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TRANSLATION OF PAPERS ON SEISMIC REFLECTION AND REFRACTION METHODS

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PREFACE

I first became acquainted with the deep seismic sounding work of Professor Puzyrev and his team in the Institute of Geology and Geophysics, during a brief visit to Novosibirsk in 1970. His description of their methods and ensuing discussion led me to make the effort to find out some more details. I am unaware of any translations into English describing their methods, and have translated some reprints of papers in Russian kindly supplied by Prof. Puzyrev, together with other papers found in the literature.

These papers give an account of the development of the theory and field techniques carried out in the Institute since the publication of the book by Puzyrev in 1959 (Ref. 4 of the first paper), a translation of which is held in the BMR library. Papers 1, 2, 3, 4, 5, 8 and 9 give detailed mathematical treatment of various aspects of the interpretational procedures and of the reasoning behind the field techniques used. Paper 6 gives a more general account of the methods, and perhaps should be read first; it also includes some numerical quantities in relation to operational procedures not given elsewhere. Paper 7 gives an illustration of the results obtained by their methods, and a comparison with more elaborate techniques of the other Russian workers.

It is probable that many of the problems discussed have been met and solved by seismic field workers in various ways in other parts of the world; however the approach of the Siberian team appears to be different in many respects from those of other groups familiar to me, particularly in the use of the time field plot in interpretation (not to be confused with the "time field" described previously by Gamburtsev, which is similar to that known as the "wave-front" technique in western literature), and the systematic use of isolated "point" soundings as a reconnaissance tool for filling in between widely separated more detailed probes.

The methods have apparently been used successfully for mapping sedimentary basins, intra-basement structure, Konrad and Mohorovicic discontinuities, and have also detected layering within the upper mantle. At a time when BMR is becoming involved in carrying out deep crustal seismic surveys, it should be worthwhile studying the Siberian methods to assess whether any aspects of them would be applicable under the conditions found in Australia, or whether they can lead to proposals for rationalizing or economizing in field procedures in this type of survey, which can be very expensive.

An additional reason for interest in these methods became apparent during a recent visit to Australia by Professor Ravich and Dr Grikurov of the Soviet Committee on Antarctic Research. They described deep seismic sounding work in the Prince Charles Mountains of Antarctica by a Russian expedition, and it transpired that the methods used were similar to those described herein, being adapted and applied to Antarctic conditions under the guidance and direction of Professor Puzyrev and Dr Mishenkin.

Notes on Terminology

In many places a distinction is made between head waves, which travel in a medium just below its upper boundary with a constant velocity, and waves which are refracted at an angle at a boundary so that their ray path penetrates into the lower medium; both these types are commonly described as "refracted" waves in English. However three terms are used in Russian - golovnoi (head), prelomlenniy, and refragiovanniy (refracted). The use of the second term does not seem to be entirely consistent in all papers; in one place all three terms are used in the same paragraph! I have endeavoured to maintain the distinction between head and refracted (non-head type) waves wherever this is required by the context.

The word "hodograph" is used for what is commonly called a travel-time curve or a time-distance curve in English. This seems to be a useful and less clumsy word, and I have in general retained its use.

Thanks are due to Mrs Helen Hughes for helping to sort out some of the more obscure Russian syntax.

Geology and Geophysics, 1960, No. 10, p. 120

N.N. Puzyrev

AN INTERPRETATION OF REFRACTION DATA IN THE PRESENCE OF A VELOCITY GRADIENT IN THE LOWER MEDIUM

Abstract

In this paper the problem of the cause of non-parallel overtaking hodographs, connected with the penetration of rays into the thickness of the lower medium, is discussed. The equation of the hodograph for a flat boundary is deduced, when the velocity in the lower medium is a linear function of depth; the method of determination of the velocity gradient and the boundary velocity is given; a method for approximate determination of the depth and for finding a composite hodograph is indicated.

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In practice in the interpretation of refraction data, it is usually assumed that the refracting layer is uniform, and that waves propagating along its upper boundary do not penetrate into a finite thickness of the layer. This assumption is valid if overtaking hodographs are parallel for corresponding waves. In some cases this condition is not fulfilled, indicating penetration of the waves (rays) into the body of the refracting layer, while the depth of penetration increases with increasing distance from shotpoint to seismograph. As is well known [1,2], there are two reasons which can explain this phenomena: convexity of the boundary and increase of velocity with depth in the second layer. In any particular case, with enough observations, it is possible to resolve uniquely which of these is the cause of the non-parallel hodographs. The influence of a convex boundary has already been adequately investigated [3] and therefore it is not considered here. In the following paper attention is concentrated on the second reason - non-parallel hodographs related to penetration of the rays into the lower layer. Apparently for practical seismic surveying, usually for the investigation of structures of relatively small amplitude, this is more important than the influence of convexity of the boundary.

1. Equation of the hodograph.

First of all, let us find the equation of the hodograph when the rays penetrate into a finite thickness of the refracting layer. This problem was discussed qualitatively by G.A. Gamburtsev [1]. Later, interest developed in finding the quantitative relation between the gradient of velocity in the second medium and the parameters of the hodographs, in order to analyse and study errors in determination of the elements of the layering and the boundary velocities, if the effect of penetration of the rays was not taken into account.

So as to avoid complex difficult mathematics, to be able to analyse and apply practically the solution obtained, let us make the following simplifying assumptions: (1) there exists only one boundary on which the velocity jumps; (2) the boundary is flat; (3) the velocity in the first medium (v_1) is constant. In the lower medium, where thickness is not great, the velocity increases with a constant gradient, i.e.

$$v_r = v_{or}(1 + \gamma z) \tag{1}$$

where z = depth from the top of the second layer

v_{or} = boundary velocity at the top of the layer.

We note that if change of velocity with depth in the refracting layer differs from a linear law, then it is necessary to increase the mean gradient γ_m, which is related to the function $v_r(z)$ by the relation [4]:

$$\gamma_m = \frac{1}{z}\,\lambda^{-1}\left(\frac{z}{v_{or}\int \frac{z\,dz}{v_r(z)}}\right)$$

where λ^{-1} denotes the inverse of the function $\lambda(x) = \frac{x}{\ln(1+x)}$. For simplicity at first, let us suppose that the boundary is horizontal (Fig. 1). Let us take as parameter the angle α between the descending ray and the vertical at the boundary. It is connected with the refracted angle i_{02} in the second medium by the relation

$$\sin\alpha = n_o \sin i_{02}; \quad \left(n_o = \frac{v_1}{v_{or}}\right).$$

Using the diagram, it is possible to deduce the equation of the travel-time curve for the refracted waves in the parametric form:

$$x = 2h \tan \alpha + \overline{AB} \tag{2}$$

$$t = \frac{2h}{v_1 \cos \alpha} + t\,(\overline{AB}) \tag{3}$$

Then, as is well known [1]:

$$\overline{AB} = \frac{2}{\gamma} \cot i_{02} = \frac{2}{\gamma} \frac{\sqrt{n_0^2 - \sin^2\alpha}}{\sin \alpha}$$

$$t(\overline{AB}) = \frac{2}{v_{or}\gamma} \operatorname{arsinh}(\cot i_{02}) = \frac{2}{v_{or}\gamma} \operatorname{arsinh}\left(\frac{\sqrt{n_0^2 - \sin^2\alpha}}{\sin \alpha}\right)$$

Substituting the values found for $\overline{AB}$ and $t(\overline{AB})$ in (2) and (3), we obtain

$$x = 2h \tan \alpha + \frac{2}{\gamma \sin\alpha}\sqrt{n_0^2 - \sin^2\alpha} \tag{2'}$$

$$t = \frac{2h}{v_1 \cos \alpha} + \frac{2}{v_{or}\gamma} \operatorname{arsinh}\left(\frac{\sqrt{n_0^2 - \sin^2\alpha}}{\sin \alpha}\right) \tag{3'}$$

Elimination of the parameter α in the general case is difficult and leads only to a complex expression. For simplification of this problem let us take into account that for comparatively small gradients, which generally occur, the point A (see Fig. 1) will change its location very little, and thus the projection O'A can be taken with sufficient approximation as a constant, and its magnitude is determined as follows:

$$O'A = 2h \tan i_{o1} = x_H, \ (\sin i_{01} = n_0).$$

Then expression (2') becomes:

$$x = x_H + \frac{2}{\gamma}\sqrt{n_o^2 \operatorname{cosec}^2\alpha - 1} \quad ,$$

where x_H = absicissa of the initial point of the deep wave.

Solving for $\sin\alpha$, we get:

$$\sin\alpha = \frac{2\,n_o}{\sqrt{4+\gamma^2\,(x+x_H)^2}}$$

For small gradient γ we can write approximately

$$\sin\alpha \doteq n_o[1-\frac{1}{8}\gamma^2\,(x-x_H)^2] \qquad (4)$$

On substituting the values found for the parameter α in expression (3), it naturally follows that the point A remains constant because to the given degree this compensates for the error which was tolerated in the calculation of $\sin\alpha$. In such a case the first term in the expression for t may be taken as approximately constant, equal to $2\,h/v_1\cos_{01}$, and the equation of the hodograph is written in the form:

$$t = \frac{2h}{v_1\cos i_{01}} + \frac{2}{v_{or}\gamma}\,\text{arsinh}\left\{\frac{\gamma\,(x-x_H)\sqrt{1-\frac{1}{16}\gamma^2(x-x_H)^2}}{2[1-\frac{1}{8}\gamma^2(x-x_H)^2]}\right\}$$

Disregarding terms in γ^3 and further in the argument of arsinh, let us rewrite

$$t = \frac{2h}{v_1\cos i_{01}} + \frac{2}{v_{or}\gamma}\,\text{arsinh}\,[\,\frac{\gamma}{2}\,(x-x_H)]$$

With further simplification, expanding the function arsinh as a series and rejecting terms in γ of the fifth and higher powers, we obtain the following approximate expression for the travel-time of the refracted wave:

$$t = \frac{2h\cos i_{or}}{v_1} + \frac{x}{v_{or}} - \frac{\gamma^2}{24v_{or}}(x-x_H)^3 \qquad (5)$$

Verification on a theoretical example showed that, despite the series of simplifying assumptions tolerated in the derivation, the travel-time equation obtained for not too large gradients is quite accurate enough. The apparent velocity of the hodograph is determined from the expression

$$v_k = \frac{v_{or}}{1-\frac{1}{8}\gamma^2(x-x_H)^2}$$

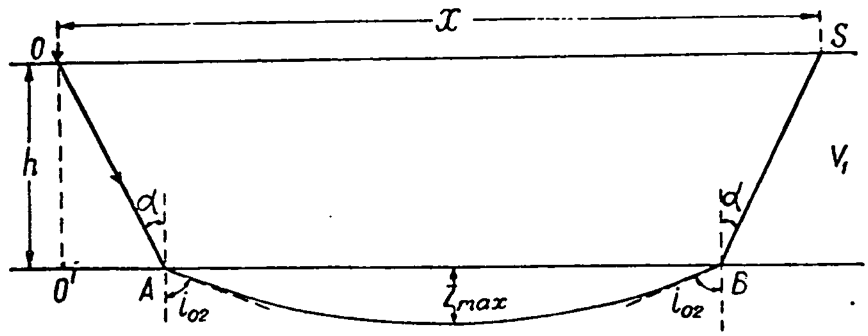

Fig. 1 FOR DEDUCTION OF THE EQUATION OF THE HODOGRAPH FOR A VELOCITY GRADIENT IN THE REFRACTING MEDIUM. CASE OF HORIZONTAL LAYERING.

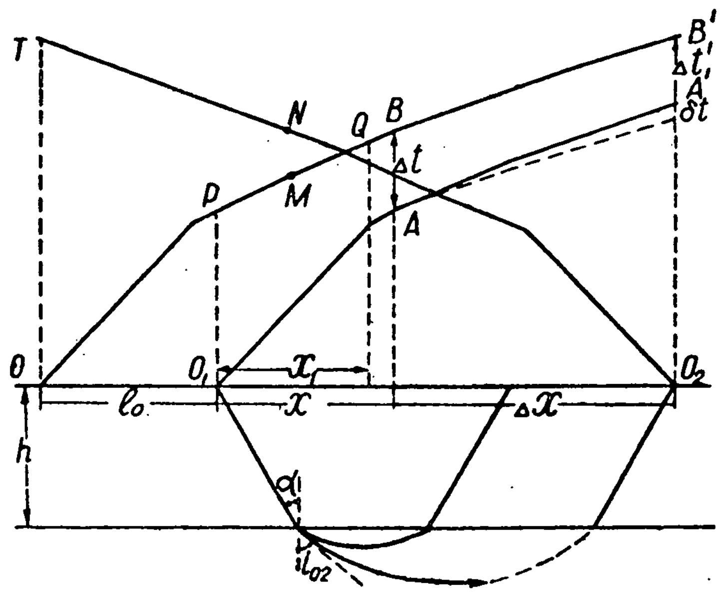

Fig. 2 REVERSE AND OVERLAPPING HODOGRAPHS OF HEAD WAVES WITH VARIABLE VELOCITY IN THE LOWER LAYER.

Hence it follows that at the initial point the apparent velocity is equal to the boundary velocity v_{or} at the top of the layer, but it will gradually increase with increasing distance from shot to seismograph.

The depth of maximal penetration of the ray into the refracting layer is determined by the equation

$$Z_{max} = \frac{1}{\gamma} \quad (\text{cosec } i_{02} - 1) = \frac{1}{\gamma} \quad (\sqrt{1 + \frac{1}{4}\gamma^2 (x-x_H)^2} \; - 1) \qquad (7)$$

From comparison of expressions (6) and (7), taking into account equation (1), it is possible to arrive at the known result that the apparent velocity of the hodograph in the case of horizontal layering of the boundary is equal to the true velocity of the medium at the depth of maximum penetration of the ray. This property remains valid for any horizontally layered medium.

The formula obtained can be generalized to the case of inclined layers, using the assumption that velocity v_{or} remains constant on the boundary and that the increase in velocity takes place in the direction of the normal to the boundary. The equation of the hodograph in parametric form can be written as follows:

$$x = x_H + \frac{1}{q\gamma} \; \frac{\sqrt{n_0^2 - \sin^2\alpha}}{\sin\alpha} \qquad (2'')$$

$$t = \frac{2h}{v_1 \cos i_{01}} - \frac{x \sin\phi}{v_1 \cos i_{01}} + \frac{2}{v_{or}\gamma} \text{arsinh} \frac{\sqrt{n_0^2 - \sin^2\alpha}}{\sin\alpha} \qquad (3'')$$

where $x_H = \dfrac{2h \sin i_{01}}{\cos(i_{01} \mp \phi)}$ is the abscissa of the initial point,

$q = \dfrac{\cos(i_{01} \mp \phi)}{\cos i_{01}}$ (upper sign corresponding to a rise).

After elimination of the parameter d and rearrangement analogous to that given above, for the case of a small gradient we obtain:

$$t = \frac{2h \cos i_{01}}{v_1} + \frac{x \sin(i_{01} \mp \phi)}{v_1} - \frac{\gamma^2 q^3 \; (x-x_H)^3}{24 v_{or}} \qquad (5'')$$

For inclined layering the hodograph becomes curvilinear with increasing apparent velocity with increasing distance between shotpoint and seismograph. This immediately follows from the formula for apparent velocity.

$$v_k \approx \frac{v_1}{\sin(i_{01} \mp \phi)} \left[1 + \frac{1}{8} \frac{\gamma^2 q^3 v_1}{v_{or} \sin(i_{01} \mp \phi)} (x - x_H)^2\right] \qquad (6'')$$

2. Determination of the gradient from overtaking hodographs

The magnitude of the gradient γ is as a rule not known beforehand, and therefore it is desirable to investigate the problem of finding the connection between γ and the degree of non-parallelism of overtaking hodographs.

Turning to fig. 2, where two overtaking hodographs are illustrated, with distance l_o between shot-points, and considering pairs of points AA' and BB' with difference in abscissae Δx, it is possible to write for the increment in time

$$\Delta t = t_B - t_A = \frac{l_o}{v_{or}} - \frac{\gamma^2}{24 v_{or}} \left[(x + l_o - x_H)^3 - (x - x_H)^3\right];$$

$$\Delta t' = t_{B'} - t_{A'} = \frac{l_o}{v_{or}} - \frac{\gamma^2}{24 v_{or}} \left[(x + l_o + \Delta x - x_H)^3 - (x + \Delta x - x_H)^3\right],$$

where x is the abscissa of the nearest of the points A and B to the shot-point.

The index of non-parallelism of the overtaking hodographs is determined from the relation

$$\theta = \frac{\delta t}{\Delta x} = \frac{\Delta t - \Delta t'}{\Delta x}$$

Using the above equations and making the necessary transformations, we obtain the following expression for the index of non-parallelism:

$$\theta = \frac{l_o \gamma^2}{8 v_{or}} [l_o + \Delta x + 2(x - x_H)] \qquad (8)$$

From here we get an approximate formula for determination of the gradient γ of increase of velocity with depth in the second layer in terms of the parameters of the hodograph

$$\gamma = 2\sqrt{\frac{2 v_{or} \theta}{l_o [l_o + \Delta x + 2(x - x_H)]}} \qquad (9)$$

An analogous formula for inclined layers has the form:

$$\gamma = 2\sqrt{\frac{2 v_{or} \theta q^3}{l_o [l_o + \Delta x + 2(x - x_H)]}} \qquad (10)$$

If the angles of inclination do not exceed say 3-5°, then it is possible to use the formula (9) for horizontal layering.

3. Determination of the boundary velocity from the reverse hodograph

It may be expected that the magnitude of the boundary velocity found by the usual methods will depend on the length of useable reverse hodographs. We suppose that boundary velocity is determined by the method of differential hodographs, and will assume at first for simplicity, that the boundary is horizontal.

For points M and N (see fig. 2) in accordance with formula (5) in the case of a relatively small gradient we may write for the difference in time on the reverse hodographs:

$$t_\Delta = t_M - t_N = \frac{2x - L}{v_{or}} - \frac{\gamma^2}{24 v_{or}} [(x - x_H)^3 - (L - x - x_H)^3], \qquad (11)$$

where L is the distance between the shot points.

Correspondingly, for the slope of the differential hodograph we may write:

$$\frac{dt_\Delta}{dx} = \frac{2}{v_{or}} - \frac{\gamma^2}{8 v_{or}} [(x - x_H)^2 + (L - x - x_H)^2] \qquad (12)$$

Calculations and graphical constructions show that changes of slope of the differential hodographs, within the limits of the region of intersecting branches of the hodographs, are generally insignificant, and it is quite acceptable to take for all intervals the boundary velocity calculated for $x = \frac{1}{2}L$. Then we can write

$$\frac{dt_\Delta}{dx} = \frac{2}{v_{or}}\left[1 - \frac{1}{8}\gamma^2 \left(\frac{L}{2} - x_H\right)^2\right] \qquad (12')$$

From this we obtain the expression for the boundary velocity:

$$v_r = 2\frac{dx}{dt_\Delta} = \frac{v_{or}}{1 - \frac{1}{8}\gamma^2\left(\frac{L}{2} - x_H\right)^2} \approx v_{or}\left[1 + \frac{1}{8}\gamma^2\left(\frac{L}{2} - x_H\right)^2\right] \qquad (13)$$

Consequently, the boundary velocity determined from the two reversed hodographs in the case under consideration of a gradient in the lower medium, will be larger than the boundary velocity at the top of the layer. Within our approximation it is equal to the true velocity at the deepest penetration of the ray. Therefore it is satisfactory to substitute $x = L/2$ in (7) and then to use formula (1).

In the general case of an inclined layer the equation for the boundary velocity will have this form:

$$v_r = \frac{v_{or}}{\frac{\sin(i_{01} \mp \phi)}{\sin i_{01}} - \frac{1}{8}\gamma^2 q^3 \left(\frac{L}{2} - x_H\right)^2} \qquad (13')$$

The corrected formulae may be used for calculation of v_{or} from a known value of v_r, since the gradient γ and other quantities may also be found from the observations.

4. Calculation of the depth

From the above exposition, it follows that if a velocity gradient in the refracting medium is not taken into account, then the calculated depths to the boundaries of the layers will be greater than the true depths. The magnitude of the discrepancy will depend on the length of the hodograph. Geometrically over-estimation of the depth follows, in particular, from the fact that the overtaking hodograph in the "blind" zone will have over-estimated apparent velocity, which also leads to increased time t_o at the shot-point.

Let us examine the problem of calculation of the true depth to the refracting boundary with allowance for the presence of a velocity gradient in the second medium, while at first, we select the simplest case of a sloping layer, when we can use equation (5'). We assume that the depths are determined by the method of t_o (arithmetic mean) from two reverse hodographs with shot-points at distance L from one to the other (see fig. 2). We select a convenient point S within the known interval in the region of intersecting hodographs. For the time $\tau = t_1 + t_2 - T$ we can write:

$$\tau = \frac{2h \cos i_{01}}{v_1} + \frac{\gamma^2}{24v_{or}} [(x-x_H)^3 + (L-x-x_H)^3 - (L-x_H)^3]$$

or after rearrangement of the square brackets and substitution of the value of x_H:

$$\tau = \frac{2h \cos i_{01}}{v_1} + \frac{\gamma^2}{24v_{or}}[3x(L-x)(L-4h \tan i_{01}) + 8h^3 \tan^3 i_{01}] \quad (14)$$

If the latter equation can be solved for h, then we obtain the desired formula for calculation of the depth to the refracting boundary. However a strict solution leads to cumbersome expressions, and considering the approximate character of this equation and the small magnitude of the second term in formula (14) in comparison with the first, it is expedient to make use of a simplified method, providing for a preliminary calculation of an approximate value of the depth h_o without taking into account the velocity gradient, according to the formula

$$h_o = v_1\tau / 2 \cos i_{01} \quad (15)$$

Using now the value of h_o for determination of the second term in (14), the depth h can be written in the following way:

$$h = h_o - \frac{\gamma^2}{48}[3x(L-x)(L-4h_o \tan i_{01}) + 8h_o^3 \tan^3 i_{01}] \tan i_{01} \quad (16)$$

The second term in this formula plays the role of a correction to the depth, calculated without taking into account the gradient of velocity in the refracting medium.

For determination of the depth at the shot points (x = 0 and x = L), the expression for h takes the form:

$$h = h_o (1 - \frac{1}{6}\gamma^2 h_o^2 \tan^4 i_{01}) \quad (16')$$

From this formula it is particularly easily seen that for a prescribed gradient the correction depends on the ratio of the velocities. The smaller the difference in the velocities of the upper and lower layers, the larger the correction. As is seen from (16), the correction depends also on the position of the points within the shot interval. It is easy to see that the first term in the square brackets takes its maximum value for $x = L/2$, i.e. the middle of the shot interval.

The determination of the depth in the general case of an inclined layer runs into computational difficulties. Moreover there appears a dependence on the angle of inclination which, although weak, is complex in form, and which necessitates application of successive approximations with preliminary solutions of the problem without taking into account the velocity gradient. Then it is possible to find approximate values of the depth h_o, and of the angle ϕ_o, which is used for finding corrections for the effect of the gradient. Without going into the calculations, based on the use of equation (5"), we obtain:

$$h = h_o + \frac{\gamma^2}{48}\Big\{[x \cos(i_{01} + \phi_o) - 2h_o \sin i_{01}]^3 + [(L-x) \cos(i_{01} + \phi_o) - 2h_o \sin i_{01}]^3 - [L \cos(i_{01} - \phi_o) - 2x \sin\phi_o \sin i_{01} - 2h_o \sin i_{01}]^3\Big\}\frac{\tan i_{01}}{\cos^3 i_{01}} \qquad (17)$$

where h_o is calculated from formula (15).

It is not difficult to ascertain that for $x = 0$ and $x = L$, i.e. for determination of the depth at the shot-points, we obtain formula (16'). This indirectly confirms the small dependence of the correction for the effect of the gradient of velocity on the angle of inclination to the horizontal. From this it can be surmised that in the overwhelming majority of cases it appears satisfactory to calculate the correction from formula (16), obtained with the supposition of a horizontal layer.

5. On the construction of composite hodographs

As is known in interpretation of data from refraction method, construction of composite hodographs is widely used. They are used frequently for finding the time t_o at the shotpoints. In the presence of a velocity gradient in the lower medium, the construction of composite hodographs is possible only after successive application of corrections for curvature of the ray, during which the magnitude of the latter can be evaluated only approximately because the exact location of the starting point of the head wave under consideration is unknown. It is possible to calculate very simply the correction ε_t for small angles of inclination by use of the formula

$$\varepsilon_t = \frac{\gamma^2}{24 v_{or}} (x - x_H)^3 \tag{18}$$

The value of x_H is estimated approximately for successive points of the section.

After revision of the hodographs the composite hodographs are constructed by the usual methods, and subsequent treatment (calculation of velocities and depths of the layer) is performed without supplementary calculations in the presence of a gradient. Frequently, with revised hodographs it is quite possible to use the time-field method in the usual modification.

It should be noted that in correction by the above formula (16') the error in the depth at the shot point is calculated under the assumption that the overtaking travel-time curve is displaced in relation to the basic one by the quantity $l_o = x_H$. In fact l_o can take any value. Calculation of the true depths to the refracting horizon in this case is much easier to carry out for the adjusted hodographs. It is also possible to use the formula for correction of t_o, the derivation of which is not presented here:

$$\Delta t_{o2} = \frac{x_1 \gamma^2}{8 v_{or}} \left(l_o + \frac{x_1}{2} - x_H\right)^2, \tag{19}$$

where x_1 is the interval of transposition of the overtaking hodograph (see fig. 2). The correction Δt_{02} must always be subtracted from the value of t_{02} obtained from the unadjusted travel-time curves.

From the foregoing it follows that, with an adequately dense system of observations, with reverse and overtaking hodographs, the distorting influence of a positive velocity gradient in the refracting medium can be sufficiently accurately predicted and eliminated, without drawing on supplementary data about the section. Reliable identification of the waves under consideration in the region investigated gives sufficient conditions for this.

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N.N. Puzyrev

ON THE QUESTION OF USING SIMPLIFIED SCHEMES OF OBSERVATION WHILE STUDYING THE FOLDED BASEMENT OF THE WEST SIBERIAN LOWLAND BY THE METHOD OF REFRACTED WAVES

Investigation of the surface of the folded basement of the West Siberian lowland has particular significance for the proper understanding of the genesis of the structural forms of the sedimentary cover and classification of the prospects of the particular region. Recently available maps of the surface of the basement, constructed on the basis of data of magnetic measurements, do not lay claim to sufficient accuracy for prospecting purposes, even for investigation of the lowland only in a regional manner. Thus the necessity arises of drawing on other geophysical methods of prospecting, capable of guaranteeing the accuracy required in practice. This problem may be solved most effectively, as shown by the work of many years of investigations, by application of the seismic method of prospecting. However, as is well known, seismic prospecting in its commonly used variations, providing continuous correlation of the waves, is an expensive method, and for investigations of a regional character, owing to the bulk of technical equipment, in practice is applied only under conditions of adequate accessibility for transport. In this connection for the region of West Siberia the problem was posed long ago of all possible simplification of the scheme of observations having regard to the peculiarities of the seismo-geological structure of the province. Recently an extensive development was achieved with operations by the method of reflected waves MRW using the technique of "point" soundings, which proved quite effective for the investigation of sedimentary cover for a series of regions and in the primary exploration of its base layer in the regional layout, and also for prospecting of structures of the second and even third order. However these reflected waves can provide only indirect evidence about the layering of the basement of the lowlands (the surface of the first and structural stages). Furthermore, it is impossible to exclude erroneous interpretation by MRW completely, for example, in relation to the recording of multiple waves. It must be noted, moreover, that in a number of regions the use of the point sounding variant of MRW is not feasible under unfavourable conditions of excitation of vibrations. Therefore it is necessary, simultaneously with improvement of the method of reflected waves, to develop methods of simplification of procedures in operations by the method of refracted waves for direct study of the basement of the lowland. Moreover we must note that the proposal for use of a simplified scheme of observations by the method of refracted waves for investigation of the basement of the West Siberian Lowland has been already put forward, in particular by the Tyumensk Geological Institute (V.K. Monastyrev, Yu. N. Grachev). In addition, a differential method of determination of the depth and boundary velocity was tested for a single linear in - line array of seismographs. More recently this method was widely tried in simple surface conditions (for example, Priaral) and gave quite effective geological results for a sufficiently extended array of seismographs.

The differential method referred to has, however, the principal disadvantage, that for calculation of the depth extrapolation of a section of the hodograph is required over a large distance, so that gross errors may be involved, particularly where there are surface inhomogeneities. Moreover, there is practically nowhere in the taiga or in the swampy regions that a long enough array of seismographs can be set out in a given direction.

In a recent paper certain simplified schemes of observations in the method of refracted waves were discussed, requiring measurement of the time of arrival of the waves only at isolated points of the investigated area. We note that application of the proposed simplified schemes assumes the possibility of discrete correlation of the waves according to dynamic and kinematic features, which we will not dwell on here. We merely point out that, if special cases related to peculiarities of formation of individual local structures are excluded, then we may consider the problem of identification of head waves corresponding to the basement as fully soluble.

Let us write the equation of the hodograph of refracted waves for a flat inclined boundary in an alternative form, introducing the depth along the normal h_m at the central point between shot-point and seismograph.

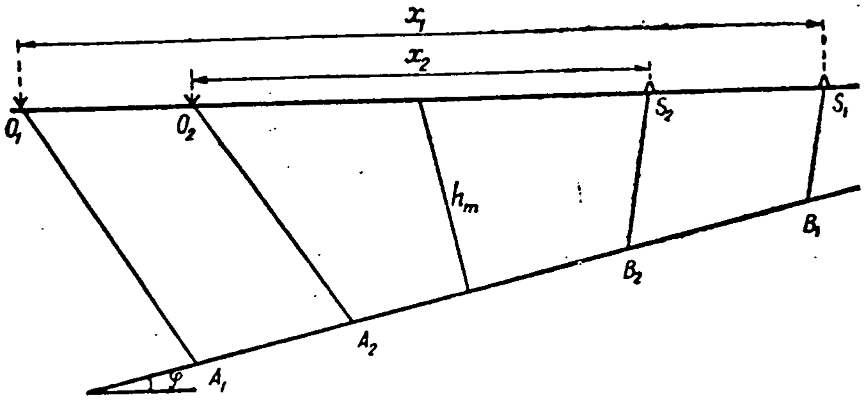
In the direction of the perpendicular

$$h_m = h - \frac{x}{2} \sin \phi$$

where h = depth at the shot point,

x = distance shot point to seismograph

ϕ = angle of inclination of the boundary.



SCHEME OF SYMMETRICAL SOUNDING

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We obtain the equation of the hodograph eventually in the form:

$$t = \frac{2h_m \cos i}{v_1} + \frac{x}{v_r} \cos \phi$$

where v_1 = mean velocity in the first medium

v_r = boundary velocity in the lower medium

$$\sin i = v_1/v_r$$

Let us now consider a symmetrical scheme with two shot points and two receiving points (see diagram). In accordance with equation (1) we may write:

$$t_1 = \frac{2h_m \cos i}{v_1} + \frac{x_1 \cos \phi}{v_r}$$

$$t_2 = \frac{2h_m \cos i}{v_1} + \frac{x_2}{v_r} \cos \phi$$

Solving these two equations for h_m and v_r, we obtain

$$h_m = \frac{v_1 (t_1 x_2 - t_2 x_1)}{2\sqrt{(x_2 - x_1)^2 - v_1^2 (t_2 - t_1)^2 \sec^2 \phi}}$$

$$v_r = \frac{x_2 - x_1}{t_2 - t_1} \cos \phi$$

In the above $v_1 = v_1(h_m)$.

For angles of inclination up to 10°, generally observed within the West Siberian Lowland, the dependence of both quantities on the angle of inclination may be disregarded, and the formulae will have the simpler form:

$$h_m = \frac{v_1(t_1x_2 - t_2x_1)}{2\sqrt{(x_2 - x_1)^2 - v_1^2(t_2 - t_1)^2}}$$

$$v_r = \frac{x_2 - x_1}{t_2 - t_1} \qquad (3)$$

Thus, the symmetrical scheme of observations considered permits, where the boundary has a relatively simple form, determination of the depth of the refracting boundary at the mean point of the baseline and calculation of the boundary velocity. If the boundary velocity varies within small limits, then it is quite satisfactory to take it as constant for the series of observations. In this case the depth h_m can be determined as a check from the formula:

$$h_m = \frac{v_1(t - \frac{x}{v_r}) \cos \emptyset}{2 \cos i} \approx \frac{v_1(t - \frac{x}{v_r})}{2 \cos i} \qquad (4)$$

If the contrast in the boundary velocity is weak, it is convenient to consider two schemes of observation - a fully symmetrical scheme, where the magnitudes of h_m and v_r are found independently, and a simplified scheme with one shot point and one receiving point (simple sounding), providing for determination of the depth only, bearing in mind that the boundary velocity is known. Observations with the full scheme are carried out for very sparse nets and have almost wholly as objective the control of the magnitude of the boundary velocity, knowledge of which helps to construct the corresponding map. Subsequently the value v_r must be extracted from this map during the final treatment of the simple soundings, which play the role of points filling in the network.

In fact schemes carried out in the field can be quite varied in accordance with local peculiarities, and with a number of simultaneously operating sets of apparatus and shot points. However any of them can always be represented as combinations of the above two schemes (symmetrical and simple soundings).

We note that the simplification of the schemes of observations in comparison with the schemes generally used, requiring continuous tracking of the waves in one region or another, is achieved in the given cases through rejection of direct straightforward determination of the parameters of the boundary characterizing its shape, including the angle of inclination. The shape of the boundary, resulting from available knowledge of the geological region, was formerly taken to be relatively simple, when during determination of average values of the parameters (h_m and v_r) at the points under consideration, it is possible to ignore it owing to its small effect. If necessary the shape of the boundary (angle of inclination and curvature) can be subsequently taken into account at a combination of the given series of points, distributed over a specified area, through introduction of a special correction. The corrections for the angle of inclination may be easily found from the formulae presented above, if we substitute in them the true angle of inclination ϕ for ϕ_o, found as a result of the construction using the approximate values of the depths. The corrections for the curvature of the boundary can be disregarded in the overwhelming majority of cases. If necessary they can always be found more easily by graphical solution of the direct problem.

An important problem in the method of observations is the choice of the length of the lay-out for observations at one station. The schemes considered above provide calculation of average parameters at separate isolated points. However in practice, obviously, at a proportion of the points it will be convenient to use the criterion of apparent velocity for more definite identification of the waves. With a set of portable multi-channel stations, this need not give rise to any particular difficulties, if a limited number of groups of geophones is used.

In a new region, where the wave map has not been studied in sufficient detail, it is advisable to set out at first a few short profiles with continuous correlation of the waves from two or several shot-points. This will allow a better basis for selection of the optimal parameters for subsequent surroundings, a study of the kinematic and dynamic features of the separate types of waves, and establishment of a firm basis for subsequent correlation of the results of the sounding operations.

Together with recording of waves in a neighbourhood of the first arrivals with soundings it is useful to look at the later part of the seismogram. There are several reasons for this, such as: (a) the presence of recordings of very intensive head or refracted waves (so-called "direct waves") permits one to be satisfied that waves from the basement are recorded in the region of the first arrivals; (b) knowing previous hodographs of "direct waves" (for example from the data of a reference profile), which change little for separate regions of the lowland in Western Siberia, it is possible to check the reliability of the distance from shot-point to seismograph taken from the map and the identification of the shot time signal; (c) in the later arrivals may be recorded various types of converted and multiple waves, which give additional information on the disposition of the basement. Thus, for example, registration of PPS type waves enables calculation of the depth under the receiving point, and registration of multiply reflected - refracted waves gives the depth under the shot-point.

If one speaks of the use only of longitudinal head waves in the neighbourhood of the first arrivals, then for ensuring the highest accuracy in calculation of the depth with a known boundary velocity, the smallest possible distance from shot-point to seismograph must be chosen. Besides identification of the waves, the length of the baseline $x_1 - x_2$ on which the waves are recorded using the symmetrical arrangement will affect the accuracy of determination of the boundary velocity. The accuracy of determinations of the depth and the boundary velocity using the approximate calculation will be deduced below.

We suppose as usual that distances in the region can be measured sufficiently accurately from a map. The error Δv_r is related to the error of measurement of the time difference $(t_2 - t_1)$ by the formula:

$$\Delta v_r = \frac{v_r^2}{x_2 - x_1} \Delta(t_2 - t_1) \qquad (5)$$

From this it follows that with $v_r \approx 5.0$ km/s and $\Delta(t_2 - t_1) \approx 0.015$s, a relative error $\Delta v_r/v_r = 2\%$ (100 m/s) can be guaranteed on a baseline with $x_2 - x_1 \approx 3.8$ km, i.e. with $0_1 0_2 = S_1 S_2 \approx 2$ km (see fig. 1).

Let us estimate the errors in depth for $\Delta v_r/v_r = 2\%$ and $\Delta t \approx 0.01$ s. From equation (4) can be obtained:

$$\Delta h_m = \frac{v_1}{2 \cos i} \sqrt{\Delta t^2 + \left(\frac{x - v_1 t \sin i}{v_r^2 \cos^2 i} \right)^2 \Delta v_r^2} \qquad (6)$$

Substituting the values $v_1 = 2.2$ km/s, x = 8 km, t = 3.2 s (with $h \approx 2$ km), we obtain:

$$\Delta h_m \approx 30 \text{ m}$$

Even if the above values of the errors Δt and Δv_r were doubled the error in depth for a single measurement will be about 60 m, and the limiting error will rarely exceed 100m. With bulk determinations of depth, the error will diminish corresponding to the density of observations and the presence of supporting points.

One of the difficulties with the basement of the West Siberian lowland investigated by the method of refracted waves, as experience shows, is the interchange of waves, due apparently to the presence of boundaries within the basement. Inasmuch as differentiation of media in the basement is small, extended zones of interference occur as a result, and for continuous tracking of the boundaries within the basement hodographs of great length and complex schemes of observations are required. Usually such schemes are difficult to realize in practice, and significant difficulties arise in their interpretation.

With the use of soundings those difficulties, of course, will not disappear, but there is reason to believe that they will not be aggravated, but possibly will occur even in lesser degree than with continuous correlation. This follows because the initial part of the hodograph will be used preferentially with soundings, as this consists of waves refracted through the basement with a minimal degree of distortion caused by penetration of the rays into the thickness of the lower medium. Moreover, if the soundings are conducted with more or less constant baselines, then the errors of the type discussed will have a systematic character to a large extent and cannot be attributed to the shape of the basement surface. Evidently, with more attention to a study of the nature of the waves arising from the upper part of the basement, persistent dynamic characteristics which help to recognize the true identity of the waves, will be successfully found.

In conclusion we remark that the method described of use of a simplified scheme was tested on the available data of the survey KMPV and showed fully satisfactory results.

Recently field operations have begun with the proposed method in separate regions of the West Siberian lowland.

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SEISMIC SOUNDINGS BY REFRACTED WAVES FOR REGIONAL GEOLOGICAL INVESTIGATIONS

Abstract

Principles of the theory of seismic soundings by the method of refracted waves are set forth. The methods, techniques and some geological results are described of regional investigations of the folded basement of the West Siberian lowland.

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The clarification of the general features of geological structure of a broad region, including the deep layers of the Earth's crust, forms an important stage in the search for useful mineral occurrences. A major role in the solution of this problem is played by the seismic method of refracted waves, inasmuch as it enables highly accurate investigations of such geologically important horizons as the surface of the basement in a platform region, boundaries within the sedimentary section, and within the crystalline crust of the Earth.

For regional operations it is required to study within a relatively short period extensive territories, often poorly assimilated and difficult of access. Seismic refraction methods used until recently are cumbersome, investigations proceed slowly, the cost is high, and the geological results are inadequate. Effective methods for regional projects must use simplified and economical schemes of observations. It is possible to simplify field observations by confining the study to the principal distinguishing features of the deep geology.

The first theoretical investigations of simplified schemes of observations were carried out by G.A. Gamburtsev [3], showing the possibility of determination of the depth of layering of a refracting boundary from a single hodograph with unknown boundary velocity.

The work of Yu. V. Riznichenko [10] is very significant; he solved the problem of interpretation of a single hodograph of a head wave with very broad assumptions about the form of the boundary (method of the lines "S"). M.M. Radzhabov [9] was also engaged in investigation along these lines.

Interesting work on simplified schemes of observations was carried out by Yu. N. Grachev [5] and V.K. Monastyrev. Reports on effective application of simplified methods are also available abroad [1, 11].

These investigations enabled essential improvement in the effectiveness of regional investigations; however they proved to be inadequate particularly in regions with complex surface conditions. Simplified schemes of observations were often used instead of full correlation systems necessary for continuous tracing of refraction boundaries, divided hodographs or pairs of hodographs, spaced at predetermined distances. The methods of isolated soundings had not an adequate theoretical foundation for investigation of one or several boundaries.

Below are set out briefly the principles of the theory of isolated and linear soundings of refracted waves, developed in the Institute of Geology and Geophysics, Siberian Division of the Academy of Sciences, USSR, in 1959-62. The feasibility of the proposed method and the features of field observations were demonstrated in the first investigations* of the surface of the folded basement of various regions of the West Siberian lowland.

Principles of the theory of soundings

A necessary condition for the application of the method of seismic sounding is the possibility of discrete correlated waves, i.e. their identification in separated regions. Identification of the waves is carried out through comparison of separate disconnected seismograms as a whole with use of secondary features of correlations, arising from features of the wave map in the given region. Such features consist of: correspondence of apparent velocities and amplitudes of the separate waves, their relative location on the seismogram for specific distances from the shot-point, frequency characteristics, and wave shape.

The method of observations is based on a combination of schemes with different degree of detail - a sparse distribution of parametric soundings and a filling network of point soundings.

* Investigations were carried out in collaboration with industrial organizations (Red Geological Administration and Novosibirsk Geophysical Trust)

Parametric soundings present their own non-extended profiles, on which are investigated in detail the wave map, definition of criteria for discrete correlation of waves, the character of non-uniformity of the overlying and refracting media, and solution of other problems. The parametric soundings form a supporting net, enabling more valid solutions of the point soundings along traverses or over areas.

A single point sounding forms the elementary scheme of observations: the time t is recorded at a single point at a distance 1 from the source of vibrations. In the case of a two-layer section with flat inclined boundaries separating two homogeneous media with velocities of transmission of elastic waves v_1 and v_r the following equation can be proved:

$$h_x = \frac{1}{2 \cos i} \left[\frac{v_1 t}{l} + \left(1 - 2\frac{x}{l}\right) \sin \phi \cos i - \sin i \cos \phi \right] \quad (1)$$

where i = critical angle, ϕ = inclination of the boundary, h_x = depth along the normal of an arbitrary point in the interval $0 \leq x \leq l$. Obviously, the least dependence of the depth on the inclination of the boundary occurs for $x = \frac{l}{2}$ when the second term in the square brackets is nil:

$$h_m = \frac{v_1}{2 \cos i} \left(t - \frac{l}{v_r} \cos \phi\right) \quad (2)$$

For small angles of inclination ($\phi < 10 - 15°$) its influence can be neglected. Consequently, with known boundary velocity the depth h_m in the centre of the interval can be determined from a single point sounding, practically without knowing the angle of inclination of the boundary.

If the boundary velocity is chosen incorrectly, an error will be introduced:

$$h_m = \frac{v_1 (v_r l \cos \phi - v_1^2 t)}{2 v_r^3 \cos^3 i} \cdot \Delta v_r$$

From this it follows that, for a given error in the boundary velocity (Δv_r), the error in the depth will be greater when the baseline 1 is greater. If the receiver is close to the critical point of the refracted wave, an erroneous choice of the boundary velocity has practically no effect on the magnitude of h_m.

If the boundary velocity was obtained previously (for example, as a result of parametric soundings), and the velocity in the first medium is known, then the relief of the boundary can be investigated by means of a net of point soundings with any (including common) baselines. For determination of the density of the network of observations we must take into account that there are certain limits to the resolving power of the method, since within the limits of each sounding the conditions are averaged out. Therefore it is not recommended that the distance between the centres of adjacent soundings should be less than $\frac{1}{2}l - h_m \tan i$.

Simultaneous determination of the boundary velocities and depths can be accomplished through use of a system of point soundings with substantially different baselines.

A strict solution of the problem for a system of two point soundings (l_1, t_1 and l_2, t_2) is possible when their centres coincide (symmetrical sounding [8]). In this case:

$$v_r = \frac{l_2 - l_1}{t_2 - t_1} \cos \emptyset \approx \frac{l_2 - l_1}{t_2 - t_1} \qquad (4)$$

For systems of three point soundings with centres distributed along a straight line at distances ΔS_{12} and ΔS_{23} respectively, we have:

$$v_r = \frac{\Delta S_{23}(l_2 - l_1) - \Delta S_{12}(l_3 - l_2)}{\Delta S_{23}(t_2 - t_1) - \Delta S_{12}(t_3 - t_2)} \qquad (5)$$

The depths in this case, as previously, are found from formula (2). A frequent case of basic interest consists of three soundings with reciprocal points (fig. 1a), when the following formulae are valid:

$$v_r = \frac{2l_3}{(1 + l_3/l_1)\, t_2 - (t_1 + (l_3/l_1)t_3)}$$

$$h_s \approx \frac{v_1(t_1 + t_3 - t_2)}{2 \cos i} \qquad (6)$$

The depth at the point S is nearly independent of the error in the boundary velocity; moreover the effects of averaging of conditions along the baseline and curvature of the boundary are much smaller. However, with large depths l_2 becomes large enough that recording the waves becomes difficult and the reliability of the results is reduced because of the influence of penetration, the possible transmission of other waves, and other factors. This defect can be overcome by means of certain complications of the scheme of observations [V.K. Monastyrev].

We now proceed to the general case of a system with an arbitrary number of points of soundings with more or less regularly alternating large and small base-lines. These soundings, distributed along a traverse, may be oriented in various directions if the media, particularly the lower medium, do not have noticeably anisotropic velocities. The distribution of depths and boundary velocities can be found in this case by the methods of the lines v_r = const or l = const.

The method of the isolines v_r is based on the fact that the same departure of the boundary velocity from the true velocity leads to different determinations of the depths for the soundings from different baselines (3): the longer the baseline, the bigger the increment in the depth. For a series of soundings with essentially contrasting baselines, formula (2) is used and a cross-section is constructed for a given v (fig. 2). The time disposition of the boundary and the value of the boundary velocity v for the profile is determined from the isolines with the least amplitude of oscillation in depths. A vertical contact of two media is indicated by the fact that on opposite sides of it, different isolines v_r have the minimal amplitude of oscillations. The requirement of minimal amplitude of oscillations of the line v_r = const follows from the nature of the geological boundaries, the curvature of which is generally not large.

In the method of the lines l = const the arrival time of the waves is referred to the centre of the soundings and a series of lines $t_i(x)$ are constructed in the coordinate system x, t, (x = distance along the profile) by interpolation, for fixed baselines l_i (fig. 3). The time increment between two arbitrary points of the profile may be considered as a time difference corresponding to a system of two symmetrically distributed soundings with baselines equal to the parameters of the isolines. Consequently the formula presented above for a symmetric sounding may be used for finding the boundary velocities and depths. Further, the presence of the time field t(x, l) permits in principle the construction of a system of overtaking, reverse, and consequently a composite hodograph (see fig. 3) and interpretation of them through the known methods [2].

For areal investigations a system of point soundings in the form of a "cluster" is convenient - the receivers of the vibrations being distributed around a fixed shot-point at alternately large and small distances (see fig. 1b). It is obvious that the scheme is not altered if one takes a

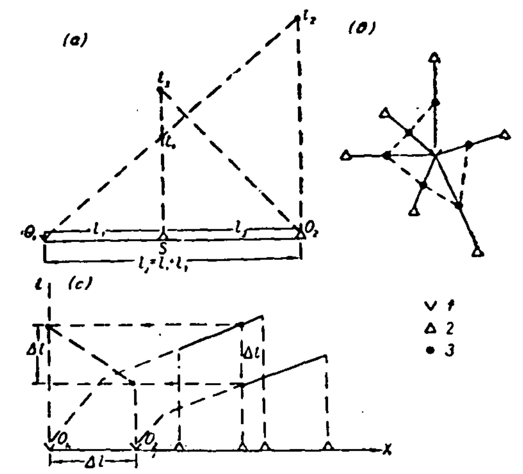

Fig. 1 SYSTEMS OF SOUNDINGS

(a) THREE POINT SOUNDINGS WITH MUTUAL POINT.

(b)"CLUSTER" SOUNDING

(c) TWO LINEAR SIMILARLY ORIENTED SOUNDINGS WITH COMMON POINT

1 SHOT-POINTS, 3 CENTRE OF SOUNDINGS

2 RECEIVING POINTS

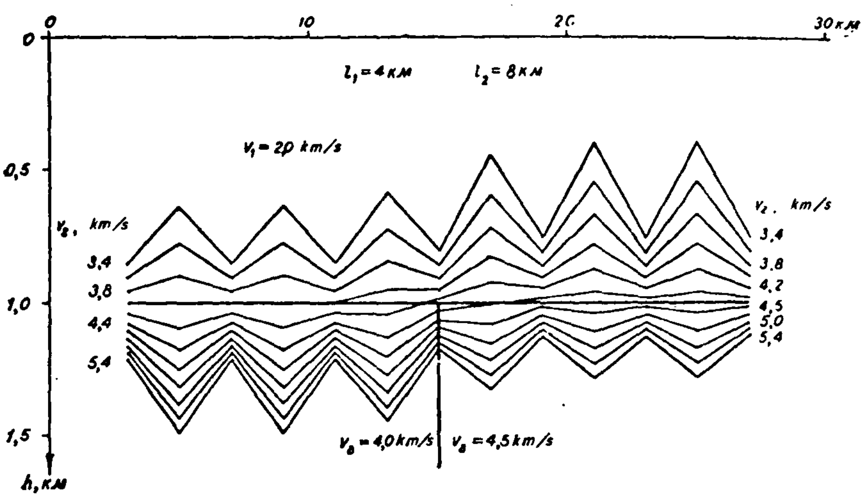

Fig. 2 THE BASIS OF THE METHOD WITH LINES v_r = const

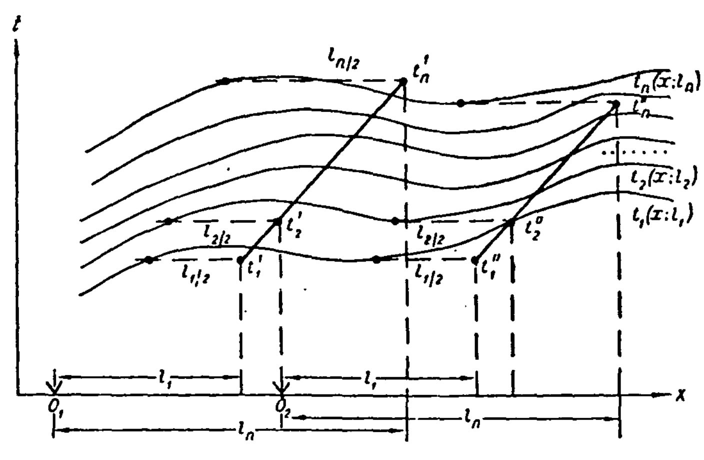

Fig. 3 THE BASIS OF THE METHOD WITH LINES l = const

receiver as fixed, and distributes the shot-points around the periphery. By unfolding the line of reading of the depths at the mid points of the unit soundings into a straight line, the methods of isolines v_r or l can be applied. After determination of the depths and boundary velocities, they are then transferred onto a plan.

In readily accessible areas with gentle relief of the earth's surface, linear soundings may prove to be efficient, when the vibrations are recorded on an array whose length (l) is adequate for reliable determination of the apparent velocity (v_k). Along a single linear sounding with known boundary velocity, a section of the boundary with length ½ l may be constructed, using formula (2).

A system of two linear soundings with mutually opposite directions of shot-point-receiver permits, in the case of a flat boundary, finding both the parameters of the layering (h_m, ϕ) and the boundary velocity. According to [10],

$$\phi = \tfrac{1}{2}[\arcsin(v_1/v_{k1}) + \arcsin(v_1/v_{k2})] \quad (7)$$

$$\frac{1}{v_r} = \tfrac{1}{2}\left(\frac{1}{v_{k1}} + \frac{1}{v_{k2}}\right) \frac{1}{\cos\phi} \quad (8)$$

With combined arrays, v_r is found by the method of the differential hodograph.

For a system of two linear soundings with the same direction from shot to receiver, a full solution of the problem is possible when the arrays of the soundings have even only one common point (see fig. 1c). Then it is possible to calculate the apparent velocity between the sources of vibrations and their arrival the same as in the case of the system with reversed orientations.

The methods of isolines v_r and l are fully applicable for the interpretation of data of an arbitrary number of linear soundings. Further the construction of composite hodographs is possible, with their subsequent treatment by the usual methods.

In the foregoing it has been assumed throughout that within the limits of a sounding the refracting surface is flat, and the upper and lower media are homogeneous. In actual conditions these suppositions are not always fulfilled sufficiently strictly and then corresponding corrections are introduced into the results obtained.

Calculation of the influence of curvature of the refracting boundary is carried out by means of successive approximations along a series of observations. Two methods of such a calculation have been developed. In the first preliminary values of the boundary velocity and depths are improved by using the angles of inclination of the boundary found from a preliminary construction of the section. The second more general method is based on successive solution of the inverse and direct problems, in the process of which are introduced adjustments to the observed times of arrival of the waves.

It can be shown that, in the presence of horizontal gradients of velocity in the overlying and refracting media, it is enough for calculation of the depths to know the values of the velocities in the centre of the baseline of the soundings. The influence of a vertical gradient of velocity in the refracting medium and the related penetration of the seismic rays into the lower thickness yields to calculation, since the time of passage of the waves penetrating into the lower medium can be represented with sufficient accuracy as the difference in time for glancing waves, and a certain correction depending on the magnitude of the velocity gradient [7]. It is necessary in this case to find the vertical gradient as a result of special investigations by parametric soundings.

The theory of soundings for a survey of a single boundary has been considered above. If several marker refracting horizons are present in a geological section, the soundings can be used for investigation of the multi-layered medium. In the most favourable circumstances, when all the basic waves are recorded simultaneously on one seismogram, the problem is solved by use of point or linear soundings. In this case, for more reliable separation of the waves in the later arrivals, the result of reproducible recording is presented in perspective, with subsequent selection of the waves according to frequency characteristics and their angle of travel to the surface.

The horizons are constructed by the usual methods in order of increasing depth of the layers. With significant velocity differentiation of the section and abrupt structural relief, the influence of the intermediate boundary is calculated [4].

In the general case the regions of recording of the various waves may not coincide, and then it is possible to consider a full study of the section with use of the parametric type of soundings. If it were known previously that the medium is horizontally layered, then in favourable circumstances one hodograph of the required length would be sufficient. When the layers are inclined then quite a complex system of reverse and overtaking hodographs is required for the usual method. However it may be shown that even in this case it is enough to be restricted to recording of one hodograph. The most rational scheme of point soundings is shown in Fig. 4. The distance shot-seismograph is successively increased keeping the centre of the sounding fixed. For multilayered media with arbitrary inclinations not greater than 15°, the hodographs (or rather the graphs of t(x)) of the separate waves practically coincide with their hodographs for horizontally layered media. The slope determines the value of velocity in the coresponding layer, and the magnitude of the intercept t_o, intersecting the time axis, determines the depth to the layer under the centre of the sounding.

For evaluation of the accuracy of the results of the soundings, an analysis was carried out of the various types of random and systematic errors. Below are presented formulae for errors in determination of depths (m_{hm}) depending on errors in measurement of time (m_t), distances (m_l), and inaccuracies in the assumed value of the boundary (m_{vr}) and medium (m_{v1}) velocities:

$$\left.\begin{aligned}
(m_{hm})_t &= \tfrac{1}{2}(v_1/\cos i)m_t \\
(m_{hm})_l &= -\tfrac{1}{2}\tan i \; m_l \\
(m_{hm})_{vr} &= \tan i \left(\tfrac{1}{2} l \cos \phi - h_m \tan i\right) m_{vr}/v_r \\
(m_{hm})_{v1} &= \frac{h_m}{v_1 \cos^2 i} \; m_{v1}
\end{aligned}\right\} \qquad (9)$$

From these relations it follows that the accuracy of determination of the depths decreases with decrease in the jump in velocity at the boundary being mapped (increased critical angle i) and with increases in depth of its layer. The latter becomes evident if we take into account that it is necessary to increase the baseline (l) of the soundings for a deep-seated buried boundary.

Expressions for the errors of determination of the boundary velocities are given only for the important case of a system of two symmetrically distributed point soundings:

$$(m_{vr})_t = \frac{\sqrt{2}\, v_r^2}{l_2 - l_1} m_t$$

$$(m_{vr})_\ell = \frac{\sqrt{2}\, v_r}{l_2 - l_1} m_l \qquad (10)$$

The accuracy of determination of v_r depends on the difference of the baselines of the soundings. It is necessary to take this into account for selection of the parameters of a scheme of observations.

The authenticity of the calculation of the influence of curvature of the boundary and inhomogeneity of the media by the methods described above, was tested on theoretical examples and was shown to be almost completely satisfactory. Thus, for the conditions of the survey of the basement of the West Siberian lowland, the departures of the results obtained from reality do not exceed 50 m in depths and 50m/s in boundary velocity.

On the whole the accuracy of the methods presented is shown to be adequate for solution of the regional problem. For example for investigation of the basement of the West Siberian lowland, it is possible to rely on determination of depths with an average error of about 3-4%, and boundary velocities 3-5% from their full magnitudes.

Testing the methods of soundings

The proposed methods of seismic soundings were applied to a large extent from 1960 to the investigation of the surface of the folded basement of various regions of the West Siberian lowland in relation to problems of oil and gas accumulation.

The folded basement is represented by strongly dislocated Palaeozoic sediments, fractured intrusives of a variety of compositions, and is ruptured by faults. Deep downwarps in it are filled with extrusive - sedimentary rocks, less dislocated and weakly metamorphosed. The surface of the basement is a marker refracting horizon with boundary velocity of transmission of elastic waves 5-6 km/s. Changes in the boundary velocity are related to changes in the material composition and the degree of metamorphism of the rocks. In many cases a continuous increase of velocity with depth is noted. The presence of zones of weathering in the surface of the basement, steeply dipping contacts and boundaries within it, commonly lead to complexity in the wave map. The sedimentary section has thickness up to 4km, and more folded porous sandy-clayey formations. The average velocity of transmission of elastic vibrations is 2-3km/s

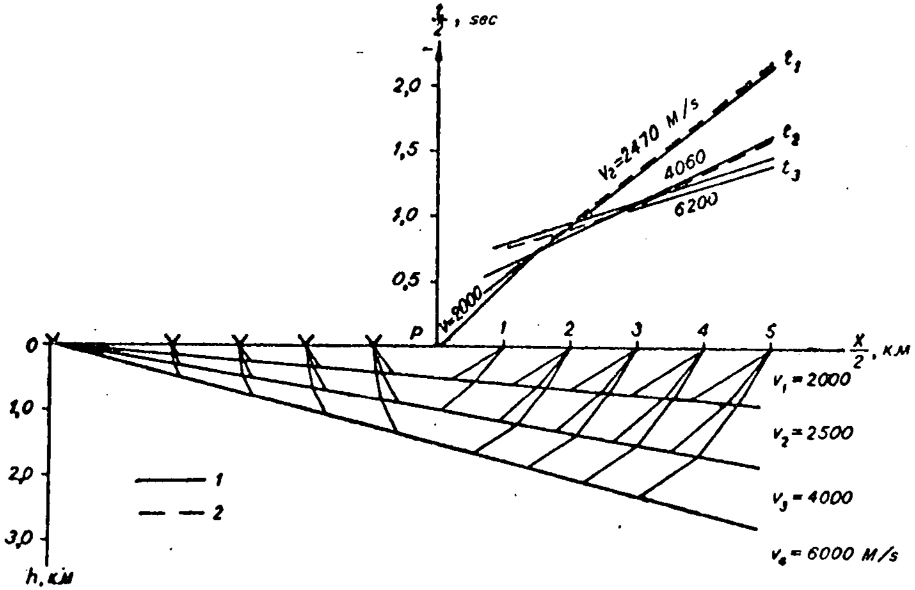

Fig. 4 THEORETICAL EXAMPLE OF SYMMETRICAL SOUNDING FOR A FOUR-LAYER SECTION

1 HODOGRAPHS FOR HORIZONTALLY LAYERED MEDIA

2 "OBSERVED" HODOGRAPHS

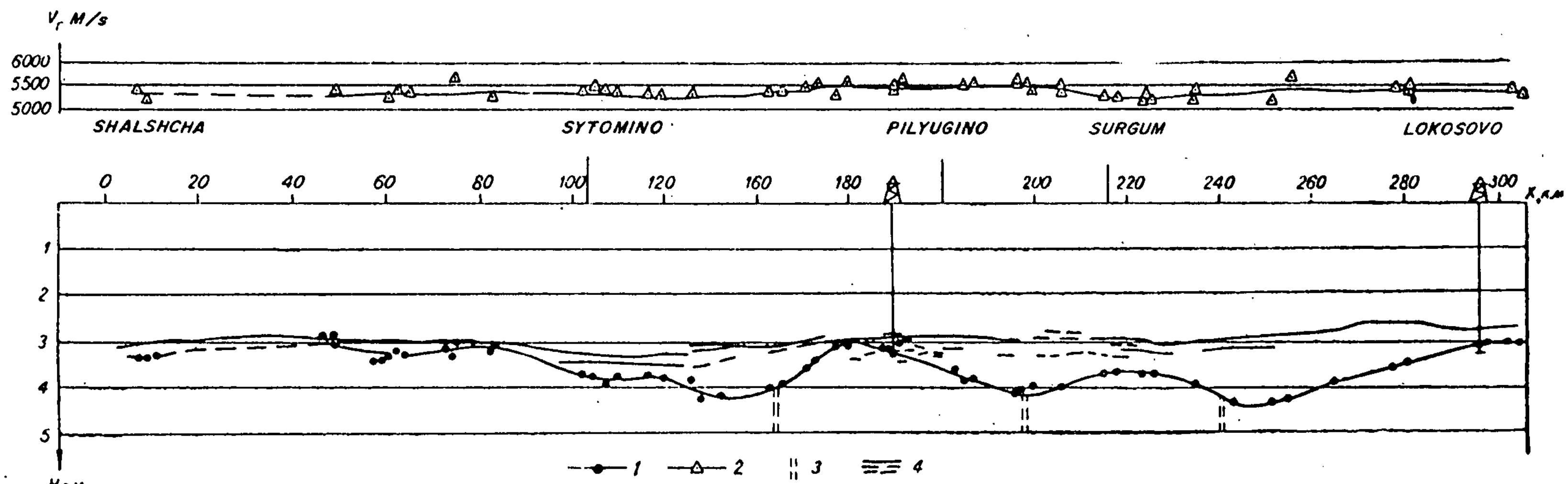

Fig. 5 SEISMIC SECTION ALONG THE LATITUDINAL ELBOW OF THE RIVER OB ACCORDING TO THE DATA OF POINT SOUNDINGS

1 BASEMENT SURFACE

2 GRAPH OF THE BOUNDARY VELOCITY

3 PROBABLE FRACTURES IN THE BASEMENT

4 REFLECTING HORIZONS WITHIN THE SEDIMENTARY SECTION.

and varies smoothly on the regional scale. Certain refracting horizons with boundary velocities from 2.2 to 4.5 km/s are found in the sedimentary section.

A large part of the territory of the West Siberian lowland is characterized by difficult surface conditions, owing to swamp and forest areas.

Many forms of observations were tested under a variety of surface and depth conditions with the use of various technical resources.

Areal investigations were conducted in the north-eastern lowland, and more recently in the northern part of the Tomsk region. Territories totalling around 30 000 sq km were surveyed. For operations in the Turukhan-Eniseisk interfluvial region, observations were carried out by a series of "cluster" soundings. Shot-points and receivers were distributed on the frozen surface of the lakes.

Observations along regional traverses were carried out in riverine and terrestrial variants. Over 1000 km of profile were carried out along the Ob (in its latitudinal elbow) and Pyarsin (in the north-eastern lowlands) Rivers, and also along two extended traverses using dirt roads (in the region of the Kolpashev Mountains).

Investigations were carried out by a scheme of linear and point soundings, supported by a sparse net of parametric observations. Parametric soundings were distributed approximately every 100 km, and single soundings at 5-10km. The length of the spreads of the linear soundings was of the order of 2km, with seismograph spacing of 150-200 m. The method of investigation using linear soundings in the region of the Kolpashev Mountains is described in detail in a paper by S.V. Krylov et al [6]. For investigations by the method of point seismic soundings along the traverse following the latitudinal elbow of the Ob River, receivers were arranged in 500 m spreads with seismograph spacing 50m. Each spread generally served for two or three single soundings. The seismographs were set out along the bank of the river, and the plant and instrumentation of the shot point on a boat. The distances between shot-points and receivers were determined from a map. A radio station was used for communication and transmission of the shot instant signal.

Where the basement was buried deeply (more than 2 km) better results were obtained using seismic stations (SS24P and SS-30-60-KMPV), limited for recording low frequencies (10-20Hz) and seismic receivers type NS-3 with characteristic frequency 4Hz.

Explosives were detonated in drill holes, natural waters (lakes, rivers), and cuts in prospecting pits. The depths of shot holes, as a rule, did not exceed 10-15m, and the explosive charge was about 5-40kg for distance of the receiving area from the shot-point up to 15-20km. For explosions in shallow lakes (Turukhan-Eniseisk region) the charge was increased 2-3 times.

The length of the baseline of the soundings was chosen as minimal, taking into account tracing the reference wave immediately after its appearance as a first arrival. On the average it was 3-4 times larger than the depth to the basement surface, and in the region under discussion worked out at 5-20km. For reliable determination of the boundary velocity from a combination of point observations, the difference between baselines of alternate soundings was chosed as 4-5km.

Discrete correlation of the waves refracted at the basement surface was carried out quite reliably in all regions. Correspondence of apparent velocities and amplitudes of the recorded waves were important features in their identification. Waves from the basement surface were characterized by a smaller amplitude of oscillation and a larger apparent velocity in comparison with waves from within the body of the sedimentary rocks. It was shown to be adequate to have seismograph spreads of not more than 500 m for reliable application of the criterion of apparent velocity. It seems that recognition of similar phases of the reference wave, if recorded as first arrivals, is almost always solved uniquely if there is a sufficient intensity of seismic records.

In a number of cases waves refracted at boundaries in the base of the sedimentary section, as well as in the body of the basement, were systematically traced (region of the latitudinal elbow of the Ob and the north-eastern lowland). With further development of the method it is possible to consider surveying simultaneously several horizons.

In order to verify the accuracy of the method the results of soundings were compared with data of drillhole surveys, showing the basement surface at depths of 2.5-3 km, and also with the results of continuous profiling by the correlation method of refracted waves. Discrepancies in the values of the depths were of the order of 100m, and in the boundary velocities, 150 to 200m/s. These estimates confirm the results of the theoretical analysis of the accuracy presented above. Also structures based on the data of linear and point soundings were compared. The results indicated these were practically identical.

The investigations were shown to be adequately effective geologically. In a short period essentially new data were obtained about the depth of layering, morphology and material composition of the surface of the basement of various regions of the lowland.

A number of previously unknown coarse structural elements (with amplitudes up to 1000m) were discovered in the region of the Kolpashev mountains [6] and of the latitudinal elbow of the Ob River. The structural plan of the Ural division and the basement surface of the Turukhan-Eniseisk interfluvial area were defined more precisely. The main important information was obtained about the structure of the Taimisk depression. There the structural features of two, and in some cases of three refracting boundaries were revealed, corresponding to the base of the sedimentary section, the surface of the basement, and a boundary within it.

It was clarified that the methods being previously used for construction of the surface of the basement by means of transformation of the depths of occurrence of reflecting horizons in the lower part of the sedimentary section could not be regarded as reliable, inasmuch as these surfaces often were scarcely conformably deposited, owing to which errors up to 1000m and more were possible. Errors of this order were often established in results of operations by the method of sounding in the Kolpashev region [6].

We dwell in somewhat more detail on the geological results of the investigations in the region of the latitudinal elbow of the Ob River* (fig. 5). The basement surface occurs here at depths of from 3 to 4.4 km and forms a few coarse structures: a monoclinal plunge to the west from northern Lokosov in the region of the villages Surgut and Pilyugino. These structural elements are bounded by depressions with the basement dipping to more than 4km. The Pilyuginsk uplift is the most abruptly expressed. Its amplitude on the western (steeper) limb reaches 1200m, while the angles of inclination of the basement surface amount to 3-5°.

The occurrence of horizontal sedimentary section in deposits of Jurassic age (according to data of previous operations by the method of reflected waves) is significantly more sloping. In these horizons the structures enumerated above merge into one very steep uplift. Unconformable disposition of these geological surfaces indirectly indicates the complex structure of the section of rocks included between them. Here one must expect wedging out of the layers, particularly in the regions of abrupt slopes in the basement surface. The zones of wedging out must be of interest for oil geology, inasmuch as they can be oil and gas traps.

*A.N. Gretskiy, P.F. Mazunin, B.P. Mishenkin, E.M. Shlyarkhter also took part in the full investigation.

Somewhat reduced boundary velocities (5.2-5.5km/s) were characteristic for this traverse on the whole. Several regions can be separated with approximately constant values of this parameter (see fig.5). The boundaries of these regions coincide with abrupt changes in the relief of the basement surface, and in the majority of cases are accompanied by intensive anomalies in the magnetic and gravitational field. It is possible that there are fractures in the body of the basement here, bounding divisions of its blocks. A similar combination of these geophysical features with the shape of the relief of the basement was noted also in other regions of the West Siberian lowland [6]. Special projects need to be organized for confirmation of the existence of deep fractures.

As a result of the testing of the proposed method its high economy and geological effectiveness were demonstrated. This now enables a systematic regional study of the basement of the West Siberian lowland as a whole. Taking as realistic productivity 800-1000km per party in a field season, this problem can be solved in 3-4 years with a strength of 4-5 parties. First priority must be given to traversal investigations along rivers, and carrying out separate isolated soundings in the most interesting and poorly studied regions. The data from seismic soundings, besides giving independent values, establish a reliable foundation for interpretation of the data of other geophysical methods.

The proposed method may prove useful also for solution of other geological problems in different regions. In particular, it seems promising to make use of it for deep seismic sounding of the Earth's crust, where in many cases it is necessary to reduce costs and simplify the field observations. In mining geology, the surface of the bedrock can be mapped, being differentiated by the boundary velocity. Small depths to prospecting boundaries allow significant improvement in the detail and accuracy of the data obtained.

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ON THE THEORY OF INTERPRETATION OF ISOLATED SEISMIC OBSERVATIONS

Abstract

An account is given of the basic conclusions of the theory of isolated seismic soundings as applied to various types of waves (reflected, head, and refracted). The methods are recommended primarily for investigations of a regional character. The possibility is stressed of using refracted waves for the study of deep geological structure.

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Seismic methods of prospecting until recently were developed mainly for detailed study of deep structures. In accordance with this, complex schemes of observations were used, guaranteeing continuous correlation of waves even when they can be traced for relatively short intervals. However as work in a number of regions shows, certain waves or groups of waves can be identified by secondary characteristics. Usually such a possibility occurs at certain stages of a geological-geophysical investigation when more or less clear principal features of deep geological structures and a characteristic pattern of the wave trace have been established.

The possibility of discrete correlation of waves permits significant simplification of the scheme of observations, particularly if the investigations have a regional character and call for a study of only the major essential pattern of the deep structure. Inevitably with this, there is some loss of information about details of structures which can be regarded as unimportant and fully compensated by the significantly improved production.

Often use of incomplete, quite simple schemes of observations are unavoidable, for example, owing to the complexity of the surface conditions. In other cases use of the full correlation system with distant sources does not allow tracing waves, the information has a scrappy character and often remains unusable owing to a deficiency in the correlation method of treatment. Mostly this is connected with unfavourable conditions of excitation and reception of vibrations.

The application of a simplified scheme of observations in recent years permitted successful solution of important geological problems [2]. Until now simplified observations were generally directed towards getting separate hodographs or their segments. Corresponding with this, methods of interpretation on the whole did not differ from those widely applied, particularly in the stage of determination of velocities. The latter did not allow all available information to be used to the best advantage.

In the present paper an account is given of the basic problem of interpretation applied to isolated observations, i.e. in the absence of hodographs. The necessity of positional observations is not denied, but the latter are used only to the extent to which they are necessary for more reliable recognition and identification of waves. The resulting t theoretical conclusions are applied to various monotype waves (reflected, head, refracted). The problem is solved of determination not only of the location of the boundary, but also of the distribution of velocity, characterising the basic kinematic features of the waves.

Reflected Waves

Let us consider a single boundary, a reflected wave from which has the characteristic feature that its monotypicity can be established by repeated traces at different points of the profile. If there is only one record of the wave (l, t) then the average depth along the normal to the boundary can be reliably determined from it at the point l/2 [4] if the calculated velocity is known. The corresponding formula has the form:

$$h_m = \tfrac{1}{2}\sqrt{v^2t^2 - l^2 \cos^2 \phi} \qquad (1)$$

The reliability is determined by the fact that dependence on the angle occurs as a cosinusoidal factor, and therefore with relatively small angles of inclination (up to 10° - 15°), it is often possible to neglect the influence of the angle of inclination in the first approximation, and thus also the shape of the boundary, particularly for small values of l. Under these conditions, h_m approximately represents the location of the point of reflection [4]*.

* The depth h is plotted at the central point of a straight line joining source and receiver, independently of how this point is situated relatively to the real surface of the observations (corrected). This remark applies equally to head and refracted waves.

At two isolated observations situated so that the intercept point P of the mean depth coincides for them (fig. 1), as well as h_m the effect of the velocity may be calculated [4] from the formula:

$$v = \sqrt{\frac{l_2^2 - l_1^2}{t_2^2 - t_1^2}} \cos \phi \qquad (2)$$

Let us examine now a system of isolated soundings with two fixed baselines l_1 and l_2, located arbitarily along a profile x. We consider the spacing of the distribution of the soundings to be satisfactory for reliable interpolation of the time at successive points. We agree to represent the observations under consideration on the plane of the profile (x, t), considering l_i as a parameter and attributing the wave arriving at this time to the mean point of the corresponding baseline. Connecting the points in the plane (x, l), corresponding to the given values l_1 and l_2, with continuous lines, we obtain two curves: l_1 = constant and l_2 = constant (fig. 2). It is obvious, that at any point $x = x_i$ of the profile, formulae (1) and (2) are now valid, as conditions of symmetry are assured. Using the lines l=const in formulae (1) and (2), the parameter ϕ need not remain as an unknown: it may be expressed in terms of the horizontal gradient corresponding to the line l=const, for which we introduce the definition:

$$\left(\frac{dt}{dx}\right)_{l=const} = \vartheta$$

Differentiating (1) with respect to t, and bearing in mind the obvious equation:

$$\frac{dh_m}{dt} = \frac{dh_m}{dx} \frac{dx}{dt} = \frac{\sin \phi}{\vartheta}$$

we obtain:

$$\frac{\sin \phi}{\vartheta} = \frac{v^2 t}{2\sqrt{v^2 t^2 - l^2 \cos^2 \phi}}$$

Solving this equation for $\cos^2 \phi$, we obtain the relation between the angle ϕ and the gradient ϑ:

$$\cos^2 \phi = \frac{1}{2\,l^2}\left[v^2t^2 + l^2 - \sqrt{(v^2t^2 - l^2)^2 + v^4 l^2 t^2 \vartheta^2}\right] \quad (3)$$

Substituting (3) in (1), we obtain:

$$h_m = h_{mo}\sqrt{\tfrac{1}{2}\left[1 + \sqrt{1 + \frac{v^2 l^2 (l^2 + 4h_{mo}^2)\vartheta^2}{16\, h_{mo}^4}}\right]} \quad (1')$$

where $h_{mo} = \tfrac{1}{2}\sqrt{v^2t^2 - l^2}$, i.e. the depth calculated under the assumption that the angle of inclination is equal to zero.

The following formula may be used approximately, if ϑ is not large, namely:

$$h_m \approx h_{mo}\left[1 + \frac{v^2 l^2 (l^2 + 4h_{mo}^2)\,\vartheta^2}{128 h_{mo}^4}\right] \quad (1'')$$

Correspondingly we obtain the following expression for velocity:

$$v = \frac{v_o}{\sqrt{1 + \frac{v_o^4 t^2 \vartheta^2}{4(v_o^2 t^2 - l^2)}}} \quad (2')$$

where

$$v_o = \sqrt{\frac{l_2^2 - l_1^2}{t_2^2 - t_1^2}}$$

If l=0 it takes the simpler form:

$$v = \frac{v_o}{\sqrt{1 + \frac{1}{4}v_o^2 \vartheta_o^2}} \quad (2'')$$

For small values of ϑ the formulae are simplified, namely:

$$v \approx v_o \left[1 - \frac{v_o^4 t^2 \vartheta^2}{8(v_o^2 t^2 - 1^2)}\right] \approx v_o \left[1 - \frac{1}{8} v_o^2 \vartheta_o^2\right] \qquad (2''')$$

We do not present the difficulties of generalizing the method of calculation of velocity and depth in the case of arbitrary variable bases, varying in the range from 1_o to 1_k. Interpolating between separate values, we obtain the field of observed time $t(x, 1_i)$, where 1_i is a parameter. To any two lines 1_i and 1_{i+j} of the field the above corrected formulae are applicable for calculation of the effective velocity.

If the gradient of the field dl/dt is known along the time axis, then in the general case for a fixed point (x, t, 1) of the field we can write:

$$v = \sqrt{\frac{1}{t}\frac{dl}{dt}} \cos \emptyset \qquad (4)$$

With a small number of observations and sufficiently complex surface conditions it is necessary to work out the structure of the field in semi-quadratic coordinates, because quadratic values for t and 1 enter into the original formulae and linear interpolation will be valid only for these. Along the axis of the ordinate in this case are plotted the values $t^2 = \tau$ and the parameter of the field will be $1^2 = \lambda$ (fig. 3). The formulae for the depth and velocity then take the form:

$$h_m = \tfrac{1}{2}\sqrt{v^2\tau - \lambda}\cos^2 \emptyset \qquad (5)$$

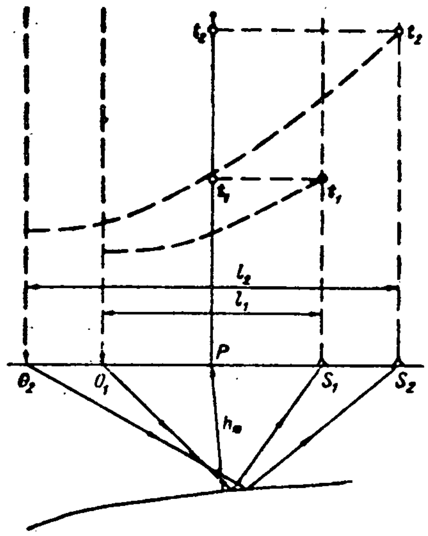
$$v^2 = \frac{\lambda_2 - \lambda_1}{\tau_2 - \tau_1} \cos^2 \emptyset \qquad (6)$$

Let us introduce a new symbol for the gradient of the curve λ = const. in coordinates (x, τ):

$$\left(\frac{d\tau}{dx}\right)_{\lambda = \text{const}} = \varepsilon$$

Evidently,

$$\frac{dh_m}{d\tau} = \frac{\sin \emptyset}{\varepsilon}$$



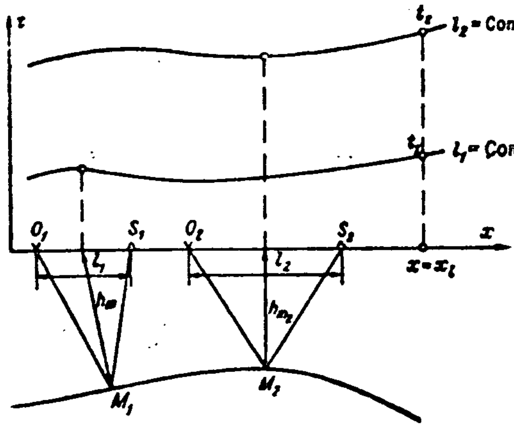


Fig. 1 SYSTEM OF TWO POINT SYMMETRICAL SOUNDINGS

Fig. 2 FIELD $t(x, l_i)$ FOR REFLECTED WAVES WITH TWO FIXED BASES OF SOUNDINGS

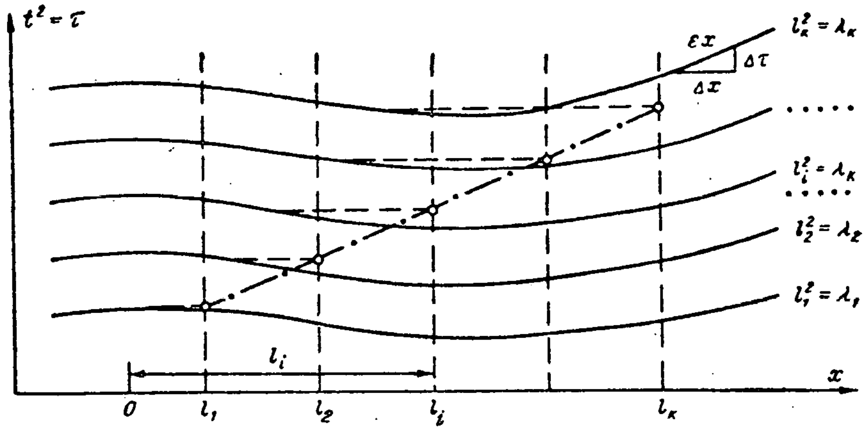


Fig. 3 FIELD $t^2(x, l_i^2)$ FOR REFLECTED WAVES

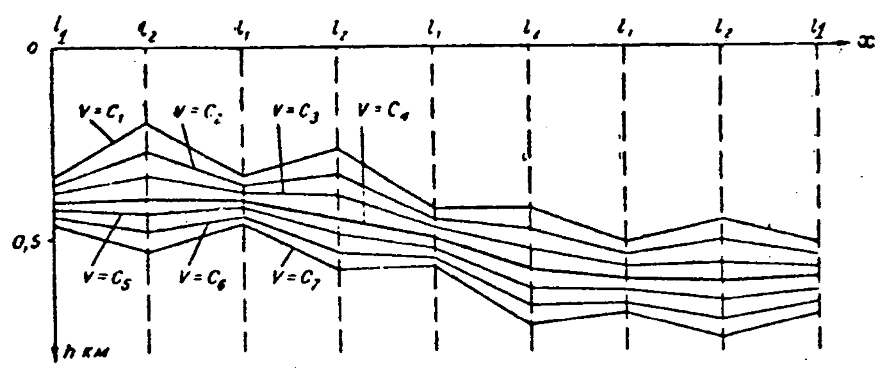


Fig. 4 FIELD OF THE SECTIONS $h_m(x, v)$. THICKENING OF THE LINE WITH MINIMAL CURVATURE AND AMPLITUDE OF OSCILLATION CORRESPONDS TO THE ACTUAL LOCATION OF THE BOUNDARY WITH $v = c_4 = v$ true

Finally we will have the following expression for the effective velocity:

$$v = \frac{v_o}{\sqrt{1 + \frac{1}{16} \frac{v_o^4 \varepsilon^2}{v_o \tau - \lambda}}} \tag{7}$$

or approximately for small gradients

$$v \approx v_o \left\{ 1 - \frac{1}{32} \frac{v_o^4 \varepsilon^2}{v_o \tau - \lambda} \right\} \tag{7'}$$

Correspondingly the expressions for the mean depth and angle of inclination are:

$$h_m = h_{mo} \sqrt{\tfrac{1}{2}\left[1 + \sqrt{1 + \frac{v^4 \lambda \varepsilon^2}{4h_{mo}^2}}\right]} \tag{8}$$

$$\cos^2 \phi = \frac{1}{2\lambda} \left[v^2\tau + \lambda - \sqrt{(v^2\tau - \lambda)^2 + \tfrac{1}{4} v^4 \lambda \varepsilon^2}\right]$$

Approximately for small gradients

$$h_m \approx h_{mo} \left(1 + \frac{1}{32} \frac{v^4 \lambda \varepsilon^2}{h_{mo}^2}\right) \tag{8'}$$

It is essential to note that the functions h_m and v are even with respect to the gradients ϑ and ε.

The influence of curvature of the boundary, as already stated earlier, is small. In practice, if the centre of the sounding is situated over one end of the curved section, then for a convex shape the influence of the curvature, obviously, will not be manifested whatever its value. For a concave shape this will be valid for all values of the curvature which do not produce cusps at the receiving points.

These considerations remain valid (approximately) also for inclined layers, as was shown earlier (N.N. Puzyrev, 1959).

Often reflected waves are recorded at very large distances from the source of the vibrations, significantly exceeding the depth of the boundary (for example, reflections from the Mohorovicic boundary in deep seismic sounding). In this case the effective velocities will essentially differ from the average and ray velocities, and the method of average velocities may not be sufficiently exact. In treatment of data of this type it is expedient to determine instead of effective velocity other parameters, characterizing the properties of the gradient of the medium in the optimum way. The method under consideration of construction of the field t(x, l) permits this to be done if certain data are available about the nature of the dependence of velocity on depth, for example, the initial velocity and the general form of the function v(z). If for example we suppose that v(z) is a linear function of depth, then for beds with boundaries close to horizontal (depth z_o), it is possible to write for a pair of soundings (l_1, t_1) and (l_2, t_2):

$$\left.\begin{aligned} l_1^2/4 + \left[z_o - (\cosh(\tfrac{1}{2} v_o \beta t_1) - 1)/\beta\right]^2 &= \sinh^2(\tfrac{1}{2} v_o \beta t_1)/\beta^2 \\ l_2^2/4 + \left[z_o - (\cosh(\tfrac{1}{2} v_o \beta t_2) - 1)/\beta\right]^2 &= \sinh^2(\tfrac{1}{2} v_o \beta t_2)/\beta^2 \end{aligned}\right\} \quad (9)$$

Solving these equations in terms of z_o and β we obtain the required data about the location of the boundary and the gradient of velocity.

A problem of this type can be solved also graphically, if the generalized ray diagram is constructed for the specific form of the function v(z). For instance, if the function is $v(z) = v_o (1 + \beta_n z)^{1/n}$, where n is a whole number, the solution consists of matching each value β_n with prescribed v_o and n, so that the points with co-ordinates $(\beta_n l_1, v_o \beta_n t_1)$ and $(\beta_n l_2, v_o \beta_n t_2)$ occur on a straight line parallel to the horizontal axis of the diagram. It is necessary to calculate the found value of β_n for several effective velocities if it turns out that the actual form of the function v(z) is essentially different from the assumed dependence. But whatever form this departure takes, the effect of curvature of the seismic rays in any case will be taken into account and the accuracy of the construction will be much higher, than with use of the method of average velocity.

It is important to note that for interpretation of the field t(x, l) the influence of the gradient in the upper medium shows up fundamentally in the depth to the location of the boundary, and does not influence its shape much, since the latter is completely determined by the character of the curve l = const. In this lies the essential advantage over calculation according to hodographs, when it is required to know the ray velocities for various distances between source and receiver.

Construction of the time field t(x,l) enables simultaneous control of the validity of discrete correlation of waves, because erroneously calculated times will not fit in to a single system.

As we saw above, the field t(x,l) permits calculation of all required parameters of the section, using very simple methods of working. But it is impossible to regard these methods as unique. Above all one must point out that any field of lines l = const. is easily converted into a system of hodographs with length $(l_n - l_1)$, with arbitrarily assigned points of excitation of the waves. For this it is enough to plot the assigned values l along the x axis and on the vertical line l_i = constant to construct from a similar curve of the field t(x,l) a point situated in the middle of the chosen starting points (see fig. 3). Since the points of excitation are not fixed in advance, then it is easy artificially to establish any system corresponding to reciprocal points. Then it is possible to interpret this system as a normal system. One must, however, remark, that transformation of the field t(x,l) into a system of hodographs for reflected waves is hardly likely to serve a useful purpose.

Another application of the analysis is not connected directly with the use of the field t(x,l) and consists of the construction of series of sections from combinations of the original data, corresponding to various values of the calculated velocity v. As a result we obtain the field of the section $h_m(x,v)$ with parameter v (fig. 4). In this it is satisfactory to calculate the mean depths (to a first approximation) according to the formula (1) with $\phi = 0$. The basic feature of this field will be the presence of extremes of various signs on areas with small or large values of the baseline. Thus, if the system of observations is such that small and large baselines are arranged in a certain order, then the lines of section have the character of oscillating curves, whose amplitude of oscillation will be bigger, the more the calculated velocity v differs from the true v_o. The phases of the oscillating process will coincide if the departure of the calculated from the true velocity has the same sign. The latter implies that in the region of the field $h_m(x,v)$ corresponding to the real position of the boundary, there must be observed a change of phase of π, and at the same time minimal amplitudes of the oscillations. In this way the most probable position of the desired boundary is determined.

It is easy to confirm the above by direct calculation in the case of a flat boundary.

Let three elementary soundings be distributed at equal intervals along the part AB of a profile (Fig. 5) with centres at points A, C, B. The two extremes of these have baselines l_1, and the middle one has a baseline l_2; we assume for clarity that $l_2 > l_1$. If the calculated velocity v is taken as equal to the true v_o, then, obviously, the points P, R, Q at the calculated depths lie on a single straight line, coinciding with the boundary. Let us assume now that $v > v_o$; then we obtain a new disposition of the points at the calculated depths. If we assume $\Delta v = v - v_o$ small, we obtain the following general expression for the increase in depth:

$$\Delta h_m = (h_m + \frac{l^2 \cos^2 \phi}{4 h_m}) \frac{\Delta v}{v_o} \qquad (10)$$

If h_m is the true value of the depth at point C, then with $v = v_o + \Delta v$ we may write the following expression for the depth at points A, B, C:

$$h'_{mA} = (h_m + \Delta x \sin \phi)(1 + \frac{\Delta v}{v_o}) + \frac{l_1^2 \cos^2 \phi}{4h_m} \frac{\Delta v}{v_o} - \frac{l_1^2 \Delta x \sin \phi \cos^2 \phi}{4h_m^2} \frac{\Delta v}{v_o}$$

$$h'_{mB} = (h_m - x \sin \phi)(1 + \frac{\Delta v}{v_o}) + \frac{l_1^2 \cos^2 \phi}{4h_m} \frac{\Delta v}{v_o} + l_1^2 \frac{\Delta x \sin\phi \cos^2 \phi}{4h_m^2} \frac{\Delta v}{v_o},$$

$$h'_{mC} = h_m(1 + \frac{\Delta v}{v_o}) + \frac{l_2^2 \cos^2\phi}{4 h_m} \frac{\Delta v}{v_o}$$

Now the points P', R', Q', on the section with calculated depths along the normals h'_{mA}, h'_{mB}, h'_{mC} will not lie on the same line. Actually,

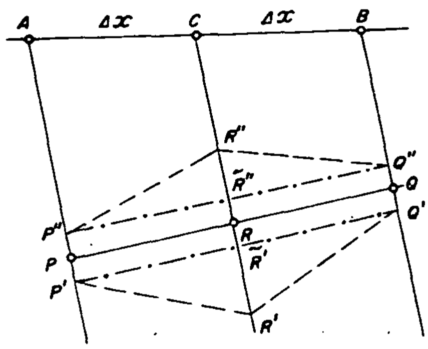


Fig. 5 IN EXPLANATION OF THE PROPERTIES OF THE FIELD $h_m(x, v)$ FOR A FLAT BOUNDARY

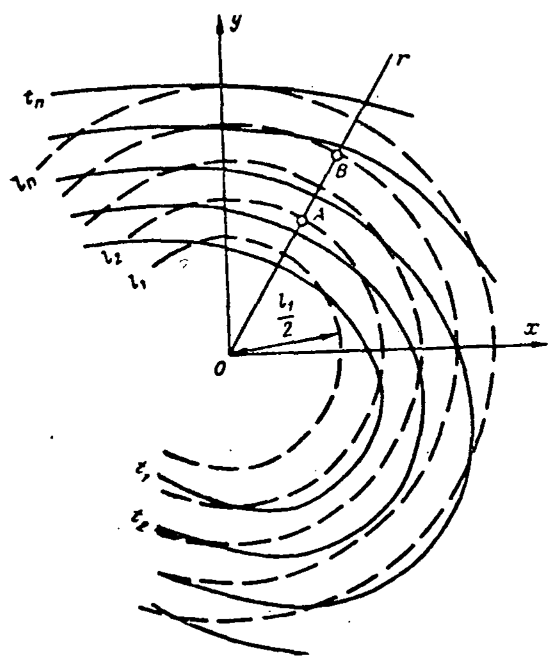


Fig. 6 BINARY FIELD $t(x, y, l)$ WITH USE OF A SINGLE SOURCE.

the average value of the depth between A and B (the point $\tilde{R}'$ on the section) is equal to

$$\tilde{h}'_{mc} = \tfrac{1}{2}(h'_{mA} + h'_{mB}) = h_m \left(1 + \frac{\Delta v}{v_o}\right) + \frac{l_1^2 \cos^2 \phi}{4h_m} \frac{\Delta v}{v_o}$$

Since the condition $l_2 > l_1$ implies $h'_{mc} > \tilde{h}'_{mc}$, the boundary has a concave form.

In an analogous way it may be shown that when $v - v_o < 0$, $h''_{mc} < \tilde{h}'_{mc}$ and consequently the boundary has a convex form.

Calculations and constructions show that the above principles remain valid also for curved boundaries.

Head Waves

For recording of head waves, as is shown in the works of G.A. Gamburtsev and N.N. Puzyrev [1,6] it is also possible to convert to a determination of mean depths. In the case of a flat boundary and two soundings with base-lines l_1 and l_2, symmetrically disposed with respect to their common centre, the medium and boundary velocities are found from the formula [6]:

$$h_m = \frac{v_1 (t_{1,2} - l_{1,2} \cos \phi / v_r)}{2 \cos i} \qquad (11)$$

and

$$v_r = \frac{l_2 - l_1}{t_2 - t_1} \cos \phi \qquad (12)$$

where $\sin i = v_1/v_r$

$t_{1,2}$ = time of arrival of the head wave

ϕ = angle of inclination of the boundary.

Just as in the case of reflected waves we can consider a system of arbitrarily distributed single soundings and construct the field t(x,l). As previously formulae (11) and (12) remain valid for a fixed point of the profile, since the symmetry of the bases is maintained.

Formulae (11) and (12) can be transformed, having expressed the angle ϕ in terms of the gradient of the line l = constant. Let us use for the gradient the former symbol $\vartheta = \left(\frac{dt}{dx}\right)_{l = const}$ and, performing those same operations as for the reflected waves, we obtain finally:

$$\sin \phi = v_1 \vartheta / 2 \cos i \qquad (13)$$

$$h_m = \frac{v_1}{2 \cos i} \left(t - \frac{1}{v_r} \sqrt{1 - \frac{v_1^2 \vartheta^2}{4 \cos^2 i}} \right) \qquad (11')$$

$$v_r = v_{ro} \sqrt{1 - \frac{v_1^2 \vartheta^2}{4 \cos^2 i}} \qquad (12')$$

where

$$v_{ro} = \frac{l_2 - l_1}{t_2 - t_1}$$

With small gradients we will have approximately:

$$h_m \approx h_{mo} + \frac{v_1^2 \, l \, \vartheta^2}{16 \cos^2 i} \tan i \qquad (11'')$$

$$v_r = v_{ro} - \frac{v_1^3 \vartheta^2}{8 \cos^2 i \sin i} \qquad (12'')$$

We note that in the case under consideration the quantities l and t enter linearly into the formulae and therefore ordinary linear interpolation between the points of the field t(x,l) is quite valid.

In calculations of the depths, the velocity v_1 is taken as known from other data, e.g. from the results of soundings with reflected waves.

An important feature of soundings with reflected waves is that for calculation of h_m the boundary is taken as flat over sufficiently large intervals of the profile, a few times greater than the depth. As calculations show, in the case of a curved boundary h_m and v_r are distorted, while the distortion is particularly obvious on the field $v_r(x)$. The problem of calculation of the curvature will be considered in a paper by C.V. Krylov and N.N. Puzyrev, and we do not continue with this problem here. We may note only that the methods of working developed permit with sufficient accuracy consideration of the influence of the low frequency component of the line of section. Irregularities having extension substantially smaller than the magnitude of the base of the sounding, can not be calculated in principle. This, naturally, can limit the application of the method of refracted waves for detailed investigations.

For interpretation of data in accordance with the scheme of treatment uder consideration, it is possible to employ the construction of a system of hodographs for a given field t(x,l) analogously with that described above for reflected waves.

In this connection it is important to note that the use of combined reversed hodographs can simplify calculation of the influence of curvature of the boundary. The method of construction of the fields of the sections remains applicable to the same extent, but the parameter of the field in this case is not the average velocity in the upper medium, but the boundary velocity. The connection between the increments Δh_m and Δv_r and the magnitude of the base lines is exressed in this case by the formula:

$$\Delta h_m = \frac{1}{2 v_1} \left(l \cos i \cos \phi - 2 h_m \sin i\right) \tan^2 i \; \Delta v_r \qquad (14)$$

In the formulae presented above it was supposed that the velocity v_1 is constant. In reality this condition, as is known, is not satisfied, and the average velocity is a function of the depth h_m. In this case the depth must be found from the general relation:

$$2\,h_m\sqrt{1-\frac{v_1^2(h_m)}{v_r^2}} \qquad = v_1(h_m)\,t_o$$

where

$$t_o = t_1 - l\cos\phi / v_r$$

For a prescribed function $v_1(h_m)$ one can draw the graph of the relation $h_m(t_o, v_r)$ and make use of this for finding depths.

Of necessity we do not present the principal difficulties of taking into account the curvature of seismic rays in the presence of a gradient of velocity in the refracting layer. For this, for instance, one can use the method described earlier by us [5].

Refracted Waves

Let the velocity vary continuously with depth in accordance with the definite law v = v(z). The equation of the hodograph of the refracted waves may be written, as is known, in the form:

$$l = 2\int_o^{z\,max} \frac{v(z)\,dz}{\sqrt{1-p^2v^2(z)}}$$

$$t = 2\int_o^{z\,max} \frac{dz}{\sqrt{1-p^2\,v^2(z)}} \qquad (15)$$

Differentiating each of these equations with respect to z and forming the ratio of the derivatives, we obtain

$$\frac{dl}{dt} = pv^2(z)$$

With the parameter $p = 1/v(z_{max})$ the gradient of the field t(x,l) is equal to the true velocity at the depth of the maximal penetration of the ray. If one operates with the terminal increment of the baselines, then a certain stratum velocity will be determined in a layer of thickness $z_2 - z_1$, where z_1 (and z_2) are the depths of penetration of the rays for baselines l_1 and l_2.

Calculation of the angles of inclination of the line v = constant for a continuous medium presents substantial mathematical difficulties and is not considered here.

It may be supposed that the influence of the angle of inclination will follow a law close to cosinusoidal, inasmuch as refracted waves through their own geometrical structure occupy a position intermediate between reflected and head waves. In the first approximation for the majority of media encountered the influence of the angle of inclination may be neglected for the calculation of the strata velocities.

It is necessary to use the values obtained for the strata velocities for the depths of penetration determined for the rays at the given point of the profile. Calculation of them is possible only under certain assumptions about the form of the function v(z). Let this function by characterized by a set of parameters α_i and, consequently, $v = v(z, \alpha_i)$. Substituting these for example, in the first of equations (15) and taking into account that

$$p = \frac{1}{v(z_{max}, \alpha_i)},$$

we obtain

$$1 = f(z_{max}, \alpha_i)$$

Since the magnitude v is known, the second equation may be written:

$$v = v(z_{max}, \alpha_i)$$

Taking all parameters except the single one (α_k) as known, we find after eliminating α_k the desired value z_{max}.

Suppose for example $v = v_o\ (1 + \beta z)$. Let us take v_o as known from the observational data, and β as some effective parameter. In this case we can write down the following system:

$$1 + \beta z_{max} = \sqrt{1 + \left(\frac{\beta 1}{2}\right)^2}$$

$$v = v_o(1 + \beta z_{max})$$

Eliminating β, we obtain the following simple expression for depth of penetration of the ray:

$$z_{max} = \frac{1}{2}\sqrt{\frac{v - v_o}{v + v_o}} \qquad (16)$$

Here 1 denotes the mean of the two base-lines l_1 and l_2, from which is determined the velocity $v = v_{pl}$. Correspondingly z_{max} is referred to the mean point of the layer.

For a check we may use the formula:

$$z_{max} = \frac{(v-v_o)t}{2 \operatorname{arsinh}\sqrt{\frac{v^2}{v_o^2} - 1}} \qquad (17)$$

where t denotes the mean time on the baselines l_1 and l_2 from which the stratum velocity was found.

Formulae (16) and (17) are not independent: l_1, and $t_{1,2}$ were already used earlier for determination of strata velocities. They are approximately valid for functions v(z) significantly departing from a linear function. This follows immediately from the method of the average gradients [4] and can be illustrated by the following example.

Let the velocity vary according to the law:

$$v = v_o e^{kz} \qquad = 2\, e^{0.25z} \quad \text{(km/s)}$$

significantly departing from linearity. A change of velocity of about this magnitude may be found in terrigenous sediments.

Let us consider symmetrical soundings with baselines $l_1=4$ and $l_2=5$ km. Times and depths of penetration of the rays are accordingly equal to:

$t_1 = 1.920$ s $\qquad$ $t_2 = 2.340$ s

$z_1 = 0.525$ km $\qquad$ $z_2 = 0.840$ km

$z_m = 0.682$ km

The magnitude of the strata velocity in this case is equal to 2.38 km/s. Calculating z_{max} from the formula (16), we obtain the value of 0.663 km. The inaccuracy for determination of the depth from the approximate formula amounts to less than 3%, which ought not to play an important role in this type of investigation. Thus, one must regard formulae (16) and (17) as first approximations in the construction of the velocity field v(x,z). In the following, after construction of the lines of strata velocity on the section, the form of the original function can be changed, which permits definition of the field of strata velocities to a second approximation.

The proposed method of construction of the fields t(x,l) for refracted waves and its transformation into a field v_{pl} (x,z) opens up ways of practical use of refracted waves for prospecting. Obviously, it is possible to use analogous procedures for treatment of seismological data. For the latter, instead of constructing the lines l = constant, one must construct surfaces l = constant and t = constant in the form of a series of charts for the data recorded from earth tremors over a long interval of time.

The surfaces or lines v = constant determined by the above method will not in general represent fully the structural features of the medium owing to the levelling effect of the vertical gradient due to gravitational compaction of the medium.

We carry out calculation of the above effect, assuming the gradients in a definite region of the medium to be constant.

Suppose we have a monocline with angle of inclination ϕ. Suppose that the medium is continuous and assuming that the load is removed, let us write the expression for the velocity function $v_1(x,z)$ in the form:

$$v_1 = v_o (1 + \gamma x + \beta z).$$

where γ and β are respectively the horizontal and vertical gradients.

If we let a be the magnitude of the gradient in the direction of the normal to the line v = constant, then we can obtain

$$\gamma = a \sin \phi, \qquad \beta = a \cos \phi$$

Suppose now that the medium is exposed to the action of gravitational compaction with gradient β_1. Then the velocity at a given point of the medium is increased to the magnitude $v_o (1 + \gamma x) \beta_1 z$ and will be equal to $v = v_1 + v_o(1 + \gamma x)\beta_1 z = v_o[1 + \gamma x + (\beta + \beta_1 + \gamma\beta_1 x) z]$

Differentiating this equation with respect to x for v = constant, and taking into account that $\frac{dz}{dx} = \tan\omega$, and $\gamma/\beta = \tan\phi$, we obtain the expression for the angle of inclination ω of the line v = constant under the combined action of lithological and compressional (gravitational) gradients:

$$\tan\omega = \frac{\tan\phi\,(1 + \beta_1 z)}{1 + \beta_1\,(1 + \gamma x)/\beta}$$

Since the initial co-ordinate was chosed arbitrarily, then for a local investigation we may assume x = 0. Then

$$\tan\omega = \frac{(1 + \beta_1 z)\tan\phi}{1 + \beta_1/\beta} \qquad (18)$$

Thus, the connection between the angles ϕ and ω is completely determined in a basic relation of compressional and lithological gradients. Here the angle of inclination ω obtained as a result of investigations with recordings of refracted waves will always be less than the actual angle. If the compressional gradient is small, then the angles ω and ϕ will be close. But in real conditions very different ratios between β and β_1 may be found, in particular, obviously, as when $\beta_1 \gg \beta$. In this case conditions will be unfavourable for application of the method of refracted waves.

The relation between the gradients β and β_1 can in practice be established by various methods. Thus for example, the value β_1 can be found by means of theoretical calculations, developed from micromodels of the medium [3, 7]. Then comparison of experimental data (sonic logging) with theoretical calculations enables the connection between the gradients β and β_1 to be found. Also the problem may be solved by means of correlation of the angles ϕ and ω in various regions investigated by the methods of reflected and refracted waves. To obtain more reliable information about the true angles of inclination it is necessary to use also geological data.

Knowledge of the relation between the gradients β and β_1 permits conversion of the angle ω into the angle ϕ and at the same time improvement of the derived representation of the medium.

The formulae for calculation of the boundary velocities from recorded data of head waves and the strata velocities from refracted waves are identical. Thus a single field t(x,l) can be constructed from the data of head (first arrivals) waves and refracted waves. From its character for various intervals of l and t it will be possible to say definitely enough in which region of the field head waves will be recorded, and in which region (in the average) refracted waves will be recorded: for refracted waves the difference in times for given differences in distance will vary regularly with the increase in absolute value, while for head waves it must remain steady for all intervals for the occurrence of the given waves.

Systems with one source

In some cases the characteristic of the observation is such that for recording of the waves it is expedient to lay out a series of detectors distributed in some region around the source. In particular, it is reasonable to establish such schemes for use in investigations of the deep structure of the Earth's crust by means of artificial explosions. In territories with difficult conditions of access or with limited opportunities for location of shot points it may also be justified to study certain areas with the use of a single source*. This is very common in seismological investigations of a regional type.

The principles of interpretation in this case remain as previously, but construction is carried out in the plane of the observations and consequently a binary field of the form t(x,y,l) must be considered. The magnitudes of l and t are the parameters of the field (fig. 6). The lines l = constant are represented as concentric circles with centres at the common shot point and radii $r_i = l_i/2$

For construction of the time field t = constant the values t_i relate to the mean point between the source and the receiver. In the case of reflected waves, in order to ensure reliable interpolation it is expedient to measure the parameters in units of l^2 and t^2, i.e. to construct the field l^2 = constant and t^2 = constant.

* Correspondingly, it is possible to consider one receiver and a moveable source.

Use of the field t(x,y,l) or $t^2(x,y,l^2)$ of head or reflected waves permits unique construction of a chart of mean depths along the normal to the corresponding boundary, and also charts of velocity (boundary for head waves and effective for reflected), using the same methods as in the case of observations along a profile. Thus for example, in the case of head waves with different distances between points A and B along the radius r with corresponding times, the boundary velocities are calculated as

$$v_{ro} = \frac{l_B - l_A}{t_B - t_A}$$

The depths are found from formula (11). It is expedient to introduce corrections for the angle of inclination from the given first approximation $h_{mo}(x,y)$.

As regards refracted waves, here also the (strata) velocity field is constructed from the source chart t(x,y,l). Further, the field of the mean depth of maximum penetration of the rays is found by use of formulae (16) and (17). Thus, the results of the interpretation will be presented in the form of a binary field v (x,y,z), i.e. two charts, together characterizing the three dimensional distribution of velocity in a certain region.

With known locations of epicentres, analogous constructions can be carried out for groups of seismological stations, in particular for reflected and head waves arising from the Mohorovicic and Conrad boundaries. For this the thickness of the corresponding layer will be reduced by half the depth of occurrence of the focus. For direct waves ($\bar{P}$, $\bar{S}$) it is necessary to apply other methods of treatment.

Conclusion

The theoretical conclusions described above were widely tested in practice with operations using reflected and refracted waves (including deep seismic soundings - D S S) and were fully justified. The method of point soundings enables quite sound information to be obtained about the deep structure using a very restricted number of observations. It can be used both with observations of a special type, and where in fully correlated systems for some reason or other the data obtained for the waves under consideration is patchy.

The wide use of refracted waves in the form of systems of soundings for regional investigations of deep structure must be considered as very promising. As is well known, until now they were used only for obtaining information about the velocity section, and the presence of a refraction (in a medium with a gradient) was often even considered as an obstacle to the effective application of the method of head waves.

The theory developed in this paper of the structure of the fields of mean values of times and depths in the use of head and refracted waves under consideration, of course, must lead to averaging of the features of deep structure, and smoothing will become more significant for deeper surfaces. For this reason the recommended schemes are intended principally for investigations of a regional character.

With more detailed investigations by the refraction method use of special simplified schemes of observations based on construction of elements of the hodograph can turn out to be effective. Such schemes were applied for example by Yu. N. Grachev [2]. Interesting surveys of this type were conducted recently by V.K. Monastyrev in the Western Siberian lowlands.*

For recording of reflected waves averaging of the details of the structures will not be occuring in principle, and the accuracy of the interpretation will be controlled completely by the reliability of discrete correlation of the waves and the density of distribution of the soundings.

It is very important in the future in the actual conditions of various regions to strive to interpret jointly the data from different types of waves. This certainly enables a substantial improvement in the reliability of the structures on account of the certain identification of the waves and the more exact knowledge of the calculated velocities.

* Data of this investigation have not yet been published.

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TRANSFORMATION OF THE TIME FIELD FOR ISOLATED SEISMIC SOUNDINGS

Abstract

An account is given of two types of transformation of the time field for isolated seismic soundings: conversion to a new line of observations, and recalculation of the field with a change of base-line. The first transformation is designed for use with waves of arbitrary type, and the second for reflected and head waves. The application of these transformations to the interpretation of seismic data is discussed.

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In interpretation of combinations of isolated seismic observations, instead of hodographs a special time field t(x,l) is examined, on which a family of lines l = const is represented as a profile along the x axis [8]. Each line characterizes a change in arrival time of waves for soundings with a fixed baseline l(distance from source to receiver). Below is derived a solution of two problems of transformation of the field t(x,l) in the two-dimensional case: conversion of observations to another level, and recalculation of the field for an alteration of the baselines. The first problem is solved for waves of arbitrary type, and the second for head and reflected waves.

Reduction of the line of observations to another level

Many applications of interpretation of the field t(x,l) suppose that the line of observations is almost straight, and the overlying medium has a relatively simple structure. Therefore it is often required to reduce the original time field to a new level surface, usually located beneath the most heteregeneous upper part of the section. In this case the structure of the medium between the line of observations and the level of reduction is considered to be known.

Besides exclusion of the surface inhomogeneities noted above, such operations result in establishment of the possible determinations of the layer velocities in multilayered media from the data of reflected waves, consideration of refraction of seismic rays at intermediate boundaries, a marked facility for use of supercritical reflections over large distances from the source, and so on.

There exists a number of methods of reduction [2,6,7] designed for interpretation of hodographs. In specified conditions they can be used also for isolated observations if the assumptions taken as a basis for these methods are fulfilled: the basic seismic surface is horizontal, and the medium between the line of observations and the level of reduction consists of a homogeneous layer with horizontal or gently sloping boundaries ($\phi \leq 3°$). Other limits included are that the stated applications developed are suitable for particular types of waves (head, or reflected). In interpretation of the field t(x,l) problems commonly arise of determination of the type of waves according to the peculiarities of this field. Thus, in the case of head waves the vertical gradient of the field $\left(\frac{dt}{dl}\right)_{x=\text{const}}$ will be constant for relatively simple structures, whereas for refracted and reflected waves the magnitude of the gradient will decrease or increase with the distance l. In this respect it is sometimes necessary, in order to arrive at a valid solution, to carry out a preliminary operation of reduction in order to exclude the distorting influence of the upper part of the section, without knowing in advance the type of waves.

The method set out below starts with the much more common model of a heterogeneous layered medium with curved boundaries. It is equally applicable for waves of various classes (head, refracted, reflected, converted). It is not required to know beforehand the type of wave.*

The essence of the proposed method consists of the fact that from the initial time field are found the angles formed by the seismic rays with the surface at the points of the source and the receiver. Then with the aid of the laws of geometrical seismology, the trajectory of the ray is projected on to the selected level of reduction. The source and the receiver

* For penetrating converted waves the boundary of conversion must be known, if it is to be included between the lines of observation and reduction.

are transferred to these points of projection.

Let us take an arbitary point t(x, l) of the time field (fig. 1). This represents the elementary sounding with base-line l, source 0, and receiver S, at the points x - ½l and x + ½l. In virtue of the principle of reciprocity the source and the receiver may change places. We consider the expression

$$\lim_{\Delta x \to 0} \frac{t(x + \frac{\Delta x}{4}, l + \frac{\Delta x}{2}) - t(x - \frac{\Delta x}{4}, l - \frac{\Delta x}{2})}{\Delta x}$$

Transforming from the time field to the hodograph [8] it is not difficult to be satisfied that this expression is the same as the inverse magnitude of the apparent velocity in the neighbourhood of the point S, if the source is located at the point 0.

In accordance with Bendorf's law,

$$\lim_{\Delta x \to 0} \frac{t(x + \frac{\Delta x}{4}, l + \frac{\Delta x}{2}) - t(x - \frac{\Delta x}{4}, l - \frac{\Delta x}{2})}{\Delta x} = \frac{1}{v_k (x + l/2)} = \frac{\sin (i_{o1} + \phi_1)}{v_o(x + l/2)}$$

where ϕ_1 = angle of inclination of the observation surface;

$v_o(x + ½)$ = velocity of elastic waves at the point S;

i_{o1} = angle between the vertical and the seismic ray at this point;

the angle ϕ here and further on is taken as positive for rising to the right and negative for falling.

In practice it is generally possible to assume that the time field is uniform in a limited region, and to convert to a finite value of Δx.

In view of this we transform the latter relation into the form:

$$i_{o1} = \arcsin \frac{v_o(x+1/2)\,[t(x+\frac{\Delta x}{4}, 1+\frac{\Delta x}{2}) - t(x-\frac{\Delta x}{4}, 1-\frac{\Delta x}{2})]}{\Delta x} - \phi_1 \qquad (1)$$

Analogously for the point 0 we obtain

$$\lim_{\Delta x \to 0} \frac{t(x-\frac{\Delta x}{4}, 1+\frac{\Delta x}{2}) - t(x+\frac{\Delta x}{4}, 1-\frac{\Delta x}{2})}{\Delta x} = \frac{1}{v_k(x-1/2)}$$

$$= \frac{\sin(i'_{o1} + \phi'_1)}{v_o(x-1/2)}$$

and

$$i'_{o1} = \arcsin \frac{v_o(x-1/2)\,[t(x-\frac{\Delta x}{4}, 1+\frac{\Delta x}{2}) - t(x+\frac{\Delta x}{4}, 1-\frac{\Delta x}{2})]}{\Delta x} - \phi'_1 \qquad (1')$$

Let us now consider the seismic rays in the underlying medium, taking it to consist of an arbitrary number of layers with curvilinear boundaries. In each layer the velocity is a function of the depth z. Taking advantage of the known expressions for a continuous medium with vertical gradient of velocity [1], we find the horizontal displacement Δx and the time interval Δt of the ray in each layer. For the nth layer we will have:

$$\Delta x_n = \int_o^{h_n} \frac{a_n v_n(z)\,dz}{\sqrt{1 - a_n^2 v_n^2(z)}}$$

$$\Delta t_n = \int_o^{h_n} \frac{dz}{v_n(z)\sqrt{1 - a_n^2 v_n^2(z)}} \qquad (2)$$

The origin of co-ordinates in each layer is referred to the point of entry of the seismic ray into it. The parameter a_n is equal to $\sin i_{on}/v_{on}$. The angle i_{on} for all layers except the first is determined from the basic law of refraction

$$\frac{\sin(i_{n-1} + \phi_n)}{\sin(i_{on} + \phi_n)} = \frac{v_{n-1}(h_{n-1})}{v_{on}}$$

whence

$$i_{on} = \arcsin \frac{v_{on} \sin(i_{n-1} + \phi_n)}{v_{n-1}(h_{n-1})} - \phi_n \qquad (3)$$

In the above expressions h_n is the vertical projection of the trajectory of the ray in the given layer, $v_{n-1}(h_{n-1})$ and v_{on} are the values of the layer with indices n-1 and n. The remaining symbols are explained in the diagram (see fig. 1).

Applying formulae (1), (2) and (3) in turn, it is possible to compute the magnitudes Δx_n and Δt_n for all layers right down to the line of reduction for the ray arriving at point S. Analogously the values of Δx_n and Δt_n are found for the ray emanating from the source 0.

Obviously, for reduction of the sounding OS to the new line of observations (O'S') the time must be altered to the value

$$\Delta T = -(\Sigma \Delta t_n + \Sigma \Delta t'_n), \qquad (4)$$

and the centre of the sounding is displaced by the quantity

$$\Delta X = \tfrac{1}{2}(\Sigma \Delta x_n - \Sigma \Delta x'_n) \qquad (5)$$

Further, the original base-line of the sounding is changed by the quantity

$$\Delta L = \Sigma \Delta x_n + \Sigma \Delta x'_n \qquad (6)$$

The summation is carried out over all layers included between the old and new observation lines. The quantity Δt is taken as positive for passage beneath and negative for passage over the line of observation, situated above the original level.

The corrections ΔT, ΔX and ΔL are calculated for the requisite number of points of the original time field, and the new field is constructed from the revised values.

In practice, instead of calculating the values of Δx and Δt from formula (2), it is expedient to make use of ray diagrams for the prescribed functions $v_n(z)$. The origin of co-ordinates of the diagram is located at the point of entry into the given layer. The ray corresponding to the angle i_{on} is found, and the time of passage and horizontal displacement of its point of intersection with the underlying layer are determined. For convenience we may plot along the rays the values of the angles formed by it with the vertical.

Let us examine some particular cases of structure of the upper part of the section, leading to considerable simplification of the calculation of the corrections:

(a) The velocities v_n are constant in each layer, the boundaries (including the lines of observation and reduction) are curvilinear. Equation (2) takes the form:

$$\Delta x_n = h_n \tan i_n$$
$$\Delta t_n = h_n / (v_n \cos i_n) \qquad (2')$$

The magnitudes of the angles i are determined as before from expressions (1) and (3);

(b) The surface of reduction and all overlying boundaries, including the ground surface, are parallel planes, and the velocities in the layers are constant. In this case, taking the x axis in the direction of the boundaries, the known relation [1] is used for determination of i_n:

$$\sin i_n = v_n / v_k \quad ,$$

were v_k is the value of the apparent velocity at the point of intersection of the ray with the surface. After substitution of this expression in (2') and simple transformations, we will have:

$$\left.\begin{aligned} \Delta x_n &= \frac{h_n v_n}{\sqrt{v_k^2 - v_n^2}} \\ \Delta t_n &= \frac{h_n v_k}{v_n \sqrt{v_k^2 - v_n^2}} \end{aligned}\right\} \qquad (2'')$$

The values v_k at the extremities of the sounding are determined from the observed field $t(x, l)$, as was explained above. We recall that the simplifying assumptions concern only the upper part of the section, above the line of observations. The distribution of velocities, shape of the boundaries and type of wave in the underlying layer remains arbitrary.

Transformation of the time field with a change of base-line

In interpretation of the field $t(x, l)$, there commonly arises the problem of calculation of the lines l = const from the given base-lines, according to an observed field with significantly different lengths of the base-lines. The solution of this problem, and practical applications in the cases of head and reflected waves, are discussed below.

Head waves

Let us assume that the waves skim along the boundary, not penetrating the refracting layer in depth, and hence the overtaking hodographs are parallel. We show that in these circumstances, assignment of two arbitrary lines l = const. permit recovery of the whole field of the head waves in the interval Δl, equal to the difference of the original base-lines.

Let l_n, and $l_{n+1} = l_n + \Delta l$, be parameters of the prescribed isolines of the field. We will show that for an arbitrary point x of the profile, the time $t(x, l_n - \Delta l)$, corresponding to the isoline $l_{n-1} = l_n - \Delta l$, is determined by the values of the initial time at the points $x - \frac{\Delta l}{2}$ and $x + \frac{\Delta l}{2}$ (fig 2a). Transforming to the generalised plane of observations (see fig 2b) where the lines l = const have the form of straight lines parallel to the axis of the abscissa, we write the obvious equation expressing the property of parallel overtaking hodographs of head waves

$$t(x, l_n + \Delta l) - t(x - \frac{\Delta l}{2}, l_n) = t(x + \frac{\Delta l}{2}, l_n) - t(x, l_n - \Delta l),$$

whence we obtain the desired relation

$$t(x, l_n - \Delta l) = t(x - \frac{\Delta l}{2}, l_n) + t(x + \frac{\Delta l}{2}, l_n) - t(x, l_n + \Delta l) \quad (7)$$

Taking this relation for a series of points of the profile, it is possible to construct the line $l_n - \Delta l$. Then, considering the lines l_n and $l_n - \Delta l$ as a starting point, the distribution of times can be formed for $l_n - 2\Delta l$. Repeating this process successively, the whole field may be re-established in the required region both in the cases of increasing and decreasing base-line.

Denoting for brevity the values of the time by t_{n-1}, t_{n-2}, t_{n-3}, ...t_{n-k}, obtained through transformation of the initial values t_{n+1} and t_n on to the upper levels l_{n-1}, l_{n-2}, l_{n-3},........,l_{n-k}, we find as a result of repeated application of formula (7):

$$t_{n-1}(x) = t_n(x - \frac{\Delta l}{2}) + t_n(x + \frac{\Delta l}{2}) - t_{n+1}(x) \quad ,$$

$$t_{n-2}(x) = t_n(x - 2\frac{\Delta l}{2}) + t_n(x) + t_n(x + 2\frac{\Delta l}{2}) - t_{n+1}(x - \frac{\Delta l}{2}) - t_{n+1}(x + \frac{\Delta l}{2}),$$

$$t_{n-3}(x) = t_n(x - 3\frac{\Delta l}{2}) + t_n(x - \frac{\Delta l}{2}) + t_n(x + \frac{\Delta l}{2}) + t_n(x + 3\frac{\Delta l}{2})$$
$$- t_{n+1}(x - 2\frac{\Delta l}{2}) - t_{n+1}(x) - t_{n+1}(x + 2\frac{\Delta l}{2}),$$

. . . .

$$t_{n-k}(x) = \sum_{j=0}^{k} t_n[x - (k - 2j)\frac{\Delta l}{2}] - \sum_{j=0}^{k} t_{n+1}[x - (k - 2j - 1)\frac{\Delta l}{2}] \quad (8)$$

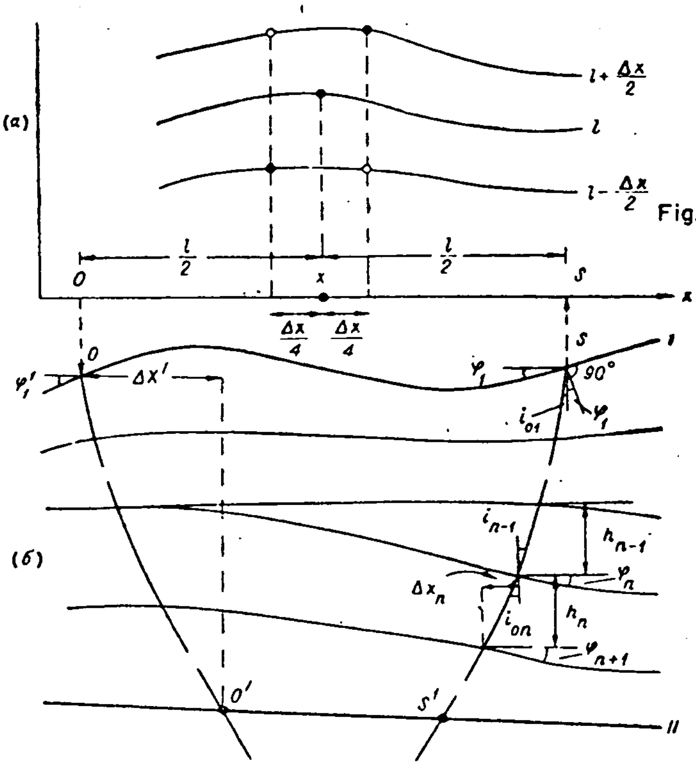

Fig. 1. AS A BASIS FOR TRANSFER TO A NEW LEVEL OF OBSERVATIONS

(a) FIELD t (x, l)

(b) CROSS-SECTION; I- LINE OF OBSERVATIONS, II LEVEL OF REDUCTION.

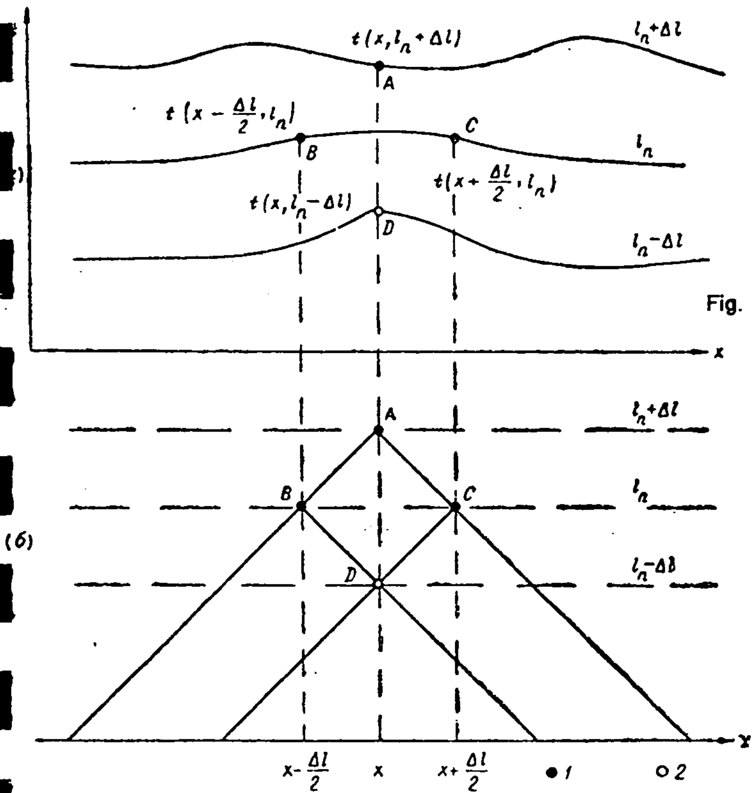

Fig. 2. AS A BASIS FOR THE PROBLEM OF TRANSFORMATION OF THE TIME FIELD OF HEAD WAVES.

(a) FIELD t(x, l)

(b) PLAN OF OBSERVATIONS ON THE GENERALIZED SURFACE,

I POINT RELEVANT TO THE COMPUTATION.

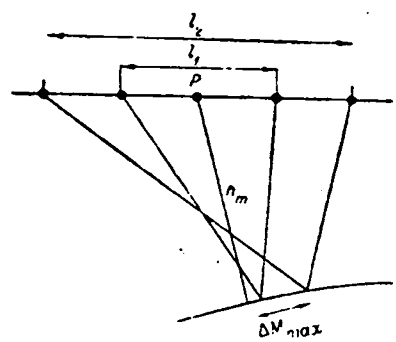

Fig. 4. AS A BASIS FOR TRANSFORMATION OF THE TIME FIELD OF REFLECTED WAVES.

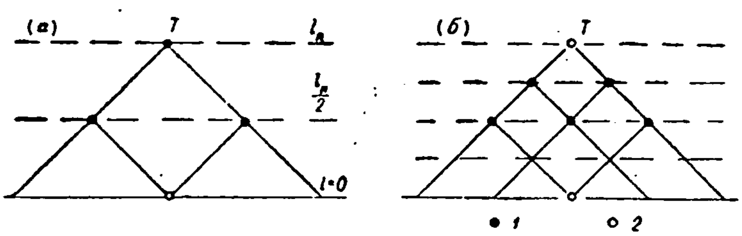

Fig. 3 SPECIAL SYSTEM OF OBSERVATIONS CALCULATED FOR DETERMINATION OF THE TIME t.

1. OBSERVATION POINT. 2. POINT RELEVANT TO THE COMPUTATION.

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Analogous expressions can be obtained also for the case of transformation of the times t_{n-1} and t_n for an increasing baseline:

$$t_{n+k}(x) = \sum_{j=o}^{k} t_n \left[x - (k - 2j)\frac{\Delta l}{2}\right] - \sum_{j=o}^{k} t_{n-1}\left[x - (k - 2j - 1)\frac{\Delta l}{2}\right] \qquad (8')$$

Let us consider some applications of the operation of transformation in the interpretation of seismic data.

Above all the limitations of the method of arbitrary systems of soundings by refracted waves, connected with the possible influence of local features of relief of the boundary and the distribution of the boundary velocities, may be appreciably mitigated. These limitations necessitated that assumptions were made about the linearity of the boundary along the baseline of the soundings for calculation of the depths and the velocities [8], and methods of calculation of the influence of the curvature of the boundary [4] were developed only for the case of structural forms with horizontal dimensions larger than the baseline of the soundings. Using the method of transformation of the field, an isoline may be found with parameter l, close in magnitude to the abscissa of the initial point x_{NT} of the head wave*. The distribution of depths obtained from this isoline gives a practically undistorted representation of the local structural forms, inasmuch as assumptions about the structure of the medium will be less strict, approximately in the same way as in the method of conjugate points. The boundary is taken to be flat only on the small interval $l - x_{NT}$. Moreover, the values of the depth in this case depend only slightly on an inaccurate choice of the magnitude of the boundary velocity [9].

Use of the reduced field extends the possibility of determination of the boundary velocity. It can be found for an interval of any extent as a multiple of Δl from the usual formulae [8]. As one of the baselines it is convenient to take $l \gtrsim x_{NT}$. In this case the value v_r will be related to the continuous portion of the boundary, but not to the two separated portions

* The magnitude x_{NT} can be evaluated with the required accuracy from the preliminary determination of the value of depth and velocity in the refracting and overlying media (v_1 and v_r) at a series of points of the profile.

as with use of base-lines of other magnitudes. At the same time it is possible to decrease the extent of the interval to which each determination of v_r relates. The latter is important in the presence of influence of local changes of boundary velocity.

If there is a significant vertical gradient of velocity in the refracting medium it may become necessary to calculate the effect of penetration of the seismic ray by means of the appropriate correction to the initial value of the time [10, 11].

Another important application of the transformation of the field is control of the accuracy of construction of discrete correlated waves by means of comparison of the observed times with the transformed ones on the same base-line.

This method of transformation of the field t(x, l) may be considered as a generalization of the special inflexible schemes of point observations of refracted waves proposed by Yu. N. Grachev [3] and V.K. Monastyrev [5]. These schemes are calculated on the observed reciprocal time T and the quantity t_o determined from the known formula [1]

$$t_o = \overrightarrow{t} + \overleftarrow{t} - T$$

In different variations of these schemes the reciprocal time is obtained directly, as shown in fig. 3a, or as a result of parallel translation of the observing points of overtaking hodographs (fig. 3b). It is obvious that in both cases the operations can be reduced to a transformation of the observing times on the datum $l = 0$, i.e. to the derived line of times $t_o(x)$. Here application of the special schemes of observations is in principle not obligatory. Arbitrary systems of soundings, ensuring the construction of two lines $l = \text{const}$, are completely satisfactory. We note that use of the line $l = 0$ is less satisfactory than the line $l \approx x_{N.T.}$, because for calculation of the depths in the first case, rectilinearity of the boundary is assumed on part of the dual seismic section. Such an assumption can give rise to appreciable distortion of the local structure, particularly with large depths of the layers and small difference in velocities v_1 and v_r. It becomes inessential for prospecting structures much longer than the duplicate seismic section.

Reflected waves

Let us consider the simplest case of rectilinear traces of the observations. We will assume that the boundary is locally flat on a number of small intervals, the maximal ength of which M_{max} is obtained from the expression [7]:

$$\Delta M_{max} = \frac{l_{max}}{2}\left(\frac{1-p^2}{1 - p^2 \cos^2 \phi - p \sin \phi \sqrt{1 - p^2 \cos^2 \phi}} - 1\right) \cos \phi$$

$$= \tfrac{1}{2}\, l_{max}\, F(p, \phi), \qquad (9)$$

where $p = l_{max}/vt_{max}$

For appreciation of the magnitude of ΔM_{max} under concrete conditions a table of the function F (p, ϕ) is presented below for various values of p and ϕ.

	ϕ =	0^o	5^o	10^o	15^o	20^o
p=	0	0	0	0	0	0
	0.2	0	0.019	0.036	0.052	0.066
	0.4	0	0.039	0.080	0.110	0.141
	0.6	0	0.066	0.138	0.182	0.234

Let us take two isolines l = const of the time field t(x, l) with parameters l_1 and l_2. We try to construct from these data a previously selected line l_3 = const.

In accordance with the conditions of the problem we write for an arbitrary point x of the field t(x, l) a system of three equations:

$$\left.\begin{aligned} v^2 t_1^2 &= l_1^2 \cos^2 \phi + 4h_m^2 \\ v^2 t_2^2 &= l_2^2 \cos^2 \phi + 4h_m^2 \\ v^2 t_3^2 &= l_3^2 \cos^2 \phi + 4h_m^2 \end{aligned}\right\} \qquad (10)$$

Eliminating from these the parameters h_m and ϕ of the section, and then reducing the resulting equality for v^2, we obtain the following proportionality:

$$\frac{t_3^2 - t_1^2}{t_2^2 - t_1^2} = \frac{l_3^2 - l_1^2}{l_2^2 - l_1^2} \tag{11}$$

or

$$t_3^2 = t_1^2 + \frac{l_3^2 - l_1^2}{l_2^2 - l_1^2} (t_2^2 - t_1^2) \tag{11'}$$

As a particular case $l_3 = 0$ and $t = t_o$. When the field is converted into the curve $t_o(x)$, we may write:

$$t_o^2 = \frac{l_2^2 t_1^2 - l_1^2 t_2^2}{l_2^2 - l_1^2} \tag{12}$$

In actual circumstances, the surface of observations can be curvilinear, and the overlying medium non-uniform, i.e. the assumptions under which equations (12') and (12) were derived are not fulfilled. In conditions such as these, before carrying out the transformation of the field it is necessary to change to a new sufficiently even line of observations so that the deviations from flatness are insignificant within the bounds of the maximal baseline of soundings.

By locating the level of reduction below the upper part of the section, which is usually the most variable, it is possible in many cases to satisfy sufficiently strictly the required conditions of uniformity in the overlying medium.

As in the case of refracted waves there is the possibility of checking the validity of the discrete correlation through convergence of the observed and transformed values of the times.

The construction of separate lines l = const can be very useful when the observations, carried out under very non-systematic distribution of the baselines, are transformed into two isolines, from which are found the distribution of velocities and depths.

In conclusion we emphasize the necessity of thorough evaluation of the primary data used in the transformation of the time field with a changed baseline. It is particularly important to satisfy this requirement in the case of refracted waves for revealing local structure. The initial lines l = const must be constructed from sufficiently reliable observations, carried out with appropriate density so as to guarantee safe interpolation between points.

This method of transformation of the time field was tested in the West Siberian lowland for investigation of the surface of the basement and of deeper boundaries in the Earth's crust.

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ON A METHOD OF RECONNAISSANCE SEISMIC INVESTIGATION OF THE EARTH'S CRUST

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Abstract

A method of deep spot sounding is suggested for the areas which are almost inaccessible or have a dense population, where continuous observations along the profiles are impossible. This technique can be also used as the first preliminary stage of crustal studies in a new area, and may be taken as the basis for selection of regions for more detailed investigations or areal observations. The technique of field work involves observations on short-range equipment by two-three stations, whose spacing from the shot-point is optimal for tracing of the primary waves. The whole system travels with a fixed baseline (distance between shot-points and seismic stations) with an interval of 20-30 km. The interpretation technique is based on discrete correlation where the combination of wave, parametric and geological indications is used for identification of the waves. Further analysis is common for all wave types and is based on plotting of special time fields t(x, 1), where x is the distance from an origin of shot-points along the profile, and 1 is the length of the field equipment baseline. The system of isolines for different 1 (not less than two) allows estimation of the topography of the main interfaces and determination of the mean boundary or layer velocities related to the wave type.

The time field can afterwards be transformed into a seismic cross-section, and the distortion effect of the overlying layers, etc., can be allowed for.

The technique outlined above has been tested during regional investigations in West Siberia. Its high efficiency and effectiveness for tracing main interfaces within the Earth's crust, including the Mohorovicic discontinuity, and for studying topography of the basement surface, have been verified.

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Seismic investigations of the Earth's crust, and in certain cases also deeper layers of the upper mantle, require always a large range. For this reason the basic volume of the work is carried out by the method of deep seismic investigations, i.e. by use of powerful artificial sources of vibrations; seismological (earthquake) data are also used to a lesser degree. The increasing role of seismic sounding is quite appropriate, in as much as only this method guarantees relatively high accuracy and reliability of results in conditions of complex distribution of elastic parameters, comparatively few jumps in the velocities of transmission of waves and densities, large absolute values of wave velocities, and other complicating factors. Among the latter, the use of the comparatively low frequency range of the wave field is particularly important.

The tendency to obtain more detailed and accurate information about the structure of the Earth's crust, particularly its "blockiness", leads to the need for applying sufficiently complex schemes of observations by continuous profiling with inclusion of reversed and overtaking hodographs and, where possible, full calculation of the dynamic features of the wave field. However this tendency often runs contrary to the need to cover with observations a sufficiently large region, commensurate with the horizontal dimensions of the first order structure.

The main aim of the present paper is the foundation of a stepwise procedure of deep seismic investigations under the conditions in areas of difficult access, and discussion of the method of reconnaissance procedures in the basic tests of these investigations in Western Siberia.

The problems which confront deep seismic sounding can be subdivided into two groups. To the first belong operations of the regional investigation of coarse enough features of the structure of the Earth's interior along extended traverses, intersecting geologically varied provinces. The second group of problems proposes undertaking significantly more detailed surveys with the objective of studying in relatively small areas the connections between different types of geological structure, zones of deep fracturing, and also solutions of questions of method (clarification of the nature of the seismic waves, their correspondence with the deep boundaries, etc.)

These groups of problems are in many ways interrelated, but their solution evidently can be divided in time into stages of reconnaissance and of detailed projects. Initially it is expedient to conduct a reconnaissance study of the region, and then using the information obtained, to select areas and methods for detailed investigations. The seismic data of the reconnaissance stage serve at the same time as a basis for interpretation of the results of other cheaper and "easier" methods, which if considered in isolation, as a rule, have to a significant degree a natural non-uniqueness in solving the inverse problem. Regions of difficult access occupy large areas. In European territories 17% consists of mountainous regions alone. In such conditions, seismic investigations by the usual method of continuous profiling along extended rectilinear traverses are extremely difficult or economically inexpedient.

The sequence of operations described above appears natural and, as is well-known, is used in practical geological and geophysical surveying. However in deep seismic sounding it is in fact not maintained. Observations are as a rule carried out in the one operation by the method of continuous profiling. In support of expansion of the amount of reconnaissance operations yet one more argument may be advanced - the need to travel to the area of investigation. Point soundings in this case can play the role of a supporting net.

It is obviously not obligatory for reconnaissance seismic investigations to produce very highly accurate and detailed results; they should be conducted by simplified methods, ensuring the study of a broad expanse of territory within a short period.

Until recently there was no general and strict foundation for methods of carrying out projects with low degree of detail. A simplified scheme of observations was brought regularly into production instead of the full correlation system needed for continuous investigation of the deep boundaries, separate hodographs or fragmentation of their tracing. Interpretation of the data was based on hodograph theory, which does not guarantee sufficiently full use of the information contained in the data of the simplified scheme of observations. A method of combined interpretation of recorded waves of various types, recorded in inextensive disconnected regions, was not effective enough.

A method which can be accepted as an effective way of carrying out reconnaissance investigations is the arbitrary system of point seismic soundings with different types of waves (reflected, refracted, head waves), developed in recent years in the Institute of Geology and Geophysics, Siberian Division of the Academy of Sciences U.S.S.R., and widely tested in conjunction with the Novosibirsk Geophysical Trust for regional studies of the Earth's crust in Western Siberia. We discuss briefly the main features and possibilities of this method.

Basically it consists of discrete correlation, i.e. a procedure for identification of analogous waves at sufficiently remote points not connected to each other. For the initial assumptions, which are based on experiments in deep seismic soundings mainly in the platform regions, a representation of the essential regional propagation of the persistent boundaries in the Earth's crust was used, to which correspond stable (reference) elastic waves. However the meaning of the term "reference wave" in this case is somewhat different from the meaning in connection with positional correlation. The stability of the wave and other parameters of the waves must be displayed in their small and smooth variability along profiles in a fixed, sufficiently narrow range of distances between source and receiver of the vibrations. Such requirements determine that the point soundings are carried out with little variation of the said distance along the profile. This feature permits allotting as reference waves those which dominate in intensity over a relatively short interval of the distance from the shot point, while not ensuring construction of a sufficiently long hodograph. In particular, typical situations commonly occur for summed vibrations of reflected and head waves near the critical point. Discrete correlations are realized through the joint use of a combination of wave, parametric and geological indicators. Wave indicators are kinematic and dynamic characteristics of the vibrations taken from the seismograms.

They are very significant at the initial stage of the process of correlation for the separation of waves. Among parametric indicators are included values and regularities of distribution of velocity of elasitc waves and other physical parameters of the medium. To the geological indicators are allotted features of the geometry of the boundaries - the relation between the shapes of the various horizons, information about inclination, curvature, depth of location of the boundary, thickness. These two groups of indicators are applied for joint examination of an aggregate of point observations, after the velocity and structure of the first variation of the section are defined. In this way, the correlation is not limited by separation of the waves on the seismograms, but is continued after critical examination of the distribution obtained for depths and velocities. In order to judge the validity of the correlation, the values obtained for wave, physical, and geological parameters are compared with distribution curves (histograms) constructed from the data of previous investigations in other regions with similar geological conditions. Further, correlations of dependence established previously between the various indicators are taken into account.

The theory of interpretation of the data of point soundings consists of a uniform approach to the various types of waves. Instead of hodographs, special time fields t(x,l) are examined, on which the times of arrival of one and the same wave along the profile (or across the area) are represented by a family of lines (or surfaces). Each line corresponds to a fixed length of the base-line or the sounding, i.e. the distance l between source and receiver of vibrations. The isolines are constructed in a system of co-ordinates t (time), X(abscissa of the centre of the sounding) by interpolation between values at the observation points. From the time field, for which the number of lines must not be less than two, the velocity of the medium and the elements of disposition of the seismic boundary are determined.

The vertical gradient of the time field, $(dt/dl)_{x = const}$, is used for determination of the velocity. The angle of inclination of the boundary can be found from the horizontal gradient of the isolines of the field $(dt/dx)_{l = const}$. From reflected wave data the distribution of effective velocities along the profile is calculated; from head waves the boundary velocities; from the records of refracted waves, the average stratified velocity at the depth of maximal penetration of the ray corresponding to the particular baseline of the soundings. In the latter case the seismic section is obtained as isolines of average stratified velocity.

An important feature of the time field is that the lines l = const have a shape close to the mirror image of the relief of the seismic boundary. This permits one to obtain a representation close to reality even in the case where there is some doubt as to the authenticity of identification of the type of the corresponding wave. As is known, with deep seismic soundings, particularly of a low detail type, the question about the nature of the wave is not always resolved uniquely, which can lead to essential inaccuracies in construction of the section if one uses not the time field, but a hodograph of substantial extent.

The time fields can be converted by various means. For head and reflected waves the whole field is uniquely determined given only two lines l = const. On this property is based a method of transforming the fields for changed baselines. Such an operation permits supplementary checking of the validity of the discrete correlation by comparison of the transformed times with those observed. Besides, in the case of head waves, a transformation into a line with parameter l, close to the magnitude of the abscissa of the critical point, makes it possible to avoid averaging of inextensive structures. The second type of transformation - converting the field to a new level of observations - can be accomplished for waves of any type. It permits one to allow for the influence of inhomogeneities in the upper part of the section (considered to be previously known) and even to reduce problems of interpretation of the waves in conditions of a multi-layered medium to the case of a two-layered section, if vibrations are recorded from the intermediate boundary.

Methods of interpretation of the point soundings include special procedures for joint interpretation of the data from waves of the various types, originating from the one boundary. In particular, a combined interpretation of head and reflected waves proves to be very effective for study of the Mohorovicic discontinuity (M), when head and reflected waves from this boundary near the critical angle are usually recorded on the one seismogram on soundings with corresponding base-lines.

Observations in the field are carried out in accordance with a system of point soundings distributed with the prescribed density along the traverse or according to an areal net. Soundings are generated from one source and receiver of vibrations, spaced at a distance optimal for separation of the specified waves or groups of waves. At the receiving point, longitudinal arrays of 6-12 groups of seismographs are laid out, with total extent as a rule not more than 1 km. Arrays of such a length are generally satisfactory for judgement about the regularity of the waves, and evaluation or the correlation of their apparent velocity, which is needed for carrying out discrete correlation. In new regions it is initially convenient to carry out parametric observations by fragmented continuous profiling with a limited number of shot points, for study of the basic features of the wave map.

For construction of the time field it is necessary to ensure alternation of the soundings along the profile with two (certainly not firmly fixed) baselines. The difference in lengths of these baselines is chosen on the basis of the required accuracy of determination of the velocity, and usually for deep investigations of the Earth's crust is made equal to 15-20 km. In regions where conditions are unsatisfactory for this disposition of source and receiver a three-dimensional system of soundings with a single shot point may be used effectively. The density of distribution of the soundings can be taken as equal to 5-10 km for study of the top layer of the consolidated crust, 20-30 km for intra-crustal boundaries, and about 50 km for investigations of the Mohorovicic discontinuity.

Firm limits are not imposed on the magnitude of the parameters of the soundings and their distribution over the area. This permits conducting investigations within the boundaries of difficult access territory, using arterial rivers and air transport. The productivity of such projects is adequately high even under difficult conditions.

We now discuss basic problems which may be solved through reconnaissance operations by the method of point seismic soundings, based on experiments in the application of this method in Western Siberia, and the results of testing it on data from continuous profiling in other regions.

Study of the surface of the basement of the platform region must appear as one of the basic problems; knowledge of this, apart from intrinsic geological interest, is needed for concrete interpretation of the waves from deeper boundaries. The basic role in this instance appertains to soundings by head waves. Regions may be excluded where great thicknesses of rocks with high velocity of transmission of elastic waves occur within the sedimentary layer (Halogen-carbonate section, traps), exhibiting a strong screening influence. In similar conditions, according to test projects on the West Siberian platform, the problems can be solved by soundings with reflected waves. Organization of projects for recording reflected waves can also be conveniently used for obtaining information about the average velocities.

Study of the Mohorovicic discontinuity is conveniently conducted by point soundings with large baselines, calculated for simultaneous registration of head waves (in the region of first arrivals), and waves reflected at the critical angle. Under favourable conditions it may be possible to change over to recording only reflected waves, which often appear with dominating intensity at small distances from the source, in the region near the critical point.

Investigations of boundaries within the consolidated crust are accomplished by one of two inter-related methods: through separation of sufficiently clear and intensive reflections in the latter part of the seismogram, and use of head waves, particularly if they have an extensive interval as first arrivals in the region concerned.

Commonly hodographs of first arrivals show a series of waves changing from one to another, differing little in their velocities and their dynamic characteristics. In such conditions separation of definite waves as material for point soundings is difficult, and the first waves are conveniently considered in the first approximation as refractions, using the corresponding methods for their interpretation (the method of KONDRATYEV, developed in the Institute of Geology and Geophysics, Siberian Division, Academy of Sciences U.S.S.R.), making possible the construction of sections with isolines of stratified velocities.

By drawing on the records of refracted waves one may obtain a total representation of the character of the section (model of the media) under very unfavourable conditions in a new region; this, besides its obvious geological interest, is important for valid execution of discrete correlation.

The data of point seismic soundings enables development of the coarse features of the structure of the Earth's crust with enough accuracy for the reconnaissance stage, differentiation of it into blocks, and clear expression of zones of deep fracturing. Comparison of the depths and velocities obtained from point and continuous soundings for the boundaries in the Earth's crust and upper mantle for the region of Central Asia and the Urals, showed that the relative difference of single determinations amounted to about 3% for average and boundary velocities, and 4-6% for depth values.

Such data can serve as a reliable basis for interpretation of data of other geophysical methods and for a well-founded selection of regions for detailed seismic investigations.

As has been said, the method of point seismic soundings was developed and tested predominantly in the plain regions. Significantly less work has been done in the mountainous regions of the geosynclinal zone. Just now operations by the sounding method are being conducted along a profile which originates in the south-eastern framework of the West Siberian platform, intersects the Tom-Kolyvansk shield zone, and goes further along the length of the axis of the Kuznetsk - intermontane basin. Thus the profile passes through very heteregeneous and complex tectonic conditions. The surface conditions here are not distinguished by great complexity; operations are conducted along the Ob and Tom Rivers. Data now available from the region Novokuzetsk-Tomsk confirms the effectiveness of the sounding method, despite the complex structural situation (abrupt change in the thickness of the sedimentary section, difference in the character of layering in the crust in separate regions etc). As a second confirmation of the possibility of a simplified scheme of deep seismic soundings in the geosynclinal zones one may quote the profile intersecting the crest of the Urals. From work previously carried out (Institute of Geophysics, Urals Branch, Academy of Science U.S.S.R., and the Urals Geological Administration) we carried out an independent interpretation of part of the experimental data with the use of the method of analysis of separate recordings. The coarse features of the structure of the crust exhibited by this interpretation were the same as by the usual interpretation from systems of hodographs.

In the mountainous structure of Eastern Europe (Carpathians, Balkans), reckoning on the worst case, we may expect the following difficulties related to the complicated orographic conditions:

(1) The impossibility of laying out traverses straight enough and with uniform orientation of the baselines predominantly in the direction of the profile;

(2) Limited possibilities in choice of shot-points and regular distribution of them along the traverse;

(3) The need for introduction of special corrections for relief in the interpretation procedures.

We discuss some aspects of the methods of surmounting these difficulties.

Specific conditions of the area make it very undesirable to repeat observations or to return to areas previously worked, if for any reason the recordings obtained do not satisfy the stated requirements. For this reason it is necessary to make recordings at stations with magnetic tape-recorders. In the past four years the apparatus AMPZChM was used, somewhat modernized in part by expanding the width of the pass band of the amplifier of the recorder in the low frequency range, increasing the coefficient of amplification, etc.

The use of apparatus with magnetic tape enables one to dispense almost completely with repeated explosions. It is necessary to have a minimum of two simultaneously operating sets of apparatus. For increased productivity it is very desirable to bring the number of sets up to 4, 5 or more. At present special small channel portable apparatus is being field-tested for regional projects using remote control (the "Taiga" equipment).

From positive results of the tests, it may prove to be suitable for use in extra difficult conditions in mountainous regions.

Problems of exciting the vibrations evidently must cause the greatest concern. This is mainly because the shot points cannot be distributed with the required density within the bounds of the traverse, particularly for excitation of waves returning from the M boundary. In this connection it is expedient to plan the distribution of sources within the boundaries of a certain band. Correspondingly, it is necessary to provide for various orientations of the baselines of the soundings in relation to the direction of the profile. Similarly the need arises to change to an areal (strip) approach to the section, which can lead to some averaging of detail, but on the other hand increases the reliability of the representation of the coarse structural shapes. In the absence of a fixed traverse, distribution of the observations in some band appears necessary, independently of the choice of shot-points.

Another way of solving the problem of excitation of vibration is the use of apparatus for accumulation of energy from the explosion of comparatively small charges (of the order of 30-100 kg). In this case it seems possible to be more flexible in the requirements for shot points, and often to use shallow natural and industrial reservoirs (lakes ?). At present it is not difficult in principle to construct such apparatus on the basis of available developments. There are certain reasons to suppose that use of an accumulator permits essential reduction in the overall charge, which is particularly important for recording of deep waves.

The possible need of changing to areal (strip) observations requires, evidently, the use of more complex three-dimensional soundings in comparison with the usual linear soundings; waves from a given source are recorded simultaneously at several stations. Investigations have shown that with certain assumptions about the medium, it is possible to calculate the elements of the disposition of the boundary and the velocity parameters from a single sounding.

The array of the seismographs can be linear, as in the plain regions, if the deviation of the azimuth of the array from the direction of the source does not exceed 45°. Parameters of the waves in this case can be safely projected on to the normal. With larger deviations and large variations in the direction to the source, it becomes necessary to use areal instead of linear arrays; the construction of these must take account of the local conditions. In some cases it may be necessary to lay out the seismographs in rugged topography, introducing appropriate corrections during calculation of apparent velocities.

We may conclude from the above that inclusion of a stage of reconnaissance in a cycle of projects for study of the Earth's crust by seismic methods, makes it possible on the one hand to increase abruptly the productivity of operations and to carry out investigations in regions of difficult access (in this instance in mountainous regions), where the usual methods of positioning observations are not applicable, and on the other hand, to select most rationally regions for detailed study of one or another feature of the structure of the crust. Thus, carrying out reconnaissance investigations by the method of point soundings, in our opinion, is appropriate in the overwhelming majority of regions for soundly based planning of a system of detailed observations aimed at the study of actual structural objects in the crust (deep fractures, wedging of layers etc.)

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STRUCTURE OF THE EARTH'S CRUST AND UPPER MANTLE ALONG THE PROFILE IZHEVSK, ISHIM, KHANTY-MANSIISK, ENISEISK, BRATSK

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Abstract

Results of computation of complex geophysical data are discussed along the sublatitudinal profile Izhevsk--Sverdlovsk--Tumen--Ishim--Khanty-Mansiisk--Krasnoyarsk ridge. Deep seismic sounding was taken here as a basis for about 3000 km long.

As results of these investigations, main features of structure of the earth's crust and some elements of structure of the upper mantle have been found out.

On the basis of all the data some ideas are given about history of development of the earth's crust and upper mantle on a large area.

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During 1962-1965 the Urals Geological Administration and the Institute of Geophysics, Ural Branch of the Academy of Science, USSR, conducted deep seismic investigations on a profile Izhevsk--Sverdlovsk--Ishim. At the same time the Novosibirsk Geophysical Trust and the Institute of Geophysics and Geology of the Siberian Division of the Academy of Sciences USSR completed deep seismic soundings along a traverse Khanti-Mansiisk--Kolpashevo--Eniseisk and further to the east along the latitudinal elbow of the Angara River (fig. 1). Results of these two profiles of total extent 3000 km permits a view of the depth structure of the Earth's crust in a variety of tectonic regions: East Russia platform, Ural shield zone, West Siberian platform, Eniseisk trough and western margin of the Siberian platform (fig. 2).

On the Sverdlovsk profile continuous profiling deep seismic sounding was used, and waves excited by large industrial explosions [2] and converted waves from distant earthquakes were recorded at large distances. To the east of Khanti--Mansiisk a method of a differential systems of point seismic soundings was used [1], based on discrete correlations of elastic waves of various types (reflected, head, refracted). This method is designed for regional studies of basic boundaries of the earth's crust with separation of the coarse zones of deep fractures and structures with dimensions of 10 km or more horizontally.

For clarification of the possibility of elucidating the deep structure with smoothed details, part of the material from the Sverdlovsk profile was re-interpreted by the procedures of the method of point seismic soundings. The characteristics of the principal deep waves and the corresponding boundaries along both traverses were shown to be close, confirming the possibility and the reliability of separating the gross features of the structure using data of point observations. Only the principal boundaries were shown on the section along the Sverdlovsk profile; therefore with smoothed detail only the principal features of deep structure are elucidated. Both profiles are shown as a continuation of each other on a joint section, although their ends are separated by about 600 km along a meridian. Such a combination of the profiles is to some extent justified by the fact that the transfer is accomplished along the trend of the anomalies of the magnetic field (fig. 2). On the sections are shown the following principal boundaries.

Φ - refracting surface with boundary velocity 5 to 6.4 km/s; this corresponds to the surface of the folded basement of the West Siberian platform.

Boundary I, lying in the depth interval of 4-11 km, is not universally present; in various regions it has apparently a different geological nature. On a large part of the Sverdlovsk traverse this is ancient basement. The boundary velocity ranges from 6 to 6.9 km/s.

Surface II, at depths of 16 to 25 km, is provisionally identified with the surface of the "basaltic" layer. For the Sverdlovsk profile the boundary velocity averages from 7 to 7.6 km/s.

Upper mantle surface M, lies at a level 34 to 47 km deep. The velocity distribution of elastic waves along it varies from 7.9 to 8.4 km/s. In the middle part of the Sverdlovsk section boundaries were isolated within the upper mantle at depths from 55 to 75 km.

The axial part of the Urals and Eniseisk trough are characterised

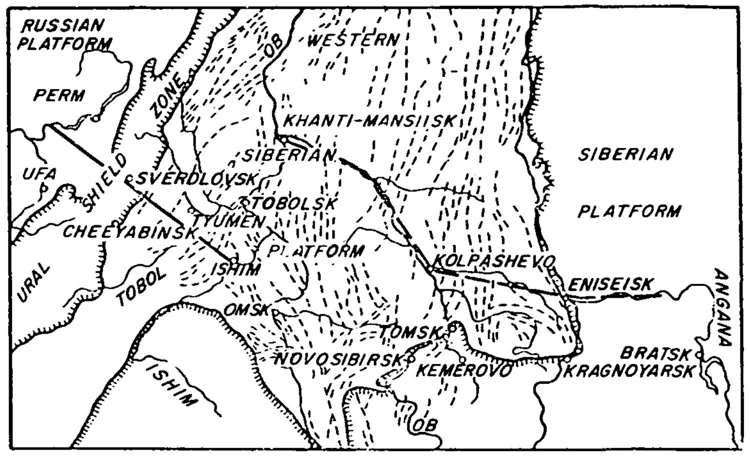

Fig. 1 PLAN OF THE DEEP SEISMIC SOUNDING PROFILE.

1 BOUNDARY OF PRINCIPAL TECTONIC STRUCTURES

2 URAL TRAVERSE

3 SIBERIAN TRAVERSE

4 AXIS OF THE MAGNETIC ANOMALY (ACCORDING TO L. Ya. PROVODNIKOV)

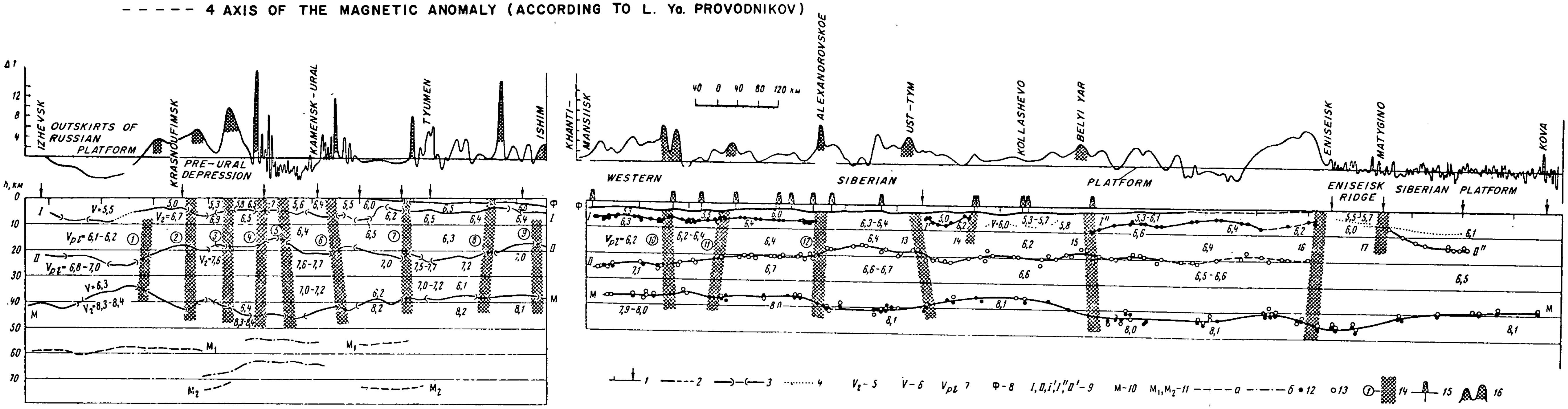

Fig. 2 CROSS-SECTION OF THE EARTH'S CRUST ALONG THE PROFILE EAST RUSSIAN PLATFORM-WEST SIBERIAN PLATFORM.

a FROM SEISMIC DATA
b FROM SEISMOLOGY (EARTHQUAKES)
1 LINE OF PROFILE WITH PROJECTIONS OF CROSS-HATCHED POINTS
2 SEISMIC BOUNDARIES
3 ZONE OF COMPLEX SEISMIC BOUNDARIES
4 LINE OF EQUAL VELOCITY FROM DATA OF REFRACTED WAVES
5 BOUNDARY VELOCITY (km/s)
6 AVERAGE VELOCITY TO A BOUNDARY (km/s)
7 LAYER VELOCITY (km/s)
8 PRE-JURASSIC SURFACE (PALAEOZOIC BASEMENT)
9 INTRA-CRUSTAL BOUNDARY
10 LOWER BOUNDARY OF THE EARTH'S CRUST (M SURFACE)
11 BOUNDARIES IN THE UPPER MANTLE
12 DEPTHS ACCORDING TO REFRACTED WAVES
13 DEPTHS ACCORDING TO REFLECTED WAVES (DATA OF POINT SEISMIC SOUNDINGS)
14 ZONES OF POSTULATED DEEP FRACTURES, SEPARATING BLOCKS OF THE EARTH'S CRUST
15 DEEP WELLS INTERSECTING BASEMENT
16 GRAPH OF THE NUCLEAR MAGNETIC FIELD

by the least persistence of the wave field and deep seismic boundaries, in comparison with the adjacent platforms.

The layered-blocky structure of the earth's crust is characteristic of the whole territory. Within each block the relief of the boundaries, the number of layers, their thickness and velocity of transmission of elastic waves, are relatively persistent. Abrupt changes in these parameters generally occur at the boundaries of the blocks, which serves as a basis for singling out here the steeply dipping zones of fractures, generally cutting through the whole thickness of the crust and penetrating into the upper mantle. The deep fractures are also marked by supplementary criteria, such as diffraction points established by detailed investigations of the Sverdlovsk section, and correlation of marked seismic features with linearly extended regional magnetic anomalies. The increased number of fractures fixed in the Sverdlovsk section in comparison with the eastern part of the traverse may be related to some extent to the different degree of detail of the seismic investigations. We examine features of the structure in the separate regions.

East Russian Platform

The principal boundaries of the earth's crust here are characterised by the following parameters. Depths to the M and K boundaries and the basement surface vary within the ranges: 34 to 43, 18 to 25, 4 to 9 km respectively. The stratified velocity in the "granite-gneiss" layer is 6.1 to 6.2 km/s, in the basaltic layer 6.7 to 6.8 km/s.

The principal structural subsurface features of the East Russian platform must be on the one hand, the conformity of the relief of the basement and K, and on the other hand, the reversal of their relief in that of M, which is expressed essentially by a change in the thickness of the basaltic layer (8 to 25 km).

Division of the eastern region of the Russian platform into three parts is proposed, i.e. the blocks: Upper Kamsk basin, Permsk - Bashkirsk arch, and the Pre-Ural downwarp.

The Upper Kamsk basin is characterized by depression of the basement to 8 or 9 km depth, depression of the basaltic layer, and uplift of the M boundary. It is separated from the Permsk-Bashkirsk arch by the Chernushinskii fracture zone, affecting the upper mantle (1 in fig. 2). The Permsk-Bashkirsk arch is characterized by uplift of the surface of the ancient basement to depths of 4 to 5 km, increase of the thickness of the basaltic layer from 8 to 10 up to 20 to 25 km, and depression of the M boundary.

The Pre-Ural depression is also bounded by deep fractures (2 and 3 in fig. 2). The surface of the basement here has a relatively local plunge and occurs at depths of 7 to 8 km. With this plunge is correlated a depression of the Konrad boundary, and a rise in the M boundary with an amplitude of 4 to 6 km.

For the eastern margin of the Russian platform, layering within the mantle is established. Practically horizontal reflection boundaries were identified at depths of 55 to 60 km.

The Ural Geosynclinal Province

The principal boundaries in the earth's crust here are characterized by the following features:

Depths to the M and K boundaries and the surface of the "granite-gneiss" complex vary within the limits: 37 to 47, 15 to 24, and 2 to 8 km. The boundary velocities here are higher: 8.2 to 8.4, 7 to 7.7, and 6.2 to 7.0 km/s respectively. For the outcropping Palaeozoic formation, a wide range of velocity is noted, from 4.5 to 7.0 km/s. This is related to the alternation of rocks - from sedimentary to igneous ultrabasic.

Geosynclinal features applying general to the Urals are the depression of the M surface ("mountain root"), uplift of the surface of the basaltic layer, and increased values of velocities of elastic waves.

The axial, and most contrasting, part of the Urals is the Tagilsko-Magnitogorsk depression and a large part of the East Ural uplift, forming a unique structure in the Earth's crust. Its characteristics are: a significant depression of the M boundary and uplift of the Konrad boundary, causing a general increase in thickness of the basaltic layer to 30 km; essentially increased velocities and density of the granite-gneiss layer, corresponding to the increase in saturation of its basic rocks; commonly a lack of clarity in tracing the M boundary, which suggests the possible presence of a crust-mantle zone. A characteristic feature of the axial part of the Ural geosynclinal province is also the increase in disruption of the Earth's crust, an increase in the amount of deep fracturing. In the upper mantle here is observed a convex boundary, whose relief is broken by the M surface. An increase in the density of the rocks of the upper mantle occurs under the Urals.

The western part of the Ural geosynclinal province has some distinctions from the axial part of the Urals, being expressed in the universal essentials here of the surface of the ancient basement of the Russian platform at a small depth: 3-5 km, relatively increased values of velocity for the upper part of the section, and decreased thickness of the basaltic layer. The western boundary of the Urals must be taken as the Mikhailovsk zone of deep fractures (3 in fig. 2).

The most valid place for the boundary of the Ural geosynclinal province is the zone of a series of blocks, through which the particular features of the Earth's crust pertaining to the Urals disappear. Thus, beyond the zone, descending to Kamyshlovsk (Kamensk-Ural) fractures (6 in fig. 2), a decrease can be noted in the thickness of the Earth's crust, density and velocity of elastic waves in it by comparison with the outcropping Urals. These parameters are close to those established for other regions of the West-Siberian platform. However, further to the east, in the western part of the Tyumensk-Kystanaisk trough, in a zone descending to the Tyumensk fracture (7 in fig. 2), a return of the Ural features is observed in the structure of the Earth's crust - an essential increase in velocity and density characteristics up to values close to the corresponding magnitudes for the Ural syncline. Further to the east the crust is again close to that studied in the region Khanti-Mansiisk (at any rate its lowest part). It must be noted that this zoning established by seismic data is correlated with the data of repeated levelling. Thus, if the Urals as a whole are characterized by a small uplift of the ancient surface, then the blocks examined are essentially being raised or sunken.

West Siberian Platform

The West Siberian platform has the broadest structure of the territory investigated; as a whole it has a somewhat diminished thickness of the Earth's crust (36 to 44 km) by comparison with the framework of its region. Minimal values are typical for the central, most submerged part of the platform.

The upper parts of the mantle within the boundaries of the platform are comprised of rocks with small variations in physical properties. The boundary velocity is contained in the range 7.9 to 8.1 km/s. From the results of combined interpretation of seismic and gravitational data it is impossible to assume significant changes in density in the upper parts of the mantle.

The earth's crust is divided by zones of fractures into blocks. Zones of relatively denser and lighter material in the earth's crust are marked out from a combination of seismic and gravitational data; these zones affect the whole of its thickness and are not reflected in the relief of the seismic boundaries. The changes in density do not exceed 0.05 g/cm^3; the corresponding changes in velocity in the earth's crust cannot be greater than 0.1 to 0.15 km/s.

Horizontal variability of the elastic properties of the rocks of the earth's crust, as in the Sverdlovsk profile, diminishes with increase in depth. The magnitude of the boundary velocities for the surface of the folded basement are characterized by a wide range of measurements: from 5 to 6.4 km/s. The average strata velocities in the "granitic" and "basaltic" layers lie within the respective ranges: 6.1 to 6.4 and 6.5 to 6.7 km/s, with a general tendency to decrease in the easterly direction.

The western province appears as a continuation of theUral folded zone under a cover of platform sedimentary Meso-Cainozoic, intersected by the Sverdlovsk profile. It has already been discussed.

The second broad province is intersected by the seismic profile within the limits of the latitudinal elbow of the Ob River, and is bounded in the east by the Omsk zone of deep fractures (12 on fig. 2). The thickness of the crust is minimal - 36 to 38 km. A distinguishing feature is the presence of sloping refracting boundaries with firm values of boundary velocity (6.2 to 6.4 km/s) at a level 6 to 8 km deep. The existence of such boundaries may indicate a relatively peaceful, conservative character for the development of the territory.

The next province is bounded by the Omsk and the Altai-Kuznetsk zones of deep fracture (12 and 15 in fig. 2). It is characterized by a more dissected relief of the deep boundaries and variability of the structure of the upper part of the consolidated crust. A block in the western part of the province is unusual; its particular structure is in many respects reminiscent of the Urals. At the surface of the basement here rocks with high velocity are found, the crust is thickened by 3 to 4 km in comparison with the adjacent regions, the basalt layer is thickened, but the granitic layer is diminished in thickness.

The eastern (Prienniseisk) part of the West Siberian plate has a thickened crust (up to 44 km). Within the Jurassic basement a refracting boundary is traced, being distinguished by abrupt oscillations in depth (4 to 11 km) and the values of the boundary velocity (6.2 to 6.6 km/s), which probably testify to its blocky character. These features suggest consideration of this seismic boundary as the surface of an ancient intensively metamorphosed basement, relatively weakly reworked by subsequent tectonic movements.

The earth's crust of the Eniseisk ridge thickens to 46 or 47 km, and does not have any clear extended boundaries within. The ridge is bounded on the east and the west by deep fractures. The average velocity throughout the whole thickness of the crust in the meridional direction was measured as 0.2 km/s more than in the latitudinal direction. No such difference was detected for the velocity through the upper mantle. These data, and the absence of persistent boundaries within the crust, indicate probably a broad development of longitudinal disjunction, being transmitted to substantial depths, but not affecting the upper mantle, where the physical properties apparently are substantially evened out.

The Siberian platform in the region of the latitudinal stretch of the Angara River is characterised by a crust of thickness 38 to 40 km. A reflecting boundary, rising towards the Eniseisk ridge, was established in the depth range 8-15 km. It apparently corresponds to the surface of the Archean foundation craton. The velocities of transmission of elastic waves in the upper half of the section of the earth's crust are from 0.3 to 0.5 km/s higher than in the Eastern Russian platform. An increase in saturation of the section of the crust in this region of the Siberian platform by rocks of basic composition may serve as a possible interpretation of this feature. The presence of traps supports this hypothesis.

Conclusions

I. As a result of the investigations the following principal features of the deep structure were established for a variety of tectonic zones:

1. The earth's crust and upper parts of the mantle of the platforms (Russian, West Siberian, and Siberian) are characterized by relatively persistent structure.

3. The folded provinces (Urals, Eniseisk Ridge) show naturally significantly more contrasting changes of physical parameters and morphology, both in the separate layers and boundaries, and for the crust as a whole in the presence of a closely spaced net of deep fractures.

4. The Urals appear to be the most anomalous zone. Increased values of velocity and density, and the presence of a mountain root, characterize its axial part, while a seismic horizon in the upper part of the mantle is uplifted.

II. Investigations must continue in the ways considered, using the method of point seismic soundings at the reconnaissance stage, so as to enable reduction in cost and increase in speed of the operations. Later, continuous profiling observations must be conducted over the more interesting areas. For investigations of the upper mantle it is expedient to make use widely of waves excited by large industrial explosions.

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N.N. Puzyrev

TWO-DIMENSIONAL TIME FIELDS FOR REFLECTED WAVES

Abstract

The theory of the time fields of monotype and exchange reflected waves is given for a boundary of arbitrary shape as a generalized system of hodographs of various types. The most important case is the field t(x, l) in which the calculation of times is related to a base point half-way between the source and receiver. It is shown that the vertical cross-section for such a field corresponds to its CDP hodograph. A new type of field is introduced for the first time, with fixed reflecting points, and the connection between the fields t(x, l) and τ (x, l) is established.

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The development of new methods of interpretation of the data from seismic surveys with the use of modern calculation techniques opens up the possibility of a substantial improvement in the geological effectiveness of the method. These possibilities have not yet been fully realized, in particular because of the inadequate development of the theoretical apparatus of interpretation.

In the majority of cases we are restricted to simple models of the media, representing boundaries as flat, while the velocities of transmission of elastic waves are constant in local regions of the space (method of average velocities). In practice in seismic prospecting and deep seismic sounding (DSS), a very complex situation often arises, when it becomes wrong to use an approximation of the media being studied by simple models. Therefore, to develop effective methods of interpretation for complex structural conditions, there is nearly always a need for more detail than has been used until now, to study features of the wave field for various models of the media, throwing light on the totality of various forms of real conditions.

In the present paper the time characteristics of reflected waves are examined on the basis of a special time field, representing its further generalization of the concept of the hodograph.

As is well-known, in the theory of interpretation of seismic prospecting and deep seismic sounding data, and also in earthquake seismology [1-6], one works with two-dimensional or three-dimensional hodographs. Nowadays with detailed investigations of the reflection method with the use of systems having considerable overlapping, it is all the more important to undertake examination of so-called common depth point (CDP) hodographs. Recently the dependence of the travel-time of reflected waves on the distance from source to receiver was presented for a symmetrical arrangement of these relative to some centre. Sometimes the term "hodograph" is used, not only for the curve $t_o(x)$ in the plane wave-front method [7], but also for examination of other types of time dependence (for example, differential hodograph, S-P hodograph in earthquake seismology, etc.). Expansion of the concept of a hodograph evidently occurs quite correctly, and this process may continue in the future, although it is absolutely necessary to establish a precise terminology for the various time dependences. However despite the various connotations of the concept of a "hodograph", it is always involved with the local transformation of the time field for a given model for fixed disposition of source, receiver, or other characteristic point (e.g. centre of baseline in the CDP method). In fact with various seismic investigations in the overwhelming majority of cases there is a combination of a large number of sources and receivers, and consequently, combinations (by systems) of hodographs. If we aim to investigate hodographs of a given type of waves for a specified model of the medium, then it is required to know in advance the disposition of the sources or the receivers. This very much hampers the theoretical investigations of complex models of the media, since the systems of observations are by no means always prescribed in advance, and often the theoretical analysis of the model is carried out in order to select the optimal disposition of the sources and receivers along a profile or over an area. In this connection it is desirable to use for the kinematics of the waves theoretical methods for which the time field being studied for the prescribed model of the medium should not be associated with the particular system of observations. We will examine primarily the special time-field $t(X, l)$ introduced in [8] and developed in [9], in the capacity of such a theoretical method. Having assigned such a field in analytical or graphical form, we can use very simple transformations to synthesize an arbitrarily prescribed system of hodographs for study of waves of any type. With this, for reflected monotype waves the vertical section of the field at any point X = const. represents its CDP hodograph.

In construction of the field $t(X, l)$ the times are counted relatively to the mid-points of the base-line from source to receiver. In certain cases it is expedient to calculate the times relative to end-points of the baselines - source or receiver. We denote the corresponding fields by $t_o(X, l)$ and $t_s(X, l)$.

Together with the three aspects of time fields two further types are examined, in which the position of the reflecting point is fixed at a given point of the profile. As distinct from the fields $t(X, 1)$, $t_o(X, 1)$, and $t_s(X, 1)$, it is possible to study this field directly only as a theoretical scheme. Problems of transfer from one field to another are considered.

The equations of the time field are given only for the case of a uniform medium with arbitrary shape of the boundary. This was done, not only with the aim of simplifying the exposition, but also because it turns out very often to be quite acceptable to regard the medium as locally uniform within specific limits.

Time field with time counted from a fixed position

(a) Monotype reflected waves. Let us consider monotype reflected waves (PP or SS) for a curved boundary, with a given equation: $z = z(x)$. Let us fix certain reflection points $M(x, z)$ on the boundary (fig. 1). Let us call the angle of reflection α, and draw in the corresponding incident and reflected rays - AM and BM. With monotype waves, we may in principle take either say as the incident one, and the other as reflected. For the purposes of the determination we will take as source throughout the point A, situated closer to the selected origin of co-ordinates.

The distance $\overline{AB}$ between source and receiver will be called 1, and the length of the normal MC will be called N. The total path of the wave will for convenience be called either vt or 2D. Obviously, as $1 \rightarrow 0$, $D \rightarrow N$.

Let us consider firstly the field $t(X, 1)$, where X denotes the abscissa of the mid-point of the baseline 1 in the selected system of co-ordinates.

The expression for the length of the normal has the form:

$$N = z\sqrt{1 + \left(\frac{dz}{dx}\right)^2} \qquad (1)$$

in which $\frac{dz}{dx} = \tan \phi$, where ϕ is the angle of inclination of the boundary at the point M.

The angle of reflection α is connected with 1, D, and ϕ by the known relation [3]:

$$\sin \alpha = \frac{1}{2D} \cos \phi \qquad (2)$$

In fig. 1 we have

$$vt = 2D = N[\sec(\alpha - \phi) + \sec(\alpha + \phi) \ldots\ldots] \cos \phi$$

therefore,

$$D = \frac{N \cos \alpha \cos^2 \phi}{\cos^2 \phi - \sin^2 \alpha} \qquad (2')$$

From (2') after transformation we finally obtain the expression for D = vt/2 in terms of the parameter x (abscissa of the point of reflection):

$$D^2 = \frac{1^2}{4} + \frac{N}{2} \left(\sqrt{N^2 + 1^2 \sin^2 \phi} + N \right) \qquad (3)$$

Here N and ϕ are expressed in terms of x in accordance with the relations presented above.

D will depend on the mean point C' of the baseline 1 (fig. 1) with abscissa X.

From the diagram we have:

$$X = x + z \tan \phi + \Delta c, \text{ where } \Delta c = CC'$$

Δc can be readily expressed in terms of D and 1 in the following form:

$$\Delta c = \frac{1}{2} - \frac{N1}{\sqrt{4D^2 - 1^2 \cos^2 \phi} + 1 \sin \phi}$$

From (2') after transformation we find:

$$\Delta c = \frac{1}{2 \sin \phi} \left(\sqrt{N^2 + 1^2 \sin^2 \phi} - N \right) \qquad (4)$$

In this way, we obtain for X the following more general expression:

$$X = x + z \tan \phi + \frac{1}{2 \sin \phi} (\sqrt{N^2 + l^2 \sin^2 \phi} - N) \qquad (5)$$

Equation (3) and (5) in conjunction present in parametric form the field D(X, l) or the corresponding t(X, l) for a given boundary z = z(x). They can be written in the following more expanded form:

$$X = x + z \frac{dz}{dx} + \frac{1}{2\, dz/dx} \left\{ \sqrt{z^2 \left[1 + \left(\frac{dz}{dx}\right)^2\right]^2 + l^2 \left(\frac{dz}{dx}\right)^2} - z \left[1 + \left(\frac{dz}{dx}\right)^2\right] \right\} \qquad (5')$$

$$D^2 = \frac{l^2}{4} + \frac{z}{2} \left\{ \sqrt{z^2 \left[1 + \left(\frac{dz}{dx}\right)^2\right]^2 + l^2 \left(\frac{dz}{dx}\right)^2} + z \left[1 + \left(\frac{dz}{dx}\right)^2\right] \right\} \qquad (3')$$

For l = 0 we will have

$$X = x + z\, dz/dx$$

$$D = N = Z \sqrt{1 + \left(\frac{dz}{dx}\right)^2} \qquad (6)$$

The latter equation has been presented formerly in reference [7].

In the particular case of a rectilinear inclined boundary $z = z_o + x \tan \phi$ it is easy to eliminate the parameter x, with the result:

$$t = \frac{1}{v} \sqrt{l^2 \cos^2 \phi + 4(X \sin \phi + z_o \cos \phi)^2} \qquad (7)$$

From this it is seen that only for l = 0 is the curve t(X) represented by a straight line, while for arbitrary l it may depart somewhat from a straight line (fig. 2), being concave upwards (i.e. in the direction of positive values of the ordinate of the field).

For a curvilinear boundary the field t(X, l), or the corresponding D(X, l), can have various forms depending on the actual conditions. The existence of a loop for the line l = const. is characteristic for a concave boundary, as can be seen in Fig. 3a, where the field has been calculated for a boundary of the concave form $z = z_o + \Delta z \; e^{-k^2 z^2}$, where Δz is the amplitude of a synclinal "fold", while the coefficient k characterizes the width of the depression.

A particular case of the reflected wave, in the kinematic sense, is diffraction from the point $M(x_o, z_o)$. Using fig. 1, it is not difficult to write the corresponding expression for the time field t(X, l):

$$vt = 2D = \sqrt{(X - x_o - \frac{l}{2})^2 + z_o^2} + \sqrt{(X - x_o + \frac{l}{2})^2 + z_o^2} \qquad (8)$$

Its general form is displayed in fig. 4.

It must be noted that the field t(X, l) may be considered as a field of CDP hodographs in any vertical section of a profile, i.e. with a fixed abscissa X of the profile. In order to obtain the equation of the CDP hodograph at the fixed point of the profile, it is enough to substitute X = const. in (3) and (5). As a result, after certain transformations we obtain:

$$\left.\begin{aligned} l^2/4 &= (X - x - z \tan \phi)\,(X - x + z \cot \phi) \\ D^2 &= l^2/4 + \frac{z}{\cos \phi}\,[(X-x) \sin \phi + z \cos \phi] \end{aligned}\right\} \qquad (9)$$

Or in another form

$$l^2/4 = (X - x - z\,\frac{dz}{dx})\;(X - x + z/\frac{dz}{dx})$$

$$D^2 = l^2/4 + z[(X - x)\,\frac{dz}{dx} + z] \qquad (9')$$

Using these general formulae, it is not difficult, for example, to obtain for a flat boundary $z = z_o + x \tan \phi$, the well-known equation for the CDP hodograph, which in our notation has the form:

$$1/4\, v^2 t^2 = D^2 = \frac{l^2}{4} \cos^2 \phi + h_m^2 \qquad (10)$$

where $h_m = z_o \cos \phi + X \sin \phi$ is the depth along normal at the selected point of the profile from the abscissa X.

For diffracted waves the expression (8) with X = const. simultaneously gives the equation of the CDP hodograph t(l).

In fig. 3b the CDP hodograph is shown for the field of fig. 3a. In the region of the loop three CDP hodographs will correspond to a given

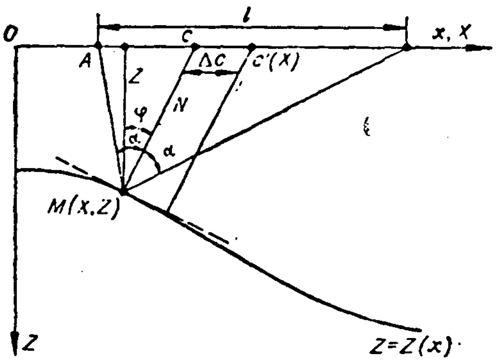

Fig.1 TOWARDS THE DERIVATION OF THE TIME FIELD OF MONOTYPE REFLECTED WAVES.

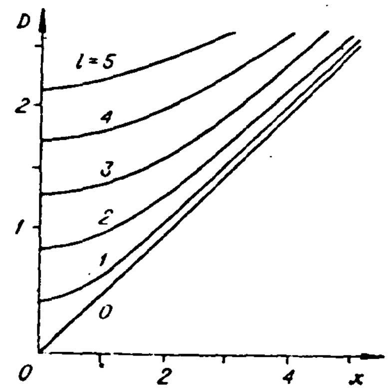

Fig. 2 THE FIELD $t(X,l)$ OF MONOTYPE REFLECTED WAVES FOR A RECTILINEAR BOUNDARY (z_o= 1km; Ø=30; v=2km/s)

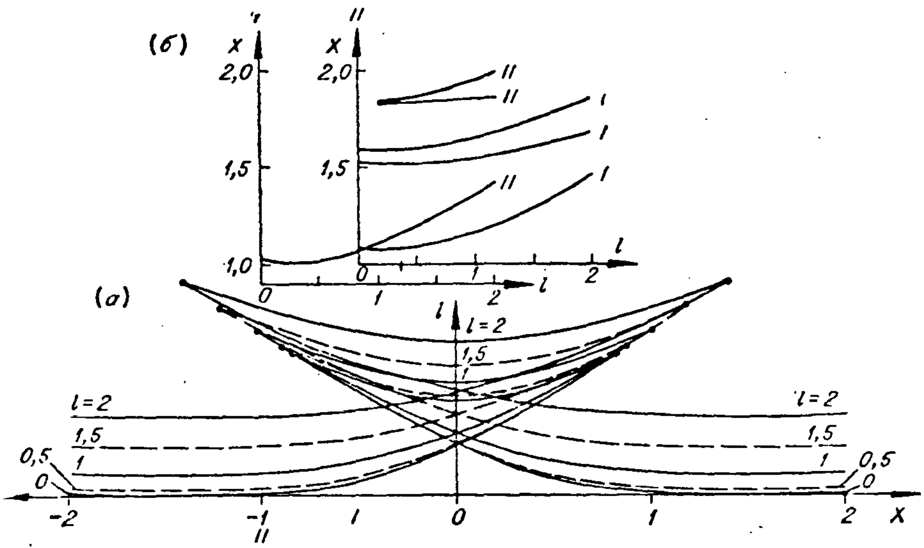

Fig. 3 (a) TIME FIELD $t(X,l)$ FOR A BELL-SHAPED BOUNDARY:

$$z=z_o+\Delta z e^{-k^2x^2}$$

(z_o= 1km; Δz=0·5km; k=2km^{-1})

(b) CDP HODOGRAPHS FOR SECTIONS I AND II OF THE FIELD $t(X,l)$

NOTE: THE t AXIS IS THE ORDINATE ON ALL GRAPHS

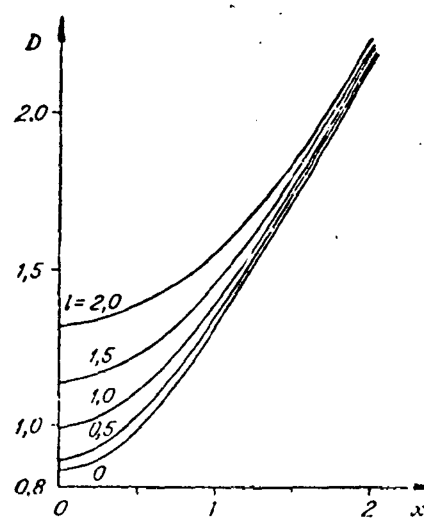

Fig. 4 THE FIELD $t(X,l)$ FOR DIFFRACTED WAVES (z_o= 1km).

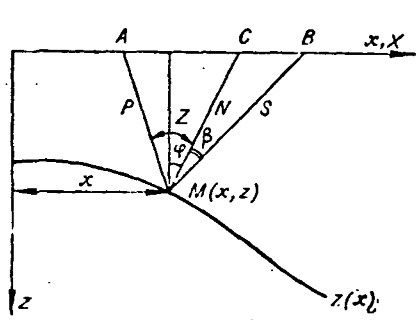

Fig. 5 TOWARDS THE DERIVATION OF THE EQUATION OF THE TIME FIELD OF CONVERTED REFLECTED WAVES.

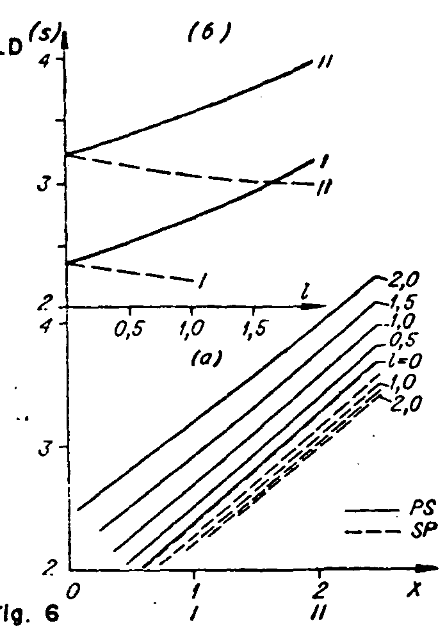

Fig. 6

(a) THE TIME FIELD $t(X,l)$ FOR CONVERTED REFLECTED WAVES PS AND SP FOR A RECTILINEAR BOUNDARY (z_o= 1km; Ø=30°; γ=0·3)

(b) CDP HODOGRAPH FOR SECTIONS I AND II OF THE FIELD $t(X,l)$

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X = const. As shown by calculation, in a number of cases the CDP hodograph can depart strongly from the hyperbolic form.

Let us write now the equation of the linear hodograph with source located at $X = X_o$. Starting from the method of construction of the field D(X, l), being based on the fact that the value of D at each step is related to the mid-point of the baseline from source to receiver, we proceed to the conclusion that the desired equation is obtained by substituting $X_o + l/2$ instead of X in the left side of (5), or correspondingly in (5'). If now this equation is solved in terms of l and (3) is incorporated in it, then we obtain as a result the parametric dependence of the equation of the hodograph for a source at the point with abscissa X_o. After the required transformations we write:

$$l = \frac{[z \tan \phi - (X_o - x)] \; [z \cot \phi + (X_o - x)]}{X_o - x + \frac{z}{2}(\cot \phi - \tan \phi)}$$

$$D^2 = \frac{l^2}{4} \; \frac{(X_o - x)^2 + z^2}{[z \tan \phi - (X_o - x)]^2} \qquad (11)$$

We note that from these equations we can obtain the corresponding equation of the hodograph, which has been presented in [3] for the particular case when the origin of co-ordinates coincides with the source. For this, it is necessary to substitute $x_1 + X_o$ for X in (11), where x_1 is the value of the parameter in the co-ordinate system whose origin coincides with the source, and when the boundary is represented by an equation of the form: $z = z(x_1)$. Then in the final stage it is possible to return again to the usual notation for the parameters.

As well as the field t(X, l) discussed, for which the times are measured relatively to the mid-point of the baseline l, the fields $t_o(X, l)$ and $t_s(l)$, in which the times are measured relatively to the source and receiving points respectively may be readily constructed.

Expression (3) for the transmission time of the waves remains unaltered. The value for the abscissa of the field is written in a very simple form:

$$X_o = X - \frac{l}{2} \; ; \quad X_s = X + \frac{l}{2},$$

where X is determined from formula (5) or (5').

In the case of a flat boundary $z = z_o + x \tan \phi$, using the usual notation for abscissa of the field,

$$v^2 t_o^2 = l^2 + 4l(z_o \cos \phi + X \sin \phi) \sin \phi + 4(z_o \cos \phi + X \sin \phi)^2$$

$$v^2 t_s^2 = l^2 + 4l[z_o \cos \phi + (X-l) \sin \phi] \sin \phi + 4[z_o \cos \phi + (X-l) \sin \phi]^2$$

The relation between the three aspects of the field for any shape of the boundary has the form:

$$t(X; l) - t_o(X - \frac{l}{2}; l) = t_s(X + \frac{l}{2}; l)$$

This enables one to transfer very simply from one field to the other. The vertical section of the field $t_o(X, l)$ and $t_s(X, l)$ for given X will be the usual hodograph for the variable l with fixed source or receiver. These will be distinguished by the direction of measurement of l.

(b) Converted PS and SP waves. For converted reflected waves, as is well known, the principle of reciprocity is not fulfilled. In this case it is necessary for full characterisation of the time field of one or the other wave, to consider for any specified model of the medium two fields differing from each other by the interchanged positions of source and receiver. In other words, for a given position of a source-receiver pair, in order to characterize completely the time field of a given converted wave, knowledge is required simultaneously along the chosen direction of both the field of the wave under consideration (let us take it as PS), and the field of the reflected wave (i.e. SP). In relation to this, formulae for calculation of both types of fields - PS and SP - are deduced below for a specified equation of the boundary $z = z(x)$. But before doing this we resolve the important question of the choice of a reference point for time in the case of converted waves.

As noted before, the time field $t(X, l)$ with measurements referred to the centre may be regarded simultaneously as the field of the CDP hodograph, since the latter is represented by the vertical section of the field for a fixed point on the X-axis. Then it is well-known that the common reflecting point in the case of monotype waves will occur only for horizontal layering of the boundary in a certain region of location of the point of reflection. If the boundary is inclined, then the reflection point moves along the boundary according to the extent of change in l, and the interval of movement can be quite significant, particularly with a large range of change in l, and significant curvatures. For waves of the converted type another position will occur, and for horizontal layering the projection of the reflecting point will be shifted relatively to the mid-point of the source-receiver approximately by the quantity:

$$\Delta S_o = \frac{l}{2} \frac{1-\gamma}{1+\gamma} \qquad (12)$$

For inclined layering the approximate expression for ΔS, obtained by using the law of tangents [10], takes the form:

$$\Delta S = \frac{l}{2} \; \frac{1-\gamma-\frac{l}{2h}(1+\gamma)\sin\phi}{1+\gamma-\frac{l}{2h}(1-\gamma)\sin\phi} \cos\phi \qquad (12')$$

Analysis of this expression, and also rigorous calculations, show that for large angles of inclination ΔS of a converted wave can be of very small amplitude. From this it follows that construction of the field t(X, l) for converted waves with time measurement relative to the centre of the baseline between source and receiver appears to be justified, although not always (e.g. for SP waves in the down-dip direction ΔS can be very large). Taking into account the fact that processing of given converted and monotype waves is usually carried out simultaneously, one may reach conclusions about the feasibility of examination of the fields t(X, l) with measurements referred to the centre of the baseline also for converted (PS and SP) waves.

For deduction of the corresponding equations we refer to fig. 5.

For PS waves we write the following relations:

$$\left.\begin{aligned} X_A &= x - z\tan(\alpha-\phi) \\ X_B &= x + z\tan[\arcsin(\gamma\sin\alpha)+\phi] \\ l = X_B - X_A &= z\{\tan(\alpha-\phi) + \tan[\arcsin(\gamma\sin\alpha)+\phi]\} \\ X = \tfrac{1}{2}(X_A+X_B) &= x + \frac{z}{2}\{\tan[\arcsin(\gamma\sin\alpha)+\phi] - \tan(\alpha-\phi)\} \end{aligned}\right\} \qquad (13)$$

$$\left.\begin{aligned} t_{ps} &= \frac{1}{V_p}\left[\sqrt{(x - X_A)^2 + z^2} + \frac{1}{\gamma}\sqrt{(X_B - x)^2 + z^2}\right] \\ \sin\beta &= \gamma \sin\alpha \end{aligned}\right\}$$

As before, the quantity x is the principal parameter, while α and β are subsidiary.

The field t(X, l) is calculated for a specified equation of the boundary z = z(x) in the following way.

From the third equation of the system (13), taking into account that $\tan \phi = dz/dx$, the angle α is found for specified values of x and l, and then $\beta = \arcsin(\gamma \sin\alpha)$. This equation is transcendental even for a flat boundary, and therefore it is solved numerically on a computer. Calculating next X_A and X_B it is easy to find the principal quantities X and t_{ps} from the formulae presented in the system (13).

The analogous formulae for SP waves are not presented here.

In fig. 6 a and b, the time fields and the CDP hodographs of PS and SP waves are presented for the particular case of a rectilinear inclined boundary $z = z_o + x \tan \phi$. It may be seen that SP and PS fields in the case of an inclined layer differ very markedly from each other. The CDP hodographs differ even more; for SP waves (or correspondingly PS with interchange of source and receiver) these lose the hyperbolic shape, and can be reversed to convex upwards.

As with the monotype waves, the fields $t_o(X, l)$ and $t_s(X, l)$ may be introduced also for converted waves; we will not dwell on this here.

<u>Fields of fixed reflection points $\tau(X, l)$</u>

For construction of the field t(X, l) as noted already above, the times of arrival of the wave for any values of l are referred to the mid-point of the baseline. In this instance, in a specified section of the field for X = const, i.e. along a CDP hodograph, different values of the co-ordinates of the reflecting point correspond to different values of l. Owing to this, the abscissa X of the time field in equation (5) turns out to be dependent on the length l of the baseline for given co-ordinates of the reflecting point (x, z). In complex structural conditions this shows that local elements of the field and the CDP hodograph will depend on the shape of the reflecting boundary.

Because of this, it is desirable to examine theoretically other types of field, in which the position of the readings of time depends only on the local elements of the reflecting boundary. For this we restrict ourselves to monotype waves only. We note that the right hand part of equation (5) consists of three terms. If we discard the latter two terms and assume X = x, then this will imply that during construction of the field the measurements of time are referred to the epicentres of the point of reflection. We find also the field τ_z (X, 1). Corresponding to (3') its equation will take the form:

$$(v\tau_z)^2 = 1^2 + 2z\left\{\sqrt{z^2\left[1 + \left(\frac{dz}{dx}\right)^2\right] + 1^2\left(\frac{dz}{dx}\right)^2} + z\left[1 + \left(\frac{dz}{dx}\right)^2\right]\right. \quad (14)$$

where we put x = X after differentiation. For the special case of a rectilinear boundary:

$$v^2\tau_z^2 = 1^2 + \frac{2z}{\cos 2\phi}\left(z + \sqrt{z^2 + \frac{1^2}{4}\sin^2 2\phi}\right), \quad (14')$$

where

$$z = z_o + X \tan\phi$$

For fixed X and variable 1 the expression (14') will represent the equation of the hodograph $\tau_z(1)$, being analogous to the CDP hodograph.

If we discard the last term in (5) and thus relate the time determined from expressions (3) or (3') to the point of emergence of the normal to the line of observations with the abscissa:

$$X = x + z\tan\phi \quad (15)$$

then we obtain another field - τ_N(X, 1), whose parametric equation is determined in the form:

$$X = x + z\frac{dz}{dx},$$

$$v^2\tau_N^2 = 1^2 + 2z\left\{\sqrt{z^2\left[1 + \left(\frac{dz}{dx}\right)^2\right] + 1^2\left(\frac{dz}{dx}\right)^2} + z\left[1 + \left(\frac{dz}{dx}\right)^2\right]\right\} \quad (16)$$

In the case of a rectilinear boundary, after eliminating the parameter x,

$$v^2\tau_N^2 = l^2 + 2\,(X \tan \phi + z_o) \cos^2 \phi\left[\sqrt{(X \tan \phi + z_o)^2 + l^2 \tan^2 \phi} + X \tan\phi + z_o\right] \qquad (16')$$

If we take into consideration the depth along the normal at the point of reflection (N = h), then we obtain:

$$v^2\tau_N^2 = l^2 + 2h(h + \sqrt{h^2 + l^2 \sin^2 \phi}\,) \qquad (16'')$$

We illustrate the passage from one field to another by the example of the transformation of t(X, l), into τ_N(X, l) in the case of a flat boundary. For this we need to know the gradients $\frac{\partial t}{\partial l}$ and $\frac{\partial t}{\partial x}$ of the field, the expressions for which, in accordance with (7), have the form:

$$\frac{\partial t}{\partial l} = \frac{l \cos^2 \phi}{v\sqrt{l^2 \cos^2 \phi + 4(X \sin \phi + z_o \cos \phi)^2}} \qquad (17)$$

$$\frac{\partial t}{\partial x} = \frac{4(X \sin \phi + z_o \cos \phi) \sin \phi}{v\sqrt{l^2 \cos^2 \phi + 4(X \sin \phi + z_o \cos \phi)^2}} \qquad (17')$$

From equations (7), (17) and (17'), it is not difficult to determine the unknown quantities ϕ, z_o and v. Substituting these now in (16'), we find the actual relation between t(X, l), and $_N$(X, l). After carrying out the required transformations we find the relation between the times $_N$ and t with fixed values of X and l:

$$\tau_N^2 = ½t.\bar{t}\ \frac{1}{2}\left[\frac{4\bar{t}\frac{\partial t}{\partial l} + l\left(\frac{\partial t}{\partial x}\right)^2}{\bar{t}^2} + 1 + \sqrt{1 + \frac{l^2}{\bar{t}^2}\left(\frac{\partial t}{\partial l}\right)^2}\ \right] \qquad (18)$$

where $\bar{t} = t - l\frac{\partial t}{\partial l}$

It is easy to see that for l = 0, and also for $\frac{\partial t}{\partial x} = 0$, the obvious condition $\tau_N = t$ is satisfied.

The analogous relation between the field t(X, 1) and τ_z(X, 1) has the form:

$$\tau_z^2 = \frac{At1}{4\bar{t}} \left\{ 1 + \frac{A}{81 \left(\frac{\partial t}{\partial 1}\right)^2} \left[1 + \sqrt{1 + \frac{16\, 1^2 \left(\frac{\partial t}{\partial 1}\right)^2 \left(\frac{\partial t}{\partial x}\right)^2}{A^2}} \right] \right\} \qquad (19)$$

where

$$A = 4\bar{t}\,\frac{\partial t}{\partial 1} + 1\,\left(\frac{dt}{dx}\right)^2 \; ; \qquad \bar{t} = t - 1\frac{\partial t}{\partial 1}$$

Other easier methods may be used for conversion of t(X, 1) into τ_N(X, 1) and τ_z(X, 1).

It follows from (5) and fig (1) that it is enough to use for this the general equation

$$t(X,1) = \tau_N(X - \Delta c; 1),$$

where Δc is determined from expression (4).

For a flat boundary, using formulae (17) and (17') for the gradients, we find

$$\Delta c = 1^2 \frac{\partial t}{\partial x} \Big/ \left[4t\left(1 - \frac{1}{t} \cdot \frac{\partial t}{\partial 1}\right) \right] \qquad (20)$$

Here Δc is always offset from the mid-point of the baseline 1 in the direction of increase, which is easily found from the value of the horizontal gradient of the field $\partial t/\partial x$.

The field τ_z(X, 1) can be conveniently obtained from the general relation

$$t(X; 1) = \tau_z(X - \Delta X; 1),$$

where $\Delta X = \Delta c + N \sin \emptyset$

For a flat boundary

$$\Delta X = \frac{1t}{\sqrt{41\left(t - 1\frac{\partial t}{\partial 1}\right)\frac{\partial t}{\partial 1} + 1^2 \left(\frac{\partial t}{\partial x}\right)^2}} \qquad (21)$$

In the general case of a curved boundary, not superimposed on the previous boundary being determined, the transformation is difficult to carry out. In practice, obviously, it is quite satisfactory as a second approximation to the actual case, to assume that the reflecting boundary is locally a curve of the second order (hyperbola, arc of a circle, etc), and to solve the problem of transformation of the field by a numerical method on the computer, using the general formula quoted above.

In conclusion, we remark that use of the time field instead of the traditional hodographs or similar methods, in our opinion, opens up new possibilities of comprehensive analysis of experimentally obtained data, and of improvement on this basis of methods of interpretation of data from seismic prospecting by the method of reflected waves. The main of advantages of operating with fields are:

1. The possibility of obtaining a clear picture of the main features of the medium being studied, from the direct form of a field for one or several boundaries; this allows a more detailed choice of a model of the object being studied.

2. For treatment of the data of a local portion of the profile (for example for treatment of a CDP hodograph) there is the possibility of taking into account the values of gradients of the field, and of obtaining from these more accurate data on the parameters of the medium.

3. The time fields permit a wide range of possibilities for various methods of transformation of the information obtained, which will undoubtedly promote new improved methods of interpretation, in particular with wide use of computing techniques.

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ON INVARIANT TIME FIELDS

Abstract

A general solution is given for the inverse kinematic problem for the fields t (x, l) applied to reflected, diffracted, and head waves, assuming homogeneous media and simple geometry of the boundaries. Invariant expressions of a rather simple form are derived; these may be kept constant for the whole field (general invariants) or for the field at a definite set of points (particular invariants).

These invariant expressions can be used for analyzing time fields which have been observed experimentally.

The conception of the time field t (x, l) was first introduced in reference [1], and was further developed in [2 to 4]. There it was shown that, given local parameters of the field, the parameters of the layering and the velocity could be found at fixed points on the profile. However, for solution of the inverse problem, as has been shown by analysis, it is necessary to use not all the local parameters, but only those of a certain set. This indicates that the parameters of the field at separate points are interrelated; it is important to bear this in mind not only in constructing the field from experimental data, but also during interpretation.

Recently invariant relations between the parameters were given particular attention, i.e. those relations which are invariant either for the whole region of the field, or for a selected set of points of the field, including a given section x = constant, i.e. the longitudinal CDP hodograph.

The following four quantities are used as 'local parameters of the time field: the baseline from source to receiver (l), the time of arrival of the waves (t), and its gradient with respect to x or l, i.e.

$$\theta = \partial t/\partial x$$
$$\eta = \partial t/\partial l$$

Thus, if the equation of the time field of any wave is known for a given model of the medium, then for an arbitrary point of this field we can write three corresponding equations, i.e.

$$t\ (x,\ l, \alpha_i) = 0;\ \theta(x,\ l, \alpha_i) = 0;\ \eta\ (x,\ l, \alpha_i) = 0$$

where α_i is a set of parameters of the model.

Reflected waves

Let us take the boundary to be flat, and specified in the form:

$$z = z_o + x \tan \phi$$

and the overlying medium as having uniform velocity of propagation V for a monotype wave (longitudinal or transverse). In accordance with [4], the equation of the field takes the form:

$$t = \frac{1}{V} \sqrt{l^2 \cos^2\phi + 4\ (x \sin \phi + z_o \cos \phi)^2} \qquad (1)$$

For an arbitrary point we may write

$$\left.\begin{aligned} Vt &= \sqrt{l^2 \cos^2 \phi + R^2} \\ V\theta &= 2\ R \sin \phi \ / \sqrt{l^2 \cos^2\phi + R^2} \\ V\eta &= l \cos^2 \phi \ / \sqrt{l^2 \cos^2\phi + R^2} \end{aligned}\right\} \qquad (2)$$

where we introduce

$$R = 2\ (x \sin \phi + z_o \cos \phi)$$

This quantity is equal to double the depth to the boundary along the normal to it through a given point on the profile, and in this case plays the role of an intermediate parameter.

By solving this system of equations, all the unknown parameters V, $\emptyset$, and R, may be expressed in terms of l, t, θ and η , while the first two of them remain unknown for all models, and the latter only for a fixed section x = const.

The corresponding formulae take the form:

$$V^2 = \frac{4t\ (t - l\eta)}{t^2\theta^2 + \frac{4t\eta}{l}\ t\ (t-l\eta)} \tag{3}$$

$$\cos^2\emptyset = \frac{4t\ (t-l\eta)}{t^2\theta^2 + 4\frac{t\eta}{l}t\ (t-l\eta)}\ \frac{t\eta}{l} \tag{4}$$

$$R^2 = \frac{4t^2\ (t-l\eta)^2}{t^2\theta^2 + 4\ \frac{t\eta}{l}\ t\ (t-l\eta)} \tag{5}$$

The expressions on the right hand sides could in principle be called invariants of the field; whereas (3) and (4) are general (i.e. valid for any points of the field), (5) is particular, being fulfilled only for a given x = const. However, such a determination of the invariants would be unreasonable, inasmuch as the corresponding expressions are quite complex. Moreover the general form of formulae (3), (4), and (5) is such that each of them can be constructed from simpler components, namely $t\eta/ l$, $t\ \theta$, $t\ (t-l\eta)$. This satisfies us that it is possible to construct much more simply not only the general, but also the particular, invariants.

Detailed analysis of the system (2) leads to the conclusion that it is appropriate to study four invariants as basic:

$$I_1 = t\eta/l = C_1 \tag{6}$$

$$I_2 = (t-l\eta)/\ (t\theta^2) = C_2 \tag{7}$$

$$I_3 = t\theta = C_3 \tag{8}$$

$$I_4 = t(t-l\eta) = C_4 \tag{9}$$

The first two of these are general, while the latter two are particular. From this

$$I_2 = I_4/I_3^2$$

and consequently only three of the invariants can be taken as independent. The constants C_i on the right hand side of the above formulae have the following values:

$$C_1 = \cos^2\phi/V^2$$

$$C_2 = V^2/(4\sin^2\phi)$$

$$C_3 = 2R\sin\phi/V^2$$

$$C_4 = R^2/V^2$$

By calculation of the parameters introduced, formulae (3), (4) and (5) may be written in the following form:

$$V^2 = 4I_2/(1 + 4I_1I_2) \quad (3')$$

$$\cos^2\phi = 4I_1I_2/(1 + 4I_1I_2) \quad (4')$$

$$R^2 = 4h_m^2 = 4I_2I_4/(1 + 4I_1I_2) \quad (5')$$

where h_m is the depth to the boundary along the normal through the given point.

We note that recently it is evidently very important in the treatment of experimental data to have the invariant I_3, which can be written also in the form:

$$t_o\,\Delta T_o = (t_o + \Delta t)\Delta T_i \quad (8')$$

where $\Delta t = t - t_o$, and ΔT_o and ΔT_i are the increments in time for an increment in baseline Δx on the time sections constructed for $l=0$ and $l=l_i$.

Often it is convenient [1,2] to study the field of reflected waves in quadratic coordinates - $t^2(x,l^2)$. In this case, instead of (2) we have the following system:

$$V^2\tau = \lambda\cos^2\phi + R^2$$

$$V^2\xi = 4R\sin\phi \quad (2')$$

$$V^2\rho = \cos^2\phi$$

where $\tau = t^2$; $\lambda = l^2$; $\xi = \frac{\partial\tau}{\partial x}$; $\rho = \frac{\partial\tau}{\partial\lambda}$

The corresponding invariants have the form:

$$I_1 = \rho = C_1$$

$$I_2 = (\tau - \lambda\rho)/\xi^2 = C_2/4$$

$$I_3 = \xi = 2C_3$$

$$I_4 = \tau - \lambda\rho = C_4$$

In principle, it is possible to study, on the basis of the source equations given in [4], the corresponding invariants for fields of other types, in particular $\tau_n(x,l)$, $\tau_z(x,l)$, on which we do not dwell in the present paper.

Diffracted waves

In accordance with (4), the equation of the field has the form

$$t = \left[\sqrt{(x - x_o - \tfrac{1}{2}l)^2 + z_o^2} + \sqrt{(x - x_o + \tfrac{1}{2}l)^2 + z_o^2}\right]/V \quad (9)$$

where x_o, z_o are coordinates of the point of diffraction.

For our purposes, without loss of generality of the solution, we can substitute $x_o=0$, i.e., take the origin as the epicentre of the point of diffraction. Then, adding to (9) the two equations for the gradients, we write:

$$Vt = \sqrt{(x - \tfrac{1}{2}l)^2 + z_o^2} + \sqrt{(x + \tfrac{1}{2}l)^2 + z_o^2}$$

$$V\theta = \frac{x - \tfrac{1}{2}l}{\sqrt{(x + \tfrac{1}{2}l)^2 + z_o^2}} + \frac{x + \tfrac{1}{2}l}{\sqrt{(x + \tfrac{1}{2}l)^2 + z_o^2}} \quad (10)$$

$$2V\eta = \frac{x + \tfrac{1}{2}l}{\sqrt{(x + \tfrac{1}{2}l)^2 + z_o^2}} - \frac{x - \tfrac{1}{2}l}{\sqrt{(x - \tfrac{1}{2}l)^2 + z_o^2}}$$

We solve this system for V, z_o, x. After the appropriate transformation we eventually get:

$$V^2 = \frac{4(l/t)\,(1 - l\eta/t)}{4\eta + (l/t)\,(\theta^2 - 4\eta^2)} \quad (11)$$

$$z_o = l/\left[(\theta_o + 2\eta)/\sqrt{4/V^2 - (\theta + 2\eta)^2} - (\theta - 2\eta)/\sqrt{4/V^2 - (\theta - 2\eta)^2}\right] \quad (12)$$

$$x = l\left\{\frac{(\theta - 2\eta)/\sqrt{4/V^2 - (\theta - 2\eta)^2}}{(\theta + 2\eta)/\sqrt{4/V^2 - (\theta + 2\eta)^2} - (\theta - 2\eta)/\sqrt{4/V^2 - (\theta - 2\eta)^2}} + \tfrac{1}{2}\right\} \quad (13)$$

The latter two equations can be written in the simpler form:

$$z_o = 1/\ (A-B) \qquad (12')$$

$$x = \tfrac{1}{2}l\ (A + B)\ /\ (A-B) \qquad (13')$$

where

$$A = (\theta + 2\eta)\ /\ \sqrt{4/V^2 - (\theta + 2\eta)^2}$$

$$B = (\theta - 2\eta)\ /\ \sqrt{4/V^2 - (\theta - 2\eta)^2}$$

The structure of the equations for V^2, z_o, and x appear difficult, and it may be impossible even in principle to present them in the form of a combination of simpler invariants, as was done for reflected waves. It is expedient to use as invariants for diffracted waves:

$$\left.\begin{aligned} &\frac{4\ t\eta + l\ (\theta^2 - 4\eta^2)}{t - l} \cdot \frac{t}{l} = C_5 = 4/V^2 \\ &l\ /\ (A-B) = C_6 = z_o \\ &\tfrac{1}{2}l\ (A + B)\ /\ (A - B) = C_7 = x \end{aligned}\right\} \qquad (14)$$

In conclusion, we examine one partial problem concerning the relation between time and horizontal gradient for a given section of the field x = const., with $l_1 = 0$ and $l_2 = l$.

The corresponding system of equations is presented in the form:

$$Vt = \sqrt{r^2 - xl + l^2/4} + \sqrt{r^2 + xl + l^2/4}$$

$$Vt_o = 2r$$

$$V\theta = (x - \tfrac{1}{2}l)\ /\sqrt{r^2 - xl + l^2/4} + (x + \tfrac{1}{2}l)\ /\sqrt{r^2 + xl + l^2/}$$

$$V\theta_o = 2x/r \qquad (15)$$

where

$$r = x^2 + z_o^{\ 2}$$

Eliminating the unknowns V, r, x from (15), we obtain the following relation between the parameters of the field at two selected points:

$$t\theta = t_o \theta_o (t_o^2 t^2 - t_o^2 \theta_o^2 l^2/4) / (t^4 - t_o^2 \theta_o^2 l^2/4) \quad (16)$$

Hence it follows that for diffracted waves $t\theta > t_o\theta_o$. As was shown earlier, in accordance with (8), for reflected waves $t\theta = t_o\theta_o$. These features of the relations between the parameters can be used to distinguish diffracted and reflected waves.

From the structure of formula (16), it can be deduced that the parameters t and θ of the field of diffracted waves do not form invariant relations.

In conclusion, we point out one important relation between the gradients θ and η, based on a property of the apparent velocity of the sloping hodographs of the diffracted waves. The apparent velocity V_k is related to the gradients θ and η for any wave by the following relation [3]:

$$1 / V_k = \tfrac{1}{2}\theta + \eta \quad (17)$$

where the gradients are determined at the point (x,l), and the apparent velocity at the point $x \pm \tfrac{1}{2}l$.

Based on this, we can write the following condition of overlapping:

$$\tfrac{1}{2}\theta (x_i, l_o) + \eta (x_i, l_i) = \tfrac{1}{2}\theta \left[x \pm \tfrac{1}{2}(l_k - l_i)_i \; l_k\right] + \eta \left[x \pm \tfrac{1}{2}(l_k - l_i)_i \; l_k\right] \quad (18)$$

where the sign attached to $\tfrac{1}{2}(l_k - l_i)$ corresponds to the direction of the overlapping hodograph: upwards - in the positive x direction, and downwards - in the negative direction.

The latter equation can be presented in the invariant form:

$$\tfrac{1}{2}\theta \left[x \mp \tfrac{1}{2}(l_k - l_i)\; {}_i l_k\right] + \eta \left[x \pm \tfrac{1}{2}(l_k - l_i)\; {}_i l_k\right] = \text{const.} \quad (18')$$

which is valid for any l_k with fixed x and l_i.

Head Waves

We assume that both media are homogeneous with velocities V_1 and V_2, and the boundary is flat and inclined: $z = z_o + x \tan \phi$

The equation of the field is written in the form:

$$t = (1/V_1)(2z_o \cos i \cos \emptyset + l \sin i \cos \emptyset + 2x \cos i \sin \emptyset) \quad (19)$$

where $\sin i = V_1/V_2$

The system of equations connecting the parameters of the field have the form:

$$\left.\begin{aligned} V_1 t &= l.\ \sin i \cos \emptyset + R.\ \cos i \\ V_1 \theta &= 2 \cos i\ .\ \sin \emptyset \\ V_1 \eta &= \sin i.\ \cos \emptyset \end{aligned}\right\} \quad (20)$$

where R has the same meaning as for reflected waves.

From this it can be seen immediately that there are two general invariants:

$$\theta = C_8 = 2 \cos i \cos \emptyset / V_1 \quad (21)$$

$$\eta = C_9 = \sin i \cos \emptyset / V_1$$

and one particular invariant

$$t - l\eta = c_{10} = R.\ \cos i / V_1 \quad (22)$$

It is not difficult to see that the latter expression represents the time t_{02} with $l = 0$.

We note further that the gradients of the field are related through the velocities V_1 and V_2 by the following equation:

$$\eta^2 + \theta^2 \tan^2 i/4 = 1/V_2{}^2 \quad (23)$$

It is known that overlapping hodographs remain parallel for head waves, even for certain types of curved boundaries and non-uniform upper medium. In this connection the condition (18') will be fulfilled.

In conclusion we remark that for non-uniform media, analysis of the dependence between parameters of the time field is possible in the majority of cases only by application of numerical methods.

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