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SEISMIC RAY TRACING IN A SPHERICAL EARTH USING COMPUTER MODELS

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PLATES

SUMMARY

Corrections for earth's curvature must be applied in long-distance seismic refraction surveys to avoid significant errors in interpretation. An incremental approach to computer ray-tracing rather than an analytical approach reveals that further corrections must be applied to Mereu's 1967 correction at recording distances greater than 600 km.

A computer program (GEOTRACE) has been developed to enable the tracing of seismic rays through a spherically symmetric earth composed of any number of homogeneous layers. Time/distance curves are plotted as output so that models may be checked against the observed data.

The program is of particular value for long-range upper mantle refraction work; apparent velocities can be matched with the observed arrivals with due regard to curvature caused by arc/chord length differences.

1. INTRODUCTION

One of the aims of both earthquake and explosion seismology is the definition of velocity-depth functions in the earth. Such functions can be related to the chemical composition of the Earth's interior through a study of the physical characteristics of minerals and their high temperature and pressure phases. Consequently such functions can assist in providing an understanding of the Earth's structure and its current tectonic processes. For these reasons numerous crustal investigations have been conducted in Australia (Dooley, 1972) and overseas. Until recently, however, few data have been available from within the mantle.

In Australia only a few long-range refraction surveys have been conducted (Denham et al., 1972; Cull, 1973) because of the scarcity of suitable large explosions, but overseas the advent of underground nuclear weapons testing has provided an increasing number of high-yield seismic sources. Data from surveys similar to Project Early Rise using chemical explosions (Green & Hales, 1968; Mereu & Hunter, 1969) can also be expected to proliferate, with added impetus provided by the recent instrumentation developments in long-plyaing unattended recorders (Simpson, 1973).

With this increasing use of high-yield seismic sources, numerous refraction recordings are being made at distances greater than 1000 km (Simpson, 1973); at these distances the flat-Earth approximations used in most interpretations are no longer valid, and corrections must be applied for the Earth's curvature (Mereu, 1967). The digital computer provides an attractive method of checking corrected models against the observed data.

In seismic refraction model studies the initial angle of incidence of a seismic ray from an energy source may be progressively increased so that the point of emergence is made to vary over the entire surface arc. This process gives rise to the complete time/distance curve including retrograde segments, and it is upon the position and shape of these retrograde segments that the uniqueness of each model is tested against observed data. Although many models may satisfy the first-arrival data, only one will completely satisfy the later events which, however, are often difficult to identify from field recordings. This ability to completely define the time/distance curve is of particular importance in the investigation of low-velocity layers because these may not be detected from the first-arrival data.

The program described here (GEOTRACE) was written in Fortran IV and was developed on a CDC 3600 computer with associated CALCOMP plotting facilities. It may be applied to simple homogeneous layered Earth models without restriction on the number of such layers, and consequently velocity gradients can be accommodated by a laminate method. Continuous travel-time curves can be drawn or, if focusing effects are required, discrete plots of arrivals can be specified. The Zoeppritz partitioning of energy to give amplitude estimates (Yacoub et al., 1968) was not investigated in these studies.

2. EARTH CURVATURE CORRECTIONS

Mereu (1967) has shown that, because of the difference in curvature between surface and subsurface horizons, apparent velocities (V) obtained using surface arc measurements are too large and must be reduced to the actual velocities (V_1) by the equation

$$V_1 = (R-H)\,V/R \qquad .....(1)$$

where R is the Earth radius and H is the depth to the refractor. However it should be noted that this expression was derived under the approximation that arc and chord lengths are not significantly different at distances up to 500 km. At the greater distances required for upper mantle studies this approximation breaks down and produces significant error; thus further corrections should be applied.

Cord lengths EF in Plate 1 are related to arc lengths $\overline{EF}$ by the equation

$$EF = 2(R-H)\ \sin\ (\overline{EF}/2(R-H))$$

(approx.)

$$= \overline{EF} - \overline{EF}^3/(24\ (R-H)^2) \qquad .....(2)$$

so the seismic travel-time for a path AEFB can be expressed as

$$T = \alpha + \beta + CD/V_1 - \overline{EF}^3/(24(R-H)^2V_1) \qquad .....(3)$$

where α and β are the usual delay times, which Mereu has shown remain constant as surface distances are increased to over 1500 km. Equation (3) differs from Mereu's previous travel time expression by the subtraction of the cubic correction term (CT) defined as

$$CT = \overline{EF}^3/(24\ (R-H)^2V_1)$$

$$= (\overline{CD} - 2\phi)^3/(24\ (R-H)^2V_1) \qquad .....(4)$$

where ϕ (=$\overline{CE}$ = $\overline{FD}$) is the usual "offset distance" commonly used in refraction seismology. For the model under consideration (Plate 1) the effect of CT can be gauged from the curves in Plate 2. It is clear that beyond 600 km the correction becomes rapidly significant and consequently apparent velocities determined without its consideration will be in error.

The magnitude of the velocity error depends upon which data are used in the least-squares analysis; first-arrival interpretations of upper mantle P data are prticularly susceptible. This can be illustrated clearly by the use of program GEOTRACE, which is developed in later chapters of this Record. The mechanics of the program will not be discussed here except to point out that the velocity profile given in Plate 3 was used to generate the time/distance plot shown also in Plate 3. Events from the P refractor (within the upper mantle) of the model in Plate 3 do not appear as first arrivals until refractor distances of 600 km are exceeded. A least-squares line fitted to the 8 first arrival P data, obtained from the model by a ray-tracing technique, gives an apparent velocity of 8.52 km/s, which from equation (1) represents an actual velocity of 8.36 km/s, 0.06 km/s above the true velocity of the model.

A corrected apparent velocity may be obtained after the application of equation (4). For the upper mantle it can be seen from Plate 2 that equation (4) may be further approximated by the expression

$$CT = (\overline{AB} - 2\emptyset)^3/24R^2V_1 \qquad .....(5)$$

After adding this correction to each of the above eight travel times the least-squares analysis was repeated and an apparent velocity of 8.46 km/s was obtained which represents an actual velocity of 8.30 km/s, the exact velocity in the model.

Besides the petrological differences which could be inferred from the above variation in velocity, major structural features could be misinterpreted because of fluctuations in the resultant intercept times. For example the intercept times obtained above from the least-squares analysis were 15.42 s and 14.21 s, which correspond to refractor depths of 146 km and 127 km, compared with the model depth of 120 km; these errors are 22% and 6% respectively.

It is therefore clearly of great importance to incorporate all significant spherical Earth corrections in any ray-tracing method, and this has been a major design consideration behind the development of program GEOTRACE described in the later chapters of this Record.

3. COMPUTER RAY-TRACING MECHANISM

The calculation of ray paths in layered media is most easily achieved in digital computers by using the concept of probe segments (Jackson, 1970). Commencing at the shot-point a seismic ray is advanced by the successive addition of arbitrarily small probe segments until a refractor boundary is intersected. At the point of intersection Snell's laws are invoked to determine the change in direction of the incident ray.

In GEOTRACE (Appendix 1) a regression process is used when an intersection is obtained. The last probe is subtracted and the ray is again advanced by the addition of a revised very much smaller probe length. Consequently the exact point of intersection can be obtained with an accuracy as great as desired. At this point Zoeppritz's equations could be used to find the relative energy of the rays; however, the early programming presented here does not include these considerations.

After a particular ray has been traced either back to the surface or else outside the range of the model, the initial angle of incidence at the shot-point is incremented and a further ray is traced. If a reflection is detected when the preceding ray had no such reflection then an option exists to provide a concentration of rays near this critical point; this has the effect of providing greater definition of cusp point regions on the time/distance curve.

The probe length can be varied but must always be less than the smallest separation between refractors. For a maximum computing speed the length should be large, but where accuracy is essential a small probe length is required. Adequate definition of all refractors can depend upon the increment in the initial angle of incidence. If the angular increment is too large it is possible that for a particular thin layer no refracted ray will be shown returning to the surface, and in some cases reflections will not appear from the underlying layer. This effect can be largely overcome by concentrating rays whenever a change in refraction/reflection modes is noted between successive rays. It should be borne in mind when models are being set up that layers thinner than the wavelength of the seismic wave may not reflect/refract physically.

4. PROGRAM GEOTRACE USAGE

The first data card following the program *RUN card is used to specify restrictions and options for the plotting routines. The variable list is:

DSC, ARCMAX, TSC, ORDEP, VREDUCE, DEPMAX, RCON, AINC, APROBE, THST, THLIM, MPT, TRA, PLT

with a card format of:

(11 F5, 21X, 2A1, A2).

DSC (km/in) and TSC (sec/in) are the distance and time scales of the output time/distance plot. DSC is also used as the scale, vertical and horizontal, of the ray path plot.

ARCMAX (km) is the maximum surface arc distance at which travel times are required - if it is left blank then rays are traced through the entire earth section.

ORDEP (km) is the depth at which the source is sited. If surface explosions are used the ORDEP is zero and may be left blank.

VREDUCE (km/s) is the reduction velocity required in the presentation of the computed travel-time plot. It may be left blank in which case a "normal" or unreduced plot results.

If only the outer layers of an earth model are to be investigated the DEPMAX (km) is used to specify the maximum depth to which rays are allowed to penetrate. If DEPMAX is left blank then a value of (ARCMAX * 0.2) is assumed.

If an estimate of focusing effects is required then the increment in the angle of incidence must be kept constant and RCON must be set greater than 1.0. If RCON is left blank then there will be an automatic reduction of the increment near the critical angles, giving a more precise t/d plot.

AINC (degrees) and APROBE (km) are the incident angle increment and the probe length respectively. The effects of these two quantities have been discussed in chapter 3. If AINC is left blank then an increment of 2 degrees is assumed while if APROBE is blank then a value of 5.0 km is used.

The range of incident angles at the source is controlled by THST and THLIM. THST (degrees) is the initial angle of incidence while THLIM (radians) is the maximum angle. If THST is left blank then the first ray will be vertically downwards; subsequent rays are incremented anticlockwise up to THLIM, which if left blank is assumed to be /2 so that all rays up to the horizontal are considered.

If a discrete travel-time plot is required then MPT must be non-blank, or otherwise a continuous travel-time curve is output; while if the ray path plot is to be suppressed (in the interests of economy) then TRA must be non-blank.

The plotter used may be specified in PLT. If it is left blank then PL (12") is assumed but if PB (31") is required then this must be specified. It should be noted that the *EQUIP card at the head of the program must correspond.

Model specification

After the preceding single data card a series of cards is used to define the model. The refractor depth (km) and velocity (km/s) are specified in that order according to the format: 2F10.2. There is no limit to the number of layers which can be specified and so velocity gradients can be specified in terms of numerous finite jumps.

The model specification cards are terminated by a 7/8 EOF followed by an *EOD card.

5. APPLICATION

In late 1972, BMR conducted a long-range seismic refraction survey (Trans-Australia Seismic Survey, TASS) over the central and southern Australian mainland (Finlayson & Drummond, 1973). Crustal and upper mantle apparent velocities were obtained from the data and it was desirable to test several features of the resulting model, notably a possible low-velocity zone (Cull, 1973).

Many configurations of velocity functions have been reported which incorporate a low-velocity layer (Simpson, 1973). In general a velocity decrease of around 5% is noted at depths between 150 and 250 km; however, there appears to be considerable lateral variation in the character of this zone. In the Canadian shield, Brune & Dorman (1963) find no actual low-velocity zone but an interval in which the longitudinal velocity does not increase with depth. Simpson (1973) has found no indication either of low-velocity or of constant-velocity intervals in the Australia shield.

The effects of the presence of a low-velocity layer on time/distance plots are shown in Plates 4 and 5. Plate 4 shows the effect of increasing the thickness of the low-velocity zone upwards from a constant depth, demonstrating that it is not until it reaches considerable thickness that the first-arrival data display a classical "shadow zone", which is seen as a break in the t/d plot. Similarly in Plate 5 the effects of increasing the thickness downwards are demonstrated.

The BMR TASS model under test using GEOTRACE is shown in Plate 6 along with the relevant data deck; the resulting travel-time curves and causative ray paths are presented in Plates 7 and 8 respectively. It is immediately apparent from Plate 7 that although the prograde segments can be explained by a vast number of models, the retrograde segments and the discontinuity are a function of the parameters of the low-velocity layer. A further feature of note is the "bending" of the prograde segments, which is caused by the arc/chord length discrepancy. For body waves the shortest time path is the chord path and it is this that is considered in the GEOTRACE computations.

By comparing the density of emergent rays along the surface arc a further verification of the interpretation may be attempted in terms of relative magnitudes of arrivals from each particular refractor. However, for any particular point on the surface arc a comparison of different phase magnitudes is not possible without the inclusion in the program of a path length decay factor, or more exactly a solution of Zoeppritz's energy partition equations at each refractor.

A further example of the use of GEOTRACE is shown in Plate 9. The model shown here is an approximation to that derived by Simpson (1973) for the Ord River blast. A major feature is the Pn velocity gradient which results in a greatly reduced retrograde travel-time segment. Because of the discrete steps required to represent this velocity gradient the GEOTRACE output contains many closely spaced cusp points; in the limit of sampling where the "steps" approach zero width these cusps disappear and consequently the P_n prograde segment becomes a smooth continuous curve.

6. EARTHQUAKE STUDIES

Because of the option to site the source at various depths, the program GEOTRACE provides a useful tool for check ng epicentres of natural events. In addition, gross structural features of the crust and mantle may be examined in terms of perturbation from a given time/ distance function.

As example of the use of the program for these purposes is given in Plate 10. Two earthquakes are considered; one above and one below a low-velocity zone. From the density of rays at the surface it is clear that earthquakes in this situation can be differentiated, both in terms of ground amplitudes and from an examination of shadow zones.

Investigations of deep mantle or of outer core velocity gradients may also be made using GEOTRACE models. The program output includes a travel-time listing and path identifier; these can be used to select a particular phase so that computed and observed travel times can be compared.

7. RECOMMENDED FUTURE DEVELOPMENT

Although GEOTRACE in its present form is useful as a check on travel times and ray paths its value could be increased by the inclusion of further functions.

As previously discussed, ground amplitudes can be estimated from ray density, but a more exact method would be through an energy estimate of each ray. By summing the ray path segments through the model the total travel path distance can be obtained from the program and can be used to compute the energy attenuation due to geometrical spending of the wave front. Such a method of energy estimation does not allow for energy loss at refractor boundaries caused by the creation of new phases; Zoeppritz's equations must be used if this source of loss is to be considered. By making various assumptions about attenuation due to non-elastic processes, the total travel path distance can also be used to estimate the attenuation due to these processes. However, correlation of theoretical and observed ground amplitudes as an interpretation technique is questionable because of local geological anomalies such as excess sediments or because of variations in ground coupling from site to site and redistribution of energy due to phase differences and interference; such studies could be useful in providing only approximate information on earthquake intensity zones or on risk in local blasting.

Further modifications to the program could include outputs of "slowness" and depth of penetration. In large seismic arrays the "slowness" dT/dD is more easily determined, and by comparing neighbouring rays in the program this value can be extracted, thus providing a further means of checking proposed models. In interpretation, the depth of penetration of a ray is related to the velocity gradient; it can be obtained by noting the depth of each successive probe segment in each ray.

For ease of presentation the program could be modified to provide a plot of the model velocity function on the same output as the time/distance curves.

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APPENDIX 1

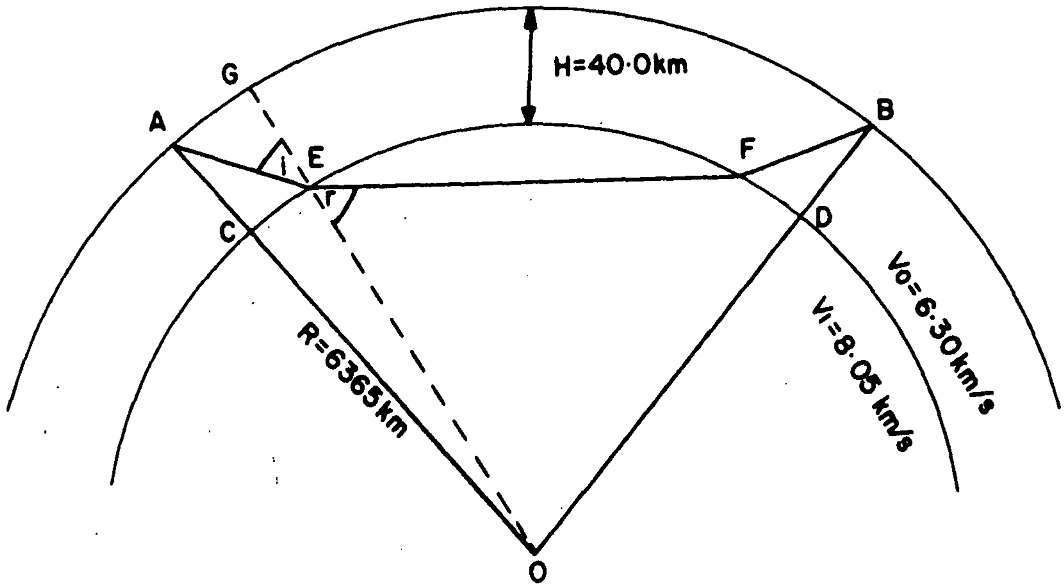
PROGRAM GEOTRACE LISTING

```
      PROGRAM GEOTRACE
C***********************************************************************
C   GEOTRACE DRAWS RAY PATHS THROUGTH EARTH MODEL OF ANY COMPLEXITY   *
C   FACILITY EXISTS FOR PLOTTING TRAVEL TIMES                         *
C   RAYS MAY BE CONCENTRATED AROUND CRITICAL RAFRACTION POINTS        *
C                       J.P.CULL                                      *
C                 REGIONAL CRUSTAL STUDIES                            *
C                 ROOM 104  BUREAU OF MINERAL RESOURCES               *
C***********************************************************************
C
      REAL LENGTH
      INTEGER PLT,TRA
      COMMON ARCMAX,VELL,RCON,TMAX,XSCZ,TSC,PLT,VREDUCE,TMIN
      DIMENSION VEL(100),DPTH(100)
      REARTH=6371.0
      PIE=3.1415927 $ PI2=1.5707963 $ DPIE=6.2831854
C
C   DEFINE PARAMETERS OF MODEL
      AXTEST=0.0
   33 CONTINUE
      READ(60,103)DSC,ARCMAX,TSC,ORDEP,VREDUCE,DEPMAX,RCON,AINC,APROBE,
     1THST,THLIM,MPT,TRA,PLT
  103 FORMAT(11F5.0,21X,A1,A1,A2)
      IF(EOF,60)34,35
   35 CONTINUE
C
      SCALE=DSC
      IF(PLT.NE.2HPB)PLT=2HPL
      IF(APROBE.LE.0.0)APROBE=5.0 $ IF(AINC.LE.0.0)AINC=2.0
      IF(THLIM.LE.0.0)THLIM=PI2
      PROBE=APROBE $ DPROBE=APROBE*0.05
      INC=THST/AINC
      AINC=AINC*0.017453
       ARCMIN=-0.001
      IF(RCON.LE.0.001)GOTO25 $ RCON=0.0 $ GOTO11
   25 RCON=1.0
   11 IF(DEPMAX.LE.0.0)DEPMAX=ARCMAX*0.2
      XSCZ=SCALE $ IF(XSCZ.LE.0.0)XSCZ=DEPMAX*0.1
      IF(TRA.NE.1H )GOTO24
      CALLPLOT(0.0,0.0,1,1) $ CALLPLOT(XSCZ,XSCZ,2,1)
      CALLPLOT (ORDEP,0.0,3,1)
   24 I=0
C
C   DEFINE EARTH MODEL IN TERMS OF DEPTHS AND VELOCITES
  100 FORMAT(F10.2,F10.2)
    1 READ(60,100)D,V $ IF(EOF,60)3,2
    2 I=I+1
      PRINT100,D,V
      DPTH(I)=D $ VEL(I)=V $ GOTO1
    3  IE=I
      VELL=VEL(1) $ IF(AXTEST.LT.1.0)CALLTDAXIS
C-----------------------------------------------------------------------
C   LOCATE ORIGIN                                                      -
      KO=-1                                                            -
      DO19I=1,IE                                                       -
      IF(ORDEP.GE.DPTH(I))GOTO19 $ KO=I-1 $ I=IE                       -
   19 CONTINUE                                                         -
      IF(KO.LE.-1)KO=IE                                                -
C-----------------------------------------------------------------------
  102 FORMAT(20X,*ARC*,8X,*TRAVEL TIME*,15X,*PATH*)
```

```
      WRITE(61,102)
C
                RF=0,0 $ RK=0,0
      LABPATH=0
C     INCREMENT INITIAL ANGLE OF INCIDENCE
    4 INC=INC+1 $ V1=VEL(KO) $TIME=0,0 $ LENGTH=0,0
      LABP=LABPATH
      LABPATH=KO $ IPF=KO $ RP=RK $ RK=0,0 $ CONCFL=0,0
      TH=INC*AINC $ IF(TH,GE,THLIM)GOTO10 $ ZO=OPDEP $XO=0,0
    5 LENGTH=LENGTH+PROBE $ ZD=ZO $ XD=XO
      DZ=LENGTH*COS(TH) $ DX=LENGTH*SIN(TH)
      ZD=ZD+DZ $ XD=XO+DX
      IF(ZD,GE,DEPMAX)GOTO9
      IF(XD,GE,ARCMAX.OR.XD,LE,ARCMIN)GOTO9
      DCENT=REARTH-ZD $ ALAD=ABS(XD)
      BOUND=SQRT((DCENT*DCENT)+(ALAD*ALAD))
      IF(BOUND,GE,REARTH)GOTO9 $ DEPTH=REARTH-BOUND
       IP=0
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     FIND VELOCITY OF RAY                                             :
      DO6I=1,IE                                                        :
      IF(DEPTH,LE,DPTH(I))7,6                                          :
    7 IP=I-1 $ I=IE                                                    :
    6 CONTINUE                                                         :
      IF(IP,EQ,0)IP=IE                                                 :
      V2=VEL(IP) $ IF(V2,EQ,V1)GOTO5                                   :
      IF(PROBE,EQ,DPROBE)GOTO17                                        :
      LENGTH=LENGTH-PROBE                                              :
      PROBE=DPROBE $ GOTO5                                             :
   17 PROBE=APROBE $ ZO=ZD $ XO=XD                                     :
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C
C***********************************************************************
C     FIND SLOPE OF INTERFACE                                          *
      RAT=V1/V2                                                        *
      IF(XD,LT,0,0)GOTO12                                              *
      SLSIN=-1,0 $ IF(ZD,GT,REARTH)SLSIN=1,0 $ GOTO13                  *
   12 SLSIN=1,0 $ IF(ZD,GT,REARTH)SLSIN=-1,0                           *
   13 SLOPE=SLSIN*ASIN(ALAD/BOUND)                                     *
C***********************************************************************
C
      O=TH $ SL=SLOPE
      IF(TH,LT,PIE)GOTO14
      O=DPIE-TH $ SL=-SLOPE
   14 WPD=SIN(O-SL) $ IF(WPD,GT,RAT)GOTO15
      WOT=ASIN(WPD/RAT) $ ABSL=ABS(SL)
      THI=SL+WOT
      IF(O,LT,PI2,AND,(-SL),GT,(PI2-O))GOTO21
      IF(O,GT,PI2,AND,SL,LT,(O-PI2))GOTO21 $ GOTO16
   21 THI=PIE+SL-WOT $ GOTO16
   15 THI=PIE-O+SL+SL $ RF=1,0 $ RK=1,0
   16 IF(TH,GT,PIE)THI=DPIE-THI
      IF(THI,GT,DPIE)THI=THI-DPIE
      IF(THI,LT,0,0)THI=THI+DPIE
      TH=THI
C
    8 TIME=TIME+LENGTH/V1
      IPL=IP $ IF(RF,GT,0,0)IPL=IPF
      LABPATH=LABPATH*1E+1+(IPL)
      IF(TRA,EQ,1H )CALLPLOT(ZD,XD,4,1)
```

```
      LENGTH=0.0 $ IF(RF.LT.1)V1=V2 $ RF=0
      IPF=IP $ GOTO5
C
    ) IF(PROBE.EQ.DPROBE)GOTO18
      LENGTH=LENGTH-PROBE
      PROBE=DPROBE $ GOTO5
   18 PROBE=APROBE
      TIME=TIME+LENGTH/V1
      IF(TRA.EQ.1H )CALLPLOT(ZD,XD,4,1)
      DEG=ASIN(ALAD/BOUND)
      IF(DCENT.LT.0.0)DEG=3.14159-DEG
      IF(XD.LT.0.0)DEG=-DEG $ ARC=REARTH*DEG
      WRITE(61,101)ARC,TIME,LABPATH
  101 FORMAT(18X,F9.2,5X,F7.2,5X,I20)
      IF(TRA.EQ.1H )CALLPLOT(ORDEP,0.0,3,1)
C   PLOT TRAVEL TIME
      IF(CONCFL.GT.0.0)GOTO20 $ IF(BOUND.LT.REARTH)GOTO23
      IF(VREDUCE.GT.0.1)TIME=TIME-ARC/VREDUCE
      IF(TIME.LT.TMIN)GOTO23
      IF(TIME.GT.TMAX.OR.ARC.GT.ARCMAX)GOTO23
      IF(MPT.EQ.1H )GOTO26
      CALLPLOT(-TIME,ARC,3,2) $ CALLTEXT(1H+,0,1,2)
      IF(RK.EQ.1.0.AND.RP.EQ.1.0)GOTO23 $ CALLTEXT(1H0,0,1,2)
      GOTO23
   26 IF(LABPATH.EQ.LABP)GOTO27 $ CALLPLOT(-TIME,ARC,3,2) $ GOTO23
   27 CALLPLOT(-TIME,ARC,4,2) $ GOTO4
   23 CONTINUE
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C                                                                        X
C              CONCENTRATE RAYS NEAR CRITICAL RAY                        X
      IF(BOUND.LT.REARTH.OR.RCON.LE.0.0)GOTO4                            X
      ONC=INC                                                            X
      THO1=(ONC-1.5)*AINC $ THO2=INC*AINC                                X
      AIL=AINC*0.05 $ CONCFL=1.0 $ INK=0                                 X
      JOIN=0.0 $ GOTO22
   20 IF(BOUND.LT.REARTH)GOTO22                                          X
      IF(VREDUCE.GT.0.1)TIME=TIME-ARC/VREDUCE
      IF(TIME.LT.TMIN)GOTO22
      IF(TIME.GT.TMAX.OR.ARC.GT.ARCMAX)GOTO22                            X
      IF(MPT.EQ.1H )GOTO29
      CALLPLOT(-TIME,ARC,3,2)                                            X
      CALLTEXT(1H+,0,1,2)                                                X
      IF(RK.EQ.1.0.AND.RP.EQ.1.0)GOTO22
      CALLTEXT(1H0,0,1,2)                                                X
      GOTO22
   29 IF(JOIN.GT.0.1)GOTO30
      CALLPLOT(-TIME,ARC,3,2) $ JOIN=1.0 $ GOTO22
   30 CALLPLOT(-TIME,ARC,4,2)
   22 INK=INK+1                                                          X
      TH=(INK*AIL)+THO1 $ IF(TH.GT.THO2)GOTO4                            X
      V1=VEL(KO) $ TIME=0.0 $ LENGTH=0.0                                 X
      LABPATH=KO $ IPF=KO $ RP=RK $ RK=0.0                               X
      ZO=ORDEP $ XO=0.0
      LABP=LABPATH $ GOTO5
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
   10 CONTINUE
      AXTEST=2.0 $ GOTO33
   34 CONTINUE
       END
```

```
      SUBROUTINE TDAXIS
      COMMON ARCMAX,VELL,RCON,TMAX,XSCZ,TSC,PLT,VREDUCE,TMIN
      INTEGER PLT
      TMAX=ARCMAX/VELL
      ALIN=10,0 $ IF(PLT,EQ,2HPB)ALIN=28,0
      IF(TSC,GT,0,()GOTO63
CTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT
C   CALCULATE TIME AXIS SCALE                                          T
      AI=ALOG10(TMAX) $ IA=AI                                          T
      IF(AI,LT,0,0)IA=IA-1                                             T
      POWER=10,0*IA $ NA=(TMAX/POWER+0,5)                              T
      IF(NA/3-1)60,61,62                                               T
   60 TSC=0,5*POWER $ GOTO63                                           T
   61 TSC=POWER $ GOTO63                                               T
   62 TSC=2,0*POWER                                                    T
   63 CONTINUE                                                         T
CLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLLL
      TMOX=TSC*ALIN
      TEX=-TMOX $ TMAX=TMOX $ TMIN=0,0
      IF(VREDUCE,LT,0,1)GOTO20
      TAX=-(ALIN*0,5-2,0)*TSC $ TMAX=TMOX+TAX $ TMIN=TAX
      TEX=-(TMAX)
   20 CALLPLOT(TEX,0,0,1,2) $ CALLPLOT(TSC,XSCZ,2,2)
      IC=0 $ DSN=XSCZ*0,2
      DO24IC=1,19
      TPOT=(IC*TSC)+TMIN
      IF(TPOT,GE,TMAX)GOTO24 $ IF(TPOT,LT,TMIN)GOTO24
   65 CALLPLOT(-TPOT,-DSN,3,2) $ ENCODE(4,107,BUF)TPOT
  107 FORMAT(F4,0)
      CALLPLOTSET(2)
   64 CALLTEXT(BUF,4,2,2)
      CALLPLOTSET(1)
   24 CONTINUE
      IC=0
   25 IC=IC+1
      DSCT=IC*XSCZ $ IF(DSCT,GT,ARCMAX)GOTO23
      CALLPLOT(0,0,DSCT,3,2)
      ENCODE(4,107,BUF)DSCT $ CALLTEXT(BUF,4,2,2)
      GOTO25
   23 CONTINUE
      IF(VREDUCE,LT,0,1)GOTO26
      CALLPLOT(0,,-(XSCZ*0,5),3,2) $ CALLTEXT(5HTT-D/,5,2,2)
      ENCODE(4,108,BUF)VREDUCE $ CALLTEXT(BUF,4,2,2)
  108 FORMAT(F4,2)
   26 CONTINUE
      RETURN
      END
```



SIMPLIFIED SPHERICAL EARTH GEOMETRY

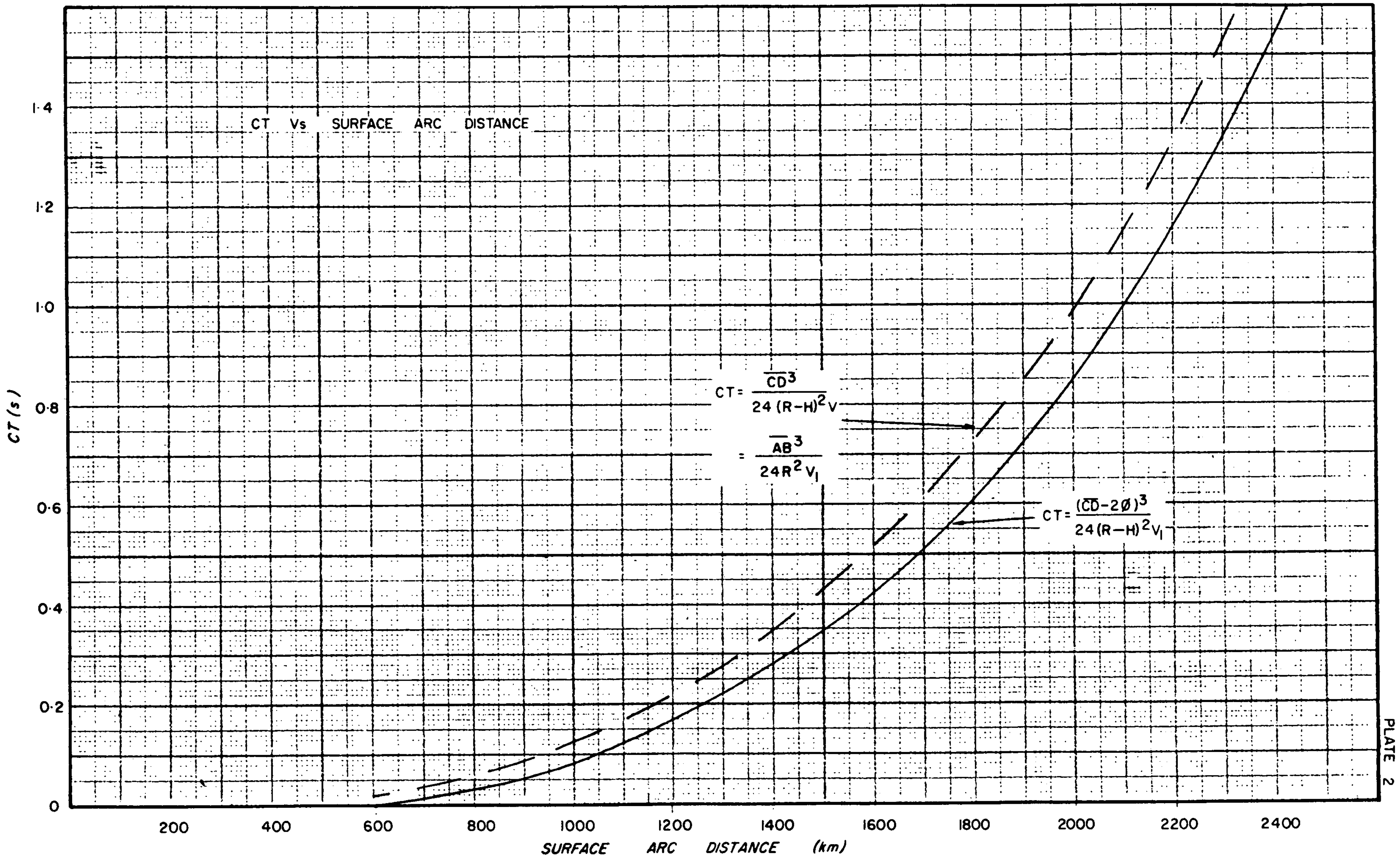
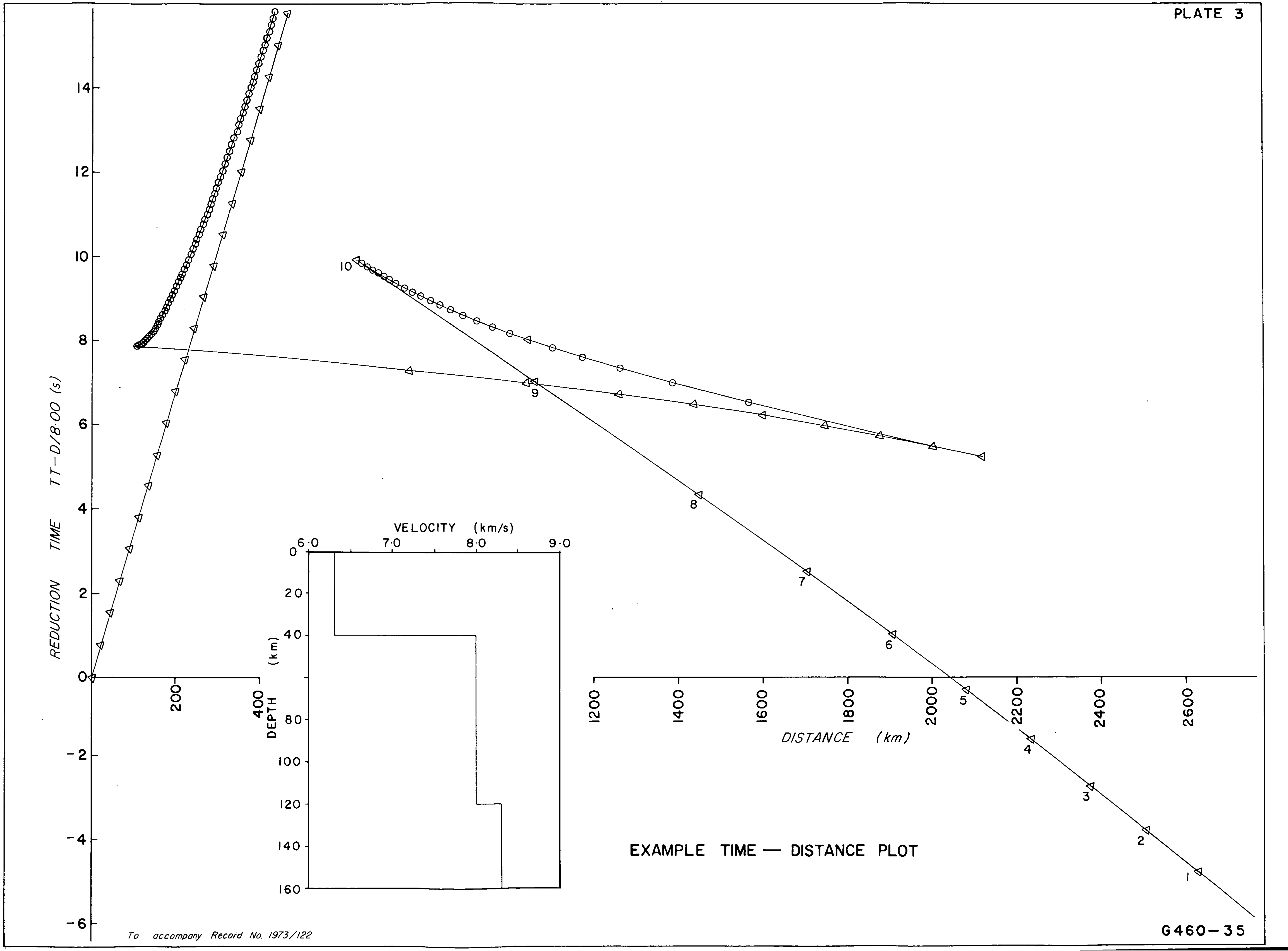


PLATE 2

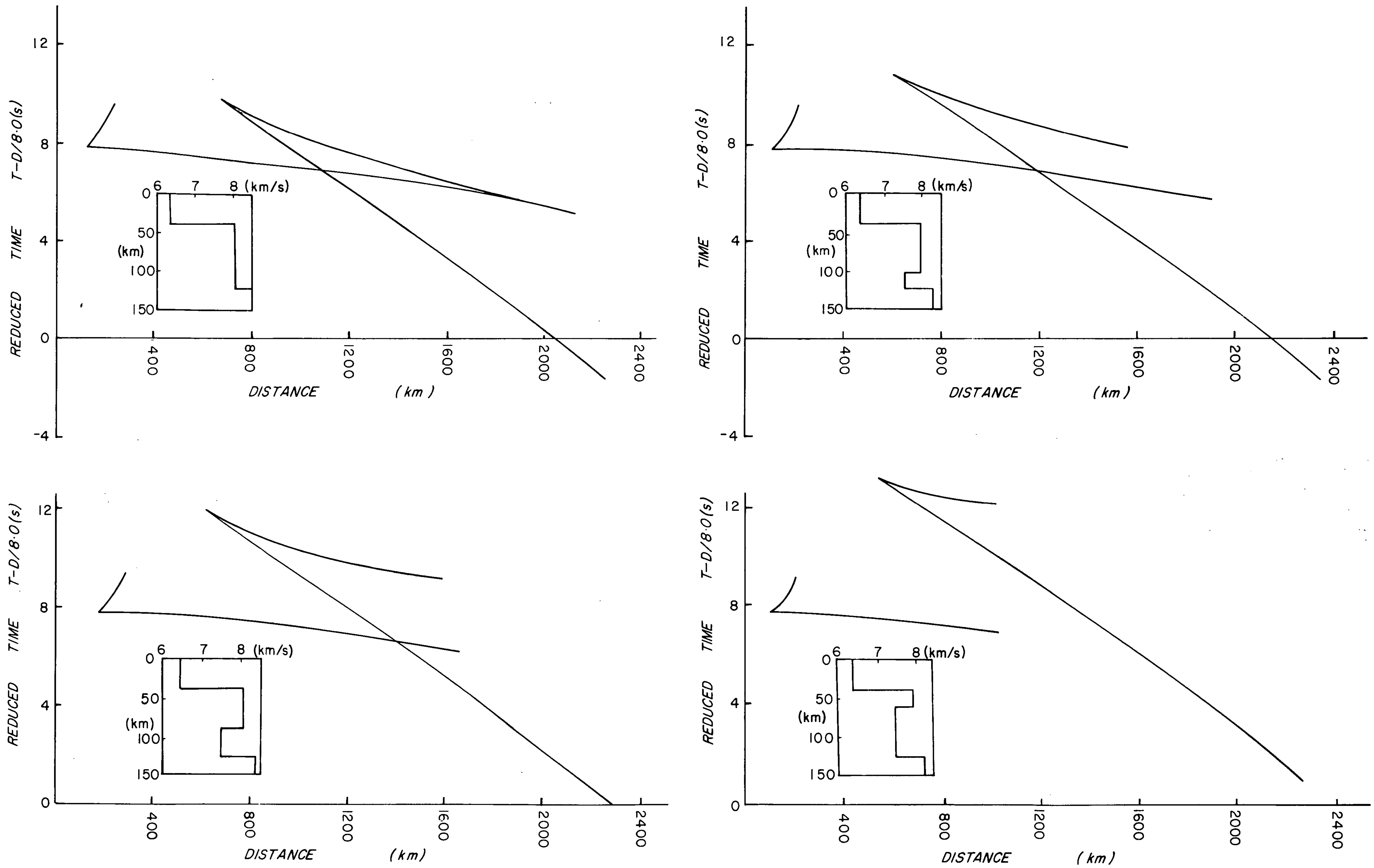
TRAVEL—TIME CORRECTION TERM (CT) AS A FUNCTION OF DISTANCE



EXAMPLE TIME — DISTANCE PLOT

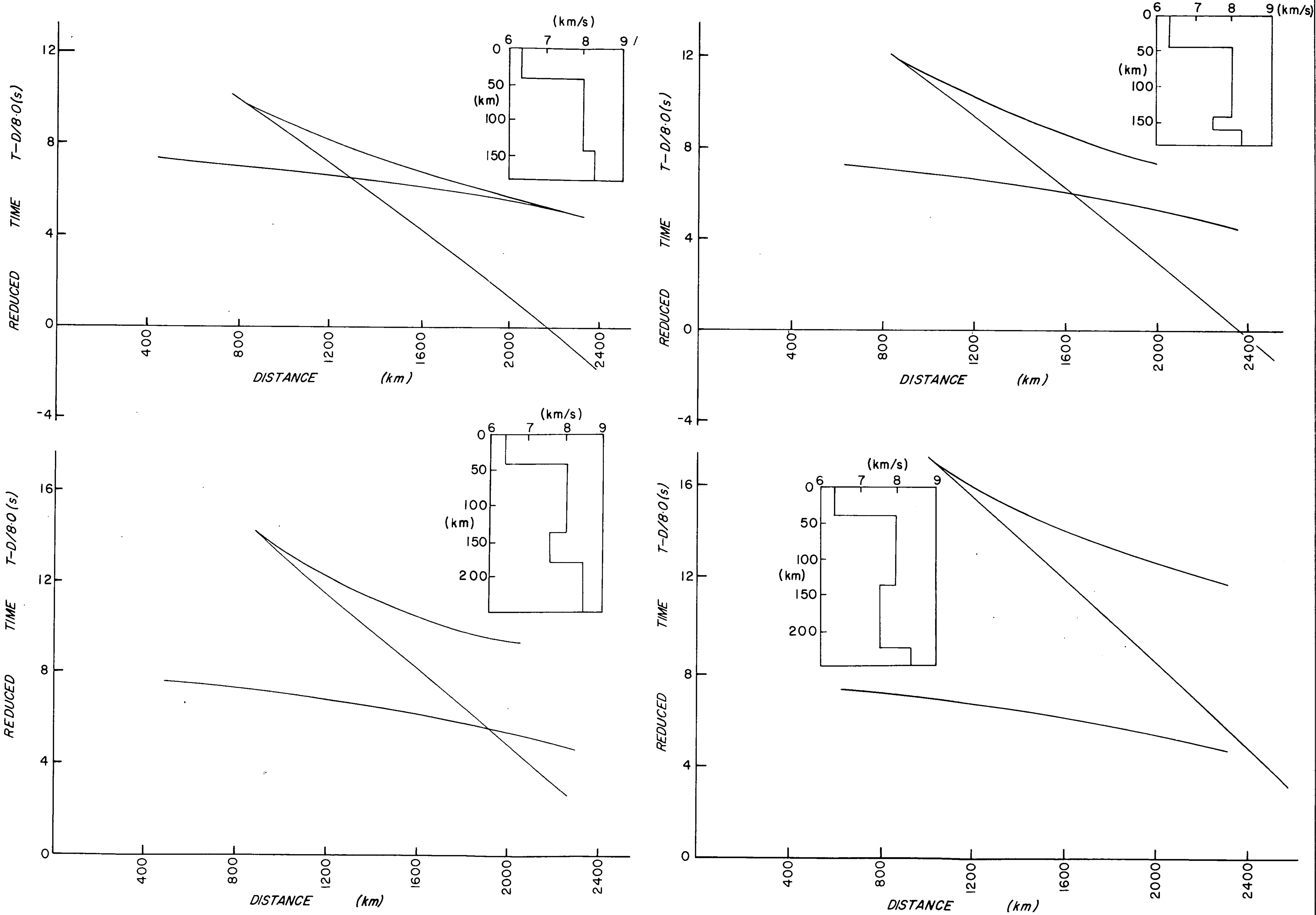
To accompany Record No. 1973/122

G460—35



EXAMPLE TIME — DISTANCE PLOTS WITH LOW VELOCITY ZONE THICKNESS INCREASING UPWARDS

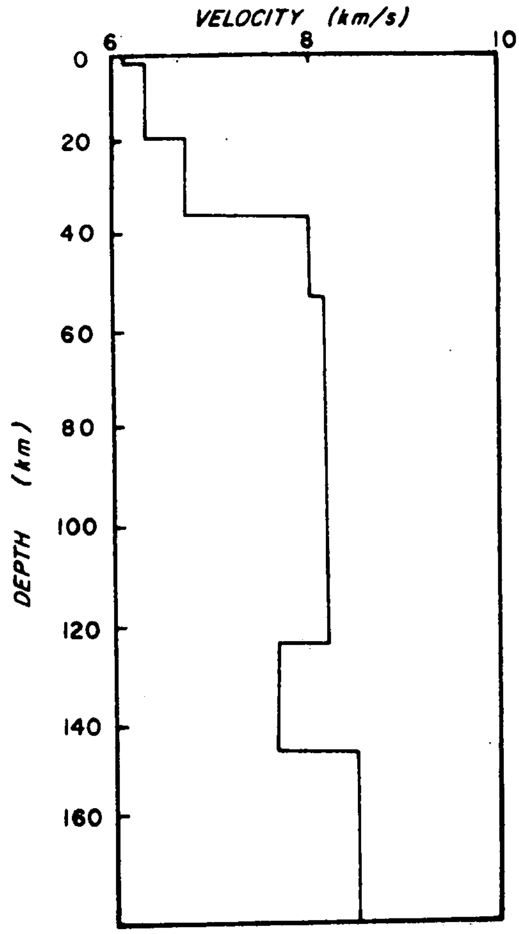
PLATE 5



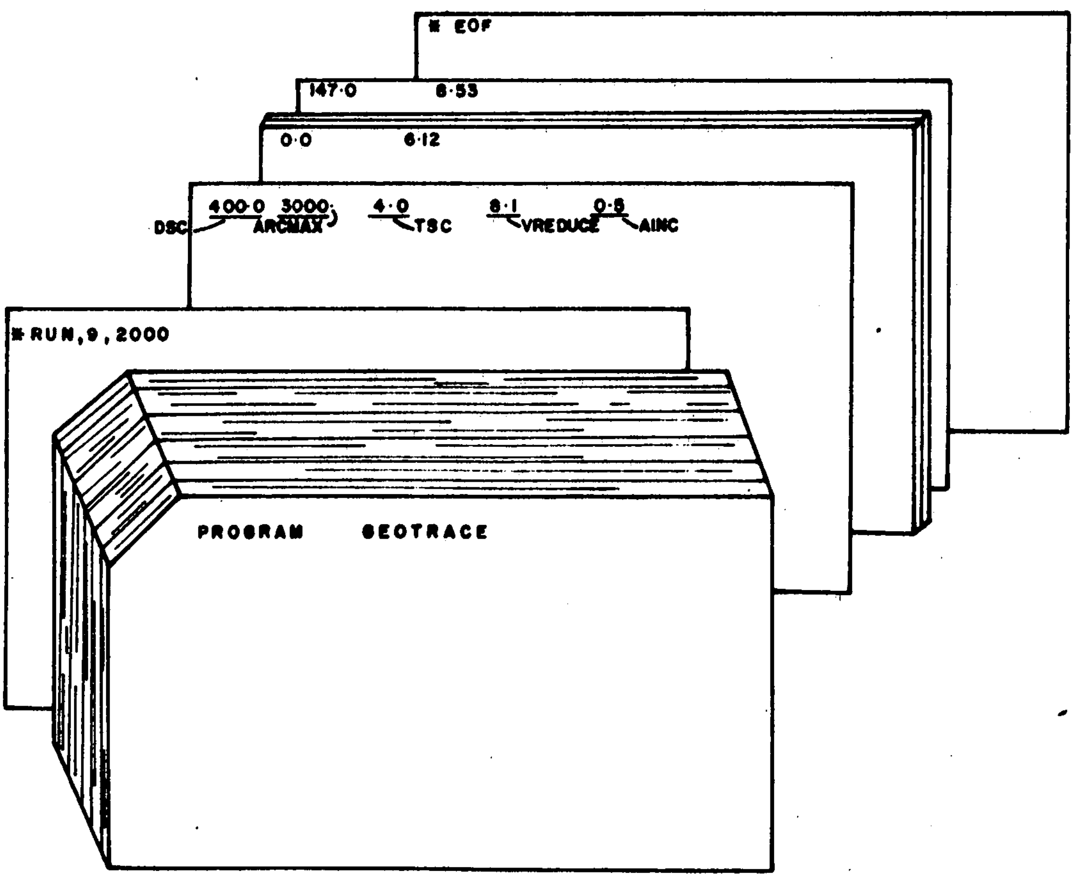
EXAMPLE TIME−DISTANCE PLOTS WITH LOW VELOCITY ZONE THICKNESS INCREASING DOWNWARDS

To accompany Record No. 1973/122

G460−37



DEPTH	VELOCITY
0	6·12
5	6·33
20	6·72
36	8·04
53	8·19
125	7·70
147	8·53



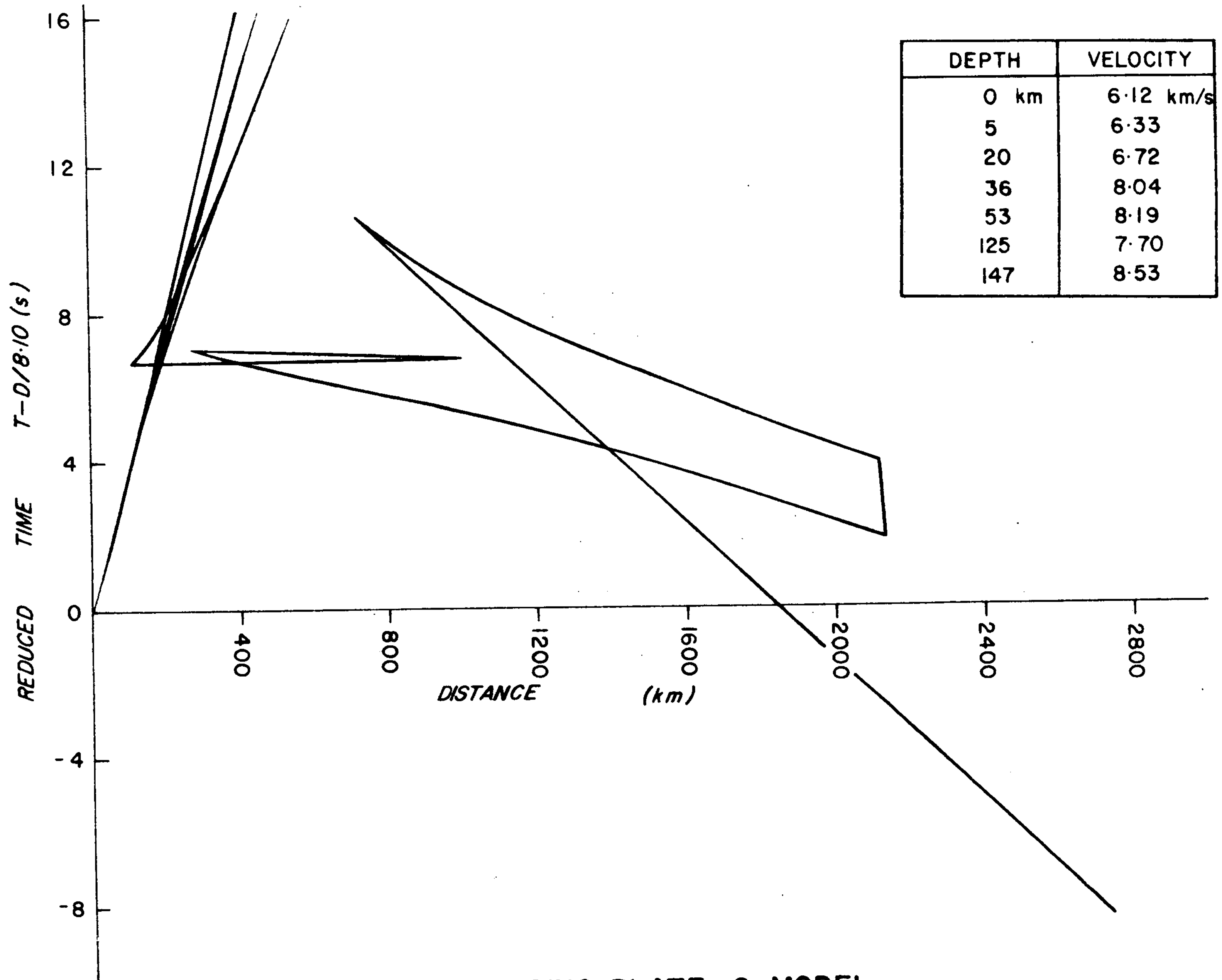
TASS VELOCITY DEPTH MODEL AND GEOTRACE CARD DECK STRUCTURE

To accompany Record No. 1973/122

G460—38A

To accompany Record No. 1973/122

PLATE 7

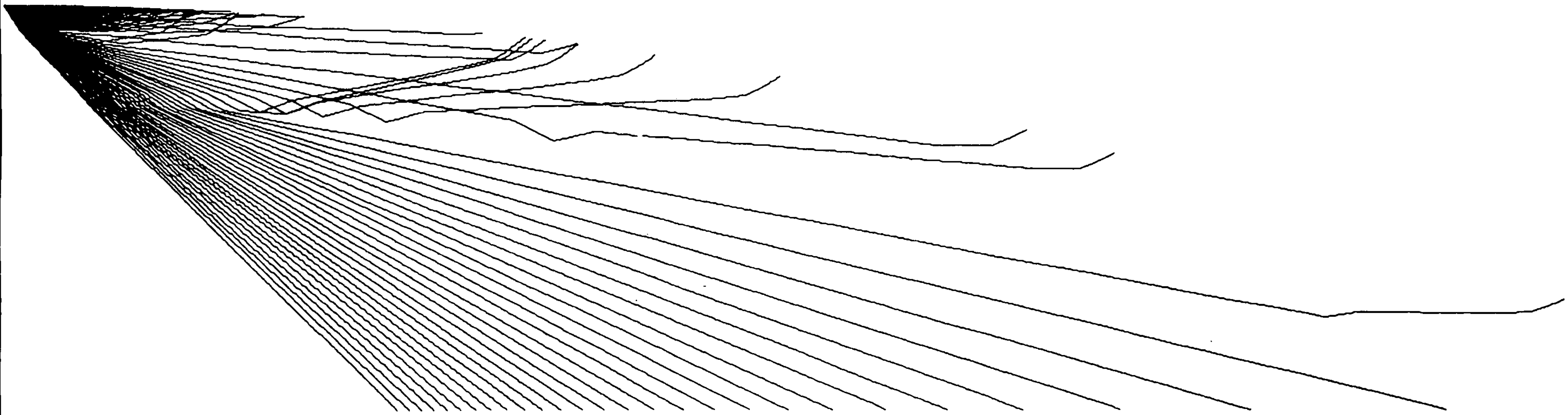


DEPTH	VELOCITY
0 km	6·12 km/s
5	6·33
20	6·72
36	8·04
53	8·19
125	7·70
147	8·53

TIME/DISTANCE PLOT USING PLATE 6 MODEL

G460—42A

To accompany Record No. 1973/122

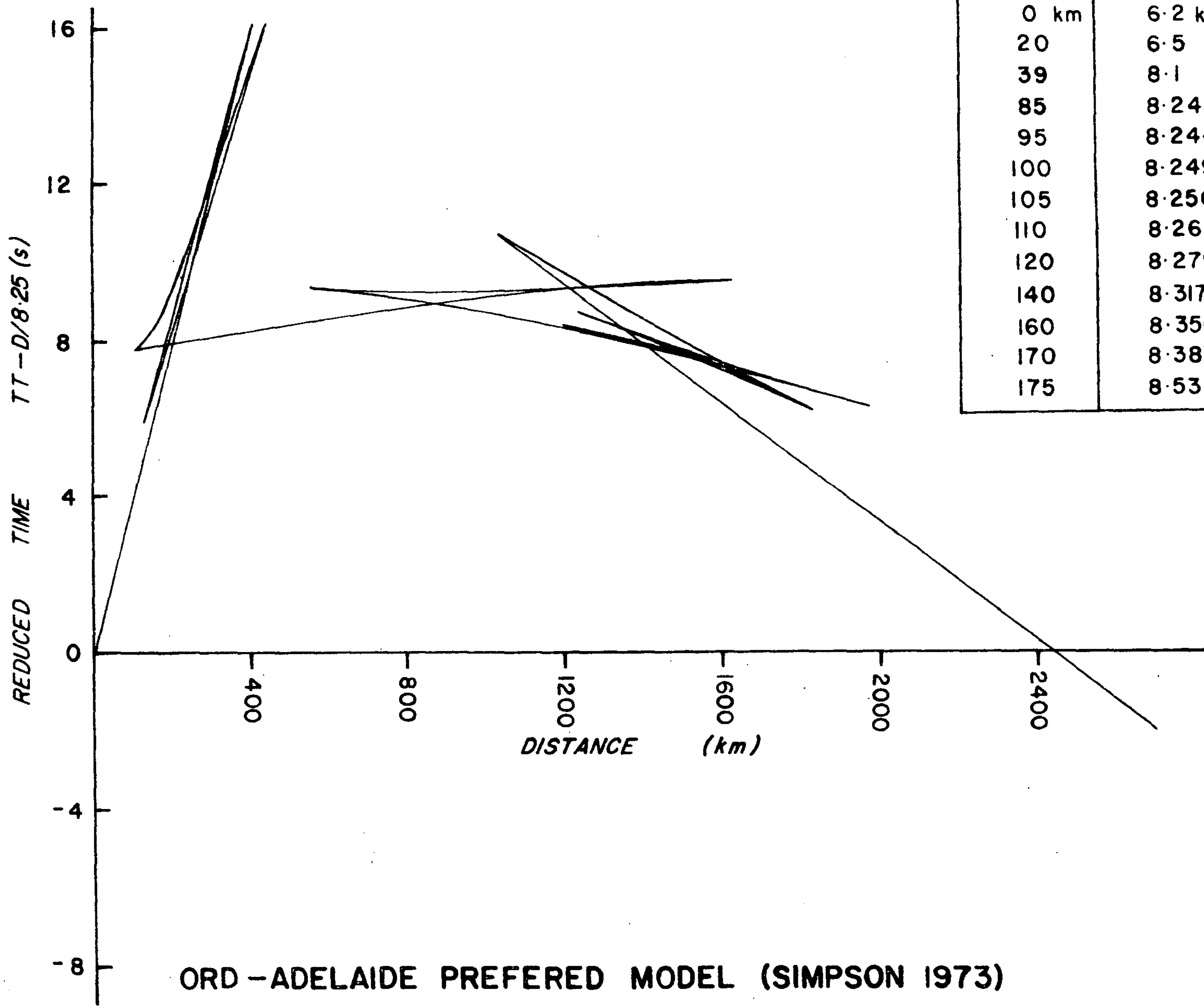


RAY PATHS FOR PLATE 6 MODEL

G460—39A

To accompany Record No. 1973/122

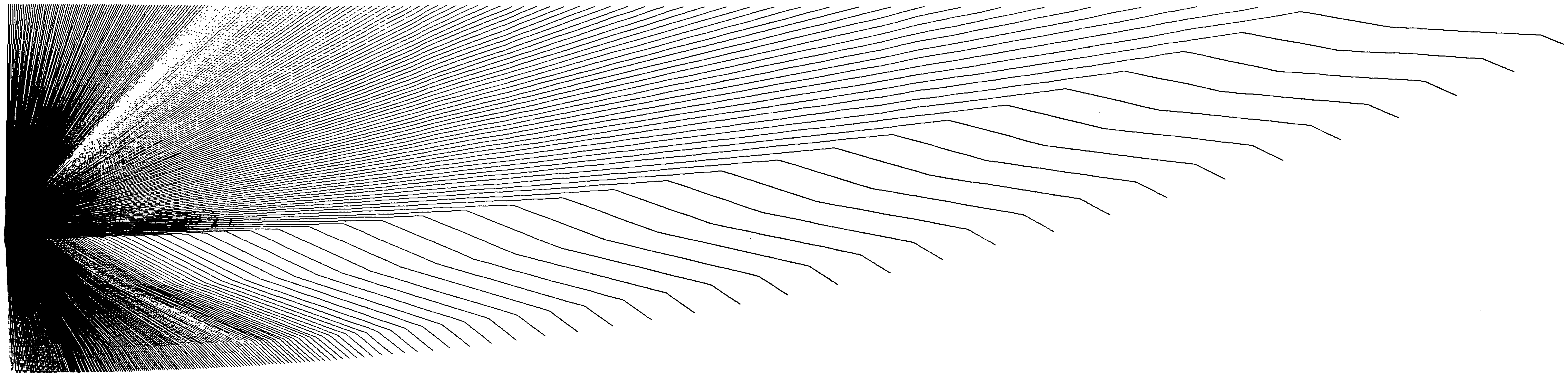
DEPTH	VELOCITY
0 km	6·2 km/s
20	6·5
39	8·1
85	8·24
95	8·2441
100	8·2497
105	8·2562
110	8·2629
120	8·2792
140	8·3173
160	8·3563
170	8·3830
175	8·5320

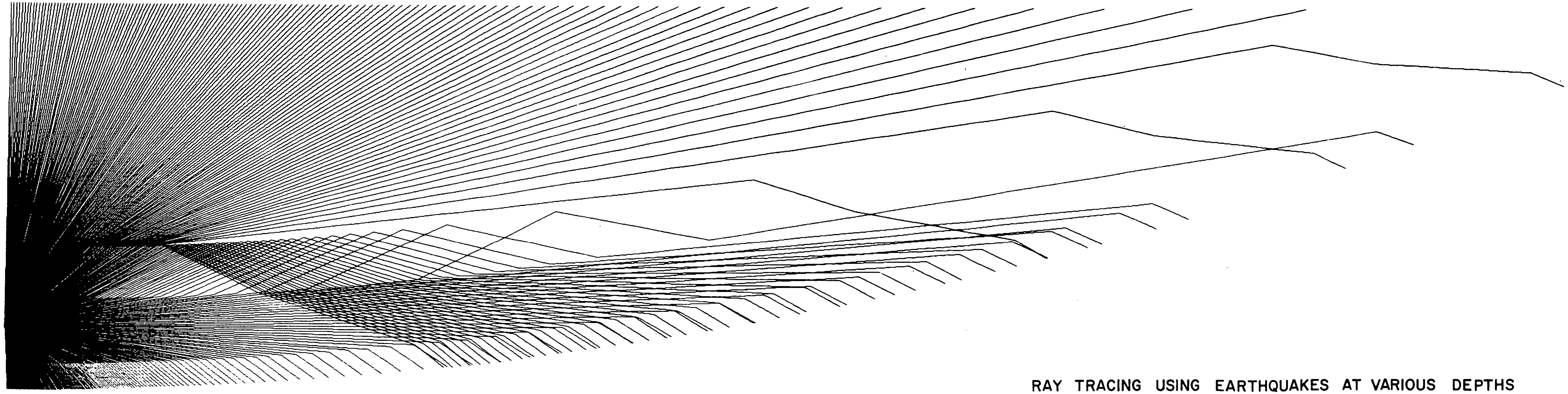


ORD−ADELAIDE PREFERED MODEL (SIMPSON 1973)

G460-40A

PLATE 9





RAY TRACING USING EARTHQUAKES AT VARIOUS DEPTHS