of centre.

10. REFERENCES

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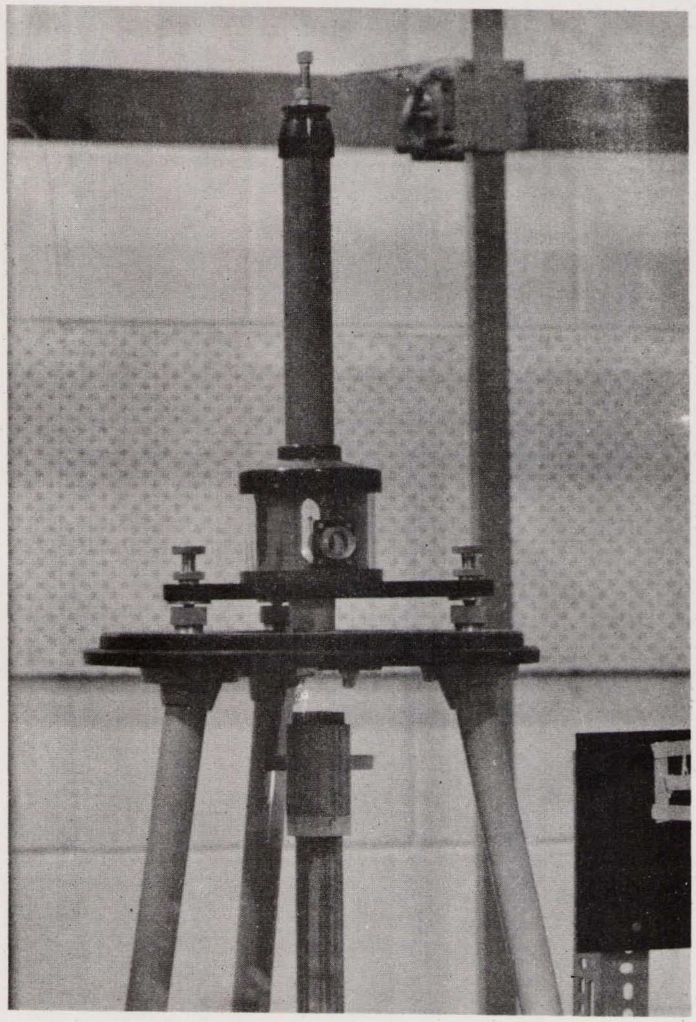

Plate 3.

Magnet system housing on tripod, with specimen in holder on top of raised piston

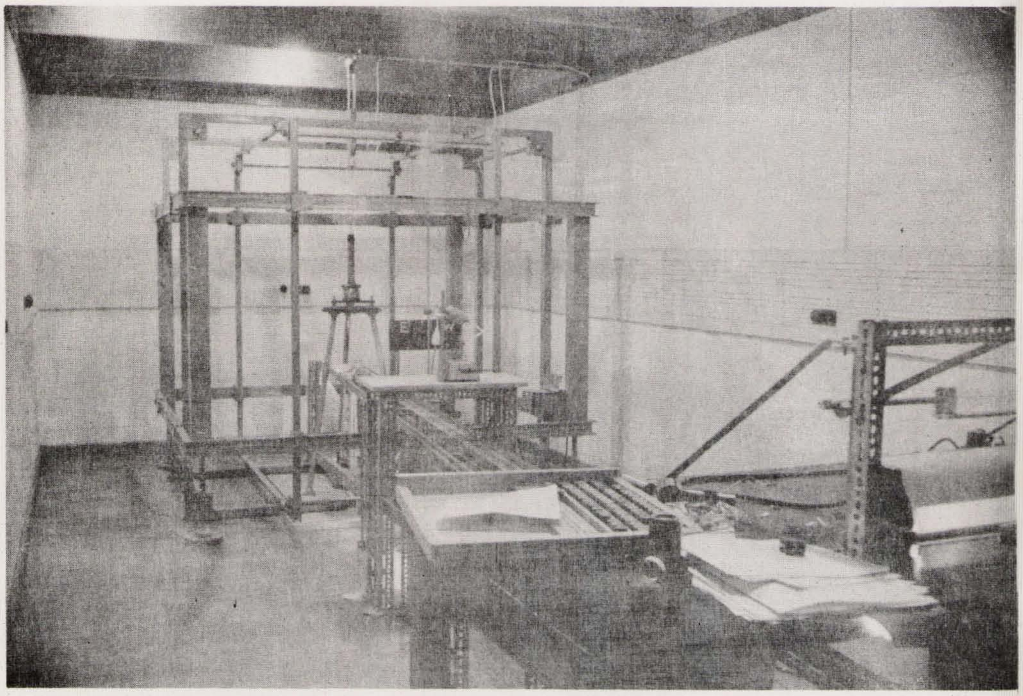

Plate 1.
General view of Magnetometer installation

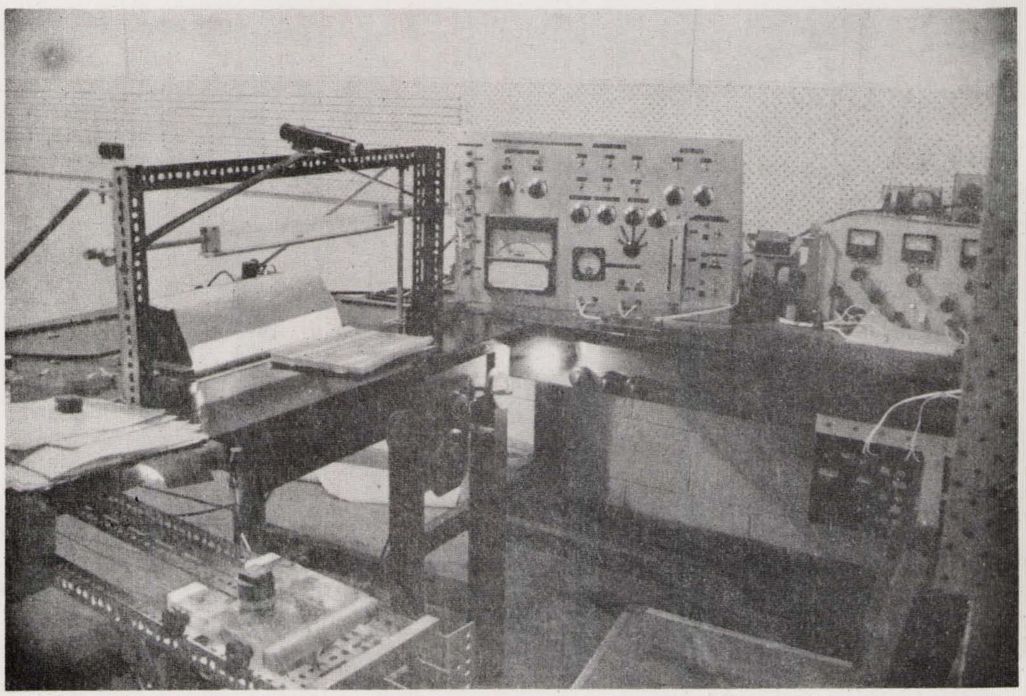

Plate 2.
Control desk and specimen trolley