012321

DEPARTMENT OF MINERALS AND ENERGY



BUREAU OF MINERAL RESOURCES, GEOLOGY AND GEOPHYSICS

RECORD 1975/4

A FORTRAN PROGRAM FOR CALCULATING THE GRAVITY EFFECT OF A THREE_

DIMENSIONAL BODY OF ARBITRARY SHAPE

bу



B.R. Spies

The information contained in this report has been obtained by the Department of Minerals and Energy as part of the policy of the Australian Government to assist in the exploration and development of mineral resources. It may not be published in any form or used in a company prospectus or statement without the permission in writing of the Director, Bureau of Mineral Resources, Geology and Geophysics.

BMR Record 1975/4 c.3 RECORD 1975/4

A FORTRAN PROGRAM FOR CALCULATING THE GRAVITY EFFECT OF A THREE.

DIMENSIONAL BODY OF ARBITRARY SHAPE

by

B.R. Spies

CONTENTS

	SUMMARY	Æ
1.	INTRODUCTION	İ
2.	MECHANISM OF COMPUTATION	2
3.	DATA INPUT	ļ
4.	PROGRAM TESTING	5
5•	DISCUSSION	5
6.	REFERENCES	7 :
	الفاق في الله المنافق ا	

APPENDIX 1 Derivation of formulae

APPENDIX 2 Program listing

ILLUSTRATIONS

- Plate 1. Program flow chart
- Plate 2. Sphere model
- Plate 3. Data sheet format
- Plate 4. Deck structure

SUMMARY

A description is given of a computer program used for calculating the gravity anomaly due to a three-dimensional body of arbitrary shape. The body is defined by a number of polygons representing depth contours of the body. By a system of interpolation between the contours the gravity anomaly caused by the three-dimensional body can be calculated to a high degree of precision.

All data are input on cards to a Cyber 76 computer and the output is a printed list of gravity values at stations along a profile or grid.

1. INTRODUCTION

Much work has been done on the computation of the gravity effects of two-dimensional bodies and computer programs for this purpose are available, for example Haigh, Pollard, Williams (1972). It is not always possible to adapt such programs to simulate geological structures encountered in the field, owing to finite strike length or complex shape. Programs for simple three-dimensional bodies such as spheres and cylinders are available (Cull, 1971) but most existing methods for calculating the gravity effect of other three-dimensional bodies involve the use of graticules or mechanical integrators (Baranov, 1953).

The program described in this Record is based on a paper by Talwani & Ewing (1960) in which formulae are given for calculating the gravity effect of a three-dimensional body of arbitrary shape. The method basically involves dividing the body up into a large number of thin laminae, determining the gravity effect for each, and adding them. Horizontal laminae are used as they can be represented by contours of depth of the body.

2. MECHAI ISM OF COMPUTATION

The brief statement which follows in this section and in Appendix 1 is derived mainly from Talwani & Ewing (Op cit.).

The three-dimensional body is first represented by contours at various depths, each contour then being replaced by a horizontal irregular n-sided polygonal lamina. By making the number of sides n sufficiently large, the polygons can be made to approximate the contour lines as closely as desired. The gravity anomaly caused by each lamina is determined analytically by the subroutine GRACI and is stored as a function of the depth of the lamina (i.e. the contour elevation). By interpolation using numerical quadrature the subroutine MUQUAD fits a continuous curve relating the heights of the laminae with their gravity anomalies. The total area under this curve gives the gravity anomaly caused by the entire body.

The co-ordinate system is given in Appendix 1. The contour at depth z is replaced by a polygonal lamina of infitesimal thickness dz. At point P the gravity anomaly due to the lamina is related to the solid angle it subtends and is given by the expression.

$$\Delta g = V dz$$
(1)

where \vee is the anomaly caused by the lamina, per unit thickness. \vee can be expressed as two line integrals, both along the boundary of the polygon.

$$V = k \rho \left[\int d\psi - \int \frac{z}{(r^2 + z^2)^{1/2}} d\psi \right] \qquad (2)$$

where

k = universal constant of gravitation

ρ - volume density of the lamina

 $z, \psi_{\bullet \bullet} r =$ the cylindrical co-ordinates used to define the boundary of the polygon.

This expression can be simplified into components in the cartesian co-ordinate system shown in Appendix 1.

The total anomaly Agraused by the entire body can be evaluated by integrating (1) between z top and z bottom, the vertical limits of the body.

$$\Delta g_{total} = \int_{Z_{bottom}}^{Z_{top}} \vee dg$$

To solve the integral the program stores V as a function of z, the depth of the contour, and by numerical quadrature fits a parabola through sets of three points. To an extent the closer the contours are spaced, the more accurate the determination of the anomaly.

The program assumes a constant density but could easily be modified to incorporate density variation with depth.

Accuracy

The accuracy of the method depends on how closely the irregular polygons fit the individual contours and on the precision of the numerical quadrature. The polygons can be made to fit the contours as closely as desired merely by increasing the number of sides but there is a corresponding increase in computer time. Obviously close fit to contours is only important when a portion of the contour boundary lies close to a point at which the computation is being made. The accuracy of the numerical quadrature is mainly governed by the contour interval. The assumption is made that V varies smoothly between contours and as long as the surface of the body varies smoothly between contours there is no advantage in interpolating depths between contours.

The most common source of error is caused by the area enclosed by a polygon differing significantly from that enclosed by the contour. If points are marked at intervals along a contour, and joined with straight lines, then the area enclosed by the polygon will in general be appreciably less than that enclosed by the contour. Care should therefore be taken to ensure that the area of the polygon is as close as possible to that of the contour.

3. DATA IMPUT

All input data are on punched cards. Contours are drawn on the body and fitted with polygons, the corners of which are specified by X and Y co-ordinates. All distances are in metres.

First data card

This card contains a list of numbers specifying parameters of the body and traverse details.

NCCNT is the number of contours defining the body, but can also include the top and/or bottom of the body. If the top and/or bottom of the body is not defined the program assumes the first and last contours define the extremities of the body, i.e., it has a flat top and/or bottom. In the general case when the top and/or bottom is not flat the depths are given on the third data card, and the top and/or bottom are defined as contours for the purpose of this data card.

NCONT must be an odd number less than 40.

DEMS is the density contrast of the body in g/om3.

FX is the X co-ordinate of the beginning of the first profile.

PY is the corresponding Y co-ordinate.

DFX is the X increment for successive points on the profile.

DPY is the corresponding Y increment.

NPTS is the number of stations on the profile.

MLINES is the number of profiles.

DIST is the distance between profiles.

FIRST DATA CARD; NCONT, DENS, PX, PY, DPX, DPY, NPTS, NLINES, DIST 1 FORFAT: 12, F8.4, 4F10.2, 2110, F10.2

Second data card

This gives a list of the number of co-ordinates in each contour, in order, from the top down. In the case of the top or bottom of the body the number may be 1. This represents a point and occurs for bodies with tops or bottoms which are not flat.

Third data card

Further data cards

There are two sets of data for each contour. The first set gives a list of X co-ordinates of the polygon, the second gives a list of corresponding Y co-ordinates in the same order. No data cards are required in the case of point contours. There may be up to 40 points for each contour, 13 fitting on a card.

Further models

Data for new models may be inserted directly after the data for the previous model. A blank card should be the last card in the data deck to signify no further models.

Deck structure

See plate 4.

Data sheet format

See plate 3.

Program listing

See Appendix 2.

Program flow chart

See plate 1.

4. PROGRAM TESTING

The gravity profile over a sphere was computed in the following example. The sphere had the following parameters; depth to centre 500 m, radius 300 m, density contrast 0.5 g/cm³. The exact gravity values are easily determined for this case, and the results (Plate 2) show that close fit is obtained by using only 3 contours with an 8-sided polygon defining them.

The top and bottom of the sphere were also defined.

The time taken on the Cyber 76 for the above model, which involved 11 observation points, was 1.9 seconds.

5. DISCUSSION

The program GRAV3D is capable of computing the gravity profile over three-dimensional bodies of complex shape. The accuracy depends on the number of contours employed and the number of sides defining the contour. However, increase in these parameters also increases computer time.

The program could be used for computing terrain corrections on a flat earth, and can be modified so that density variations with depth can be accommodated.

It was not considered warranted to include a PLOT facility in the program because of the extra computing time that would be involved.

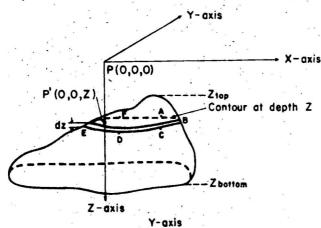
6. REFERENCES

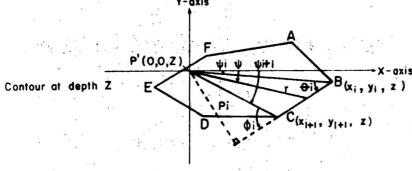
- BARANOV, V. 1953 Sur le calcul de l'influence gravimetrique des structures definies par les isobathes.

 Geophys. Prosp. 1(1), 36-43.
- CULL, J.P., 1971 A fortran program for calculating the gravity effect of vertical cylinders. Bur. Miner. Resour.

 Aust. Rec. 1971/74 (unpubl).
- HAIGH, J.E., POLLARD, P.C., & WILLIAMS, J.P., 1972 a computer program for calculation of gravity and magnetic curves for two-dimensional bodies of arbitrary cross-sections. <u>Bur.Miner. Resour. Aust. Rec.</u> 1972/49 (unpubl).
- TALWANI, M., & EWING, M., 1960 Rapid computation of gravitational attraction of three-dimensional bodies of arbitrary shape. Geophysics, 25(1), 203-25.

APPENDIX I DERIVATION OF FORMULAE





P= point of computation

Z = depth to lamina of thickness dz representing contour

Z top = depth of top of body

Z bottom= depth of bottom of body

ABCDEF = polygonal lamina

P'= projection of P on Plane A.B.C.D.E.F.

r = radius vector in plane A.B.C.D.E.F.

Gravity effect of lamina
$$V = k\rho$$
. $\left[\oint d\psi - \oint \sqrt{\frac{Z}{r^2 + Z^2}} d\psi \right]$

where k = universal constant of gravitation

p = volume density of the lamina

$$z, \psi \text{ and } r \text{ are cylindrical coordinates.}$$

$$r = \frac{Pi}{\sin(\varphi_i + \psi - \psi_{i+1})}$$

Second integral for segment BC =
$$\arcsin \frac{Z \cos \Theta_i}{\sqrt{P_i^2 + Z^2}}$$
 - $\arcsin \frac{Z \cos \Phi_i}{\sqrt{P_i^2 + Z^2}}$

Total contribution from all sides is

$$V=k\rho\left[\sum_{i=1}^{n}\left\{\psi_{i+1}-\psi_{i}-\arcsin\frac{Z\cos\theta\,i}{\sqrt{Pi^{2}+Z^{2}}}\right.\right.+\left.\arcsin\frac{Z\cos\theta\,i}{\sqrt{Pi^{2}+Z^{2}}}\right\}\right]$$

G 29 -222A

11

APPENDIX 2

```
PROGRAM GRAVED (INPUL, SUTPUL, TAPECO- IMPUL, TAPECI-OUTPUT)
                   THIS PHOGRAM COMPUTES THE VENTICAL COMPONENT OF THE GRAVITATIONAL EFFECT OF A 3-D BOLY ALONG A SENIES OF PAHALLEL PROFILES.
                   THE BODY IS DESCRIBED BY POLYGONS REPRESENTING DEPTH CONTOURS.
                  ALL DISTANCES ARE IN METHES. DENSITY IS IN OMS/CC.
                   CONTOURS ARE LISTED FROM THE TOP DOWN.
                           DIMENSION DEPTH(40), DELGA(40), XLIST(40,40), YLIST(40,40), NLIST(40) FORMAT(18, F8, 4, 4F10, 2, 2110, F10, 2) FORMAT(4012)
                 I FORMAT(IE, F8. 4, AFIO.2, 2110, FIO.2)

POMAT(4012)

FORMAT(13F6.1)

FORMAT(
                         IL.50)
FOMAT(15X, F8.1, 10X, F8.1, 17X, F9.3)
     JOT= 0
1000 CONTINUE
JOT- JOT- 1
                 HEAD I.NCONT. DENS. PX. PY. DPX. DPY, NPTS. NLINES, DIST
FIRST DATA CARD GIVES THE FOLLOWING...

NCONT.... THE NUMBER OF CONTOURS REPRESENTING THE BODY. THIS
INCLUDES ANY CONTOURS WHICH MAVE DEGENERATED TO A
SINGLE POINT. NCONT NUST BE AN ODD NUMBER GREATER
THAN I AND LESS THAN O.

DENS.... THE DENSITY OR DENSITY CONTRAST OF THE BODY.
PX. PY.... THE X AND Y STARTING COORDINATES OF THE FIRST
PROFILE.

LPX. LPY... THE X AND Y INCREMENTS FOR SUCCESSIVE POINTS ON A
PROFILE.
NITS.... THE NUMBER OF POINTS ON EACH PROFILE.
VLINES... THE NUMBER OF PROFILES REQUIRED.
DIST.... THE DISTANCE BETWEEN ADJACONT PROFILES.
000000000
c
                   TEST TO SEE IF THERE ARE ANY FURTHER MODELS. .
                           IF(NCONT. EQ. 0) GO TO 9999
PHINT 4. JOT, NCONT, DENS, PX, PY, DPX, DPY, NPTS, ML INES, DIST.
                  READ 2, (NLIST(NL), NL=1, NCONT)
SECOND DATA CARD GIVES A LIST OF THE NUMBER OF SIDES OF EACH POLYGON
C
                  HEAD 3, (DEPTH(ND), ND-1, NCONT)
THIS STATEMENT KEADS IN A LIST OF THE DEPTHS TO THE CONTOURS,
INCLUDING ANY WHICH HAVE DECEMERATED TO A SINGLE POINT
                           DO 2003 I=1,NCONT
NUM=NL[ST(I) )
IF(NUM-EQ.1) PRINT 5,I,DEPTH(I)
IF(NUM-EQ.1) GO TO 2003
                   READ 3, (XLIST(I, IX), IX=1, NUN)
READ 3, (YLIST(I, IY), IY=1, NUM)
THESE STATEMENTS READ IN SETS OF COORDINATES FOR THE APICES
OF EACH POLYGON.
NO COORDINATE PAIR MUST BE PRESENT FOR SINGLE POINT CONTOURS.
                                    F(NUM.NE.I) PRINT 6.I.DEPTH(I).NLIST(I).(XLIST(I,IX).YLIST(I.IX).
     11X=1, NUM)
2003 CONTINUE
NOTE=0
NCONTL1=NCONT-1
                NCONTLI=NCONT-1

THIS LOOP TESTS WHETHER CK NOT ANY OF THE CONTOURS ARE AT THE SAME DEPTH- IF SO, SUBROUTINE ERROR! PRINTS AN ERROR MESSAGE AND THE MODEL IS ABANDONED.

DO 99 I=1,NCONTL!

IFFODETH(I)-EQ. DEPTH(I+1))CALL ERROR!(I, DEPTH,NOTE)

99 CONTINUE

IFFODETH(I)-EQ. 999) OD TO 1000

DO 3904 [=1,NCONT
NUM=NLIST(I)

IFFOUN-EQ. 1) GO TO 3904

THIS LOOP PUTS XLIST AND YLIST RELATIVE TO AN ORIGIN AT THE FIRST POINT ON THE FIRST PROFILE.

DO 2444 MN=1,NUM
XLIST(I,NM)=XLIST(I,NM)-PX
YLIST(I,NM)=YLIST(I,NM)-PY
44 CONTINUE
```

```
BEARNG-90.0

IF (DPY-NE.O.0) BEARNG-ATAN(DPX/DPY)+180.0/3,1415927

XUN-SQRT(DISTN-2/CDPX-PP-1)+DPY-8XII

C THIS LOOP MOVES TO THE NEXT LINE.

DO 3901 N=1,NLINES

IF (N.Fg.) 00 TO 2004

PX-PX-XUDDY
PY-PY-XUDDY
PY-PY-XUDDY
PY-PY-XUDDY
ON THIS LOOP MOVES TO THE NEXT LINE.

DO 7505 1=1,NCOVT

NIT-NLIST(1)

IF (NIT-BL.) 00 TO 7650

DO 7506 (*1,NC)

THIS TILE YLIST(1,K)+DXXX
YLIST(1,K)+XLIST(1,K)+DXXX
YLIST(1,K)+XLIST(1,K)+DXXX
YLIST(1,K)+XLIST(1,K)+DXXX
YLIST(1,K)+YLIST(1,K)+DXXX
YLIST(1,K)+YLIST(1,K)+DXXX
YLIST(1,K)+XLIST(1,K)+DY

FOO 3POINT 8.N.PX,PY,EEANO
PAINT 9

DO 3900 J=1,NPTS

IF (J.EQ.1) GO TO 1834

C THESE TWO LOOPS CAUSE AN INCREMENT OF ONE STATION ALONG THE PROFILE.

DO 1001 IN=1,NCOVT

KLIS-MLIST(1)

IF (NLIS-1,K)-XLIST(1,LIX)-DPX
YLIST(1,LIX)-XLIST(1,LIX)-DPX
THE MEMORIAN EFFECT OF THE CONTOURS AND PRIVIS THE RESULT.
CALL GRADICALIST, Z.XLIST, YLIST, TOTEQ, NC, DPX)
DEL GANVINGONT, DPT THE DEL GA, TO TAL G)
GRAVITATIONAL EFFECT OF THE CONTOURS AND PRIVIS THE RESULT.
CALL NURDINGONT, DPT THE DEL GA, TO TAL G)
GRAVITATIONAL EFFECT OF THE CONTOURS AND PRIVIS THE RESULT.
CALL NURDINGONT, DPT THE DEL GA, TO TAL G)
GRAVE TATIONAL EFFECT OF THE CONTOURS AND PRIVIS THE RESULT.
CALL NURDINGONT, DPT THE DEL GA, TO TAL G)
GRAVITATIONAL EFFECT OF THE CONTOURS AND PRIVIS THE RESULT.
CALL NURDINGONT, DPT THE DEL GA, TO TAL G)
GRAVE TATIONAL EFFECT OF THE CONTOURS AND PRIVIS THE RESULT.
CALL NURDINGONT, DPT THE DEL GA, TO TAL G)

GO TO 1000

GO TO 1000

GO T
```

APPENDIX 2

```
SUBMOUTIVE NUCUAD(NOONT, DEPTH, DELGA, TOTALG)

THIS SUBMOUTIVE DIES A VUMENICAL QUADMATURE FOR SUMMING VALUES OF

DELGA AT DIFFERENT DEPTHS.

NCONT, DEPTH + DELGA ARE TYPUT DATA.. TOTALG IS OUTPUT.

DIMENSION DEPTH(40), DELGA(40)

TOTALG=0.0

IA-NCONT-2

DO 6000 [=1,IA,2
2 |= DEPTH(1)
2 = DEPTH(1)
3 = DEPTH(1)
6 = DELGA(1)
6 = DELGA(1)
6 = DELGA(1)
6 = DELGA(1)
6 = DELGA(1-1)
6 = DELGA(1-2)

IF(G1 = ELU..AND.G3 = E0.0) G = 0.

EQ9A=G1=(21-23)/(21-22)=(3+2-3-22-1)

EQ9B=G=(21-23)=3/(2-23)=(3+2-21)

EQ9C=G3=(21-23)=3/(2-22)=(3+2-21-2-2)

DOTTALG=TOTALG=EQ9

NOOO CONTINUE

RETURN

END
                          SUBROUTINE ERRORICI, DEPTH, NOTE)
THIS SUBROUTINE PRINTS AN ERROR MESSAGE IF ANY ADJACENT CONTOURS ARE
AT THE SAME DEPTH, AND SETS NOTE=999 TO STOP EXECUTION OF
THE MODEL,
DIMENSION DEPTH(40)
1 FORMATCIX, 4(/)* ILLEGAL DATA ENTRY. CONTOURS*13* AND*13* ARE BOT
IH AT DEPTH*F8.1)
J=1*1
PRINT 1.1, J. DEPTH(1)
VOTE=999
RETURN
END
SUBROUTINE GRADI(NLIST, Z, XLIST, YLIST, TOTEQ, NC, EPX)

C THIS SUBROUTINE COMPUTES THE GRAVITY EFFECT OF ONE POLYGONAL LAWINA

C MLIST IS THE NUMBER OF COORDINATES

C XLIST ARE THE X COORDS, YLIST ARE THE Y COORDS.

C Z IS THE DEPTH TO THE LAWINA

C TOTEQ IS THE RESULT

DIM ENSION NLIST(40), XLIST(40, 40), YLIST(40, 40)

TOTEQ-0.0

XI=XLIST(NC,1)

YI=YLIST(NC,1)

IF(X1:EQ.0.AND-Y1:EQ.0.) XI=-DPX/10000.

KI=SQRIY(XLIST(A), YLIST(40, 40), YLIST(40, 40)

C THIS LOOP TAKES SUCCESSIVE PAIRS OF COORDS AROUND POLYGON AND

MAKES NEW XI, YL EQUAL TO OLD X8, YR ETC.

NLISTN=NLIST(NC, 1)

LICUP EXCEEDS NLIST, THEN SET LVP=1

IF(LVP-NLISTN) 3000, 3000, 2009

2999 XIPI=XLIST(NC, 1)

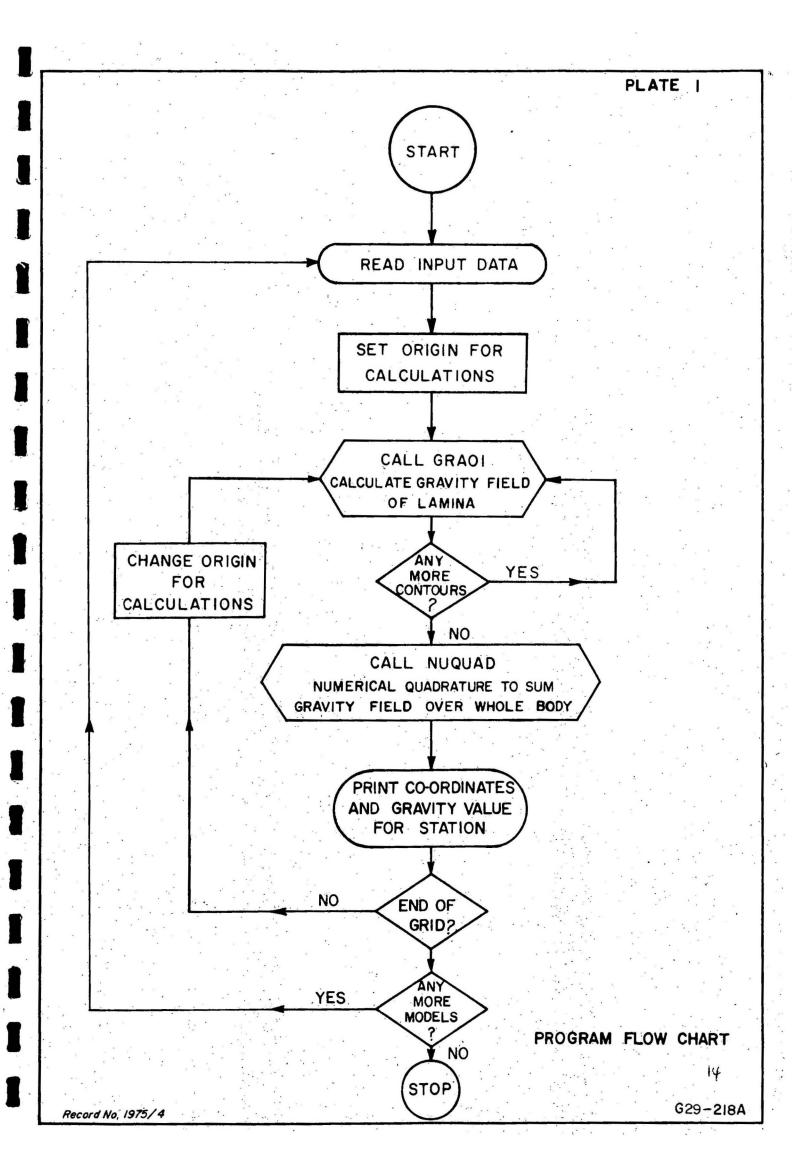
YIPI=YLIST(NC, 1)

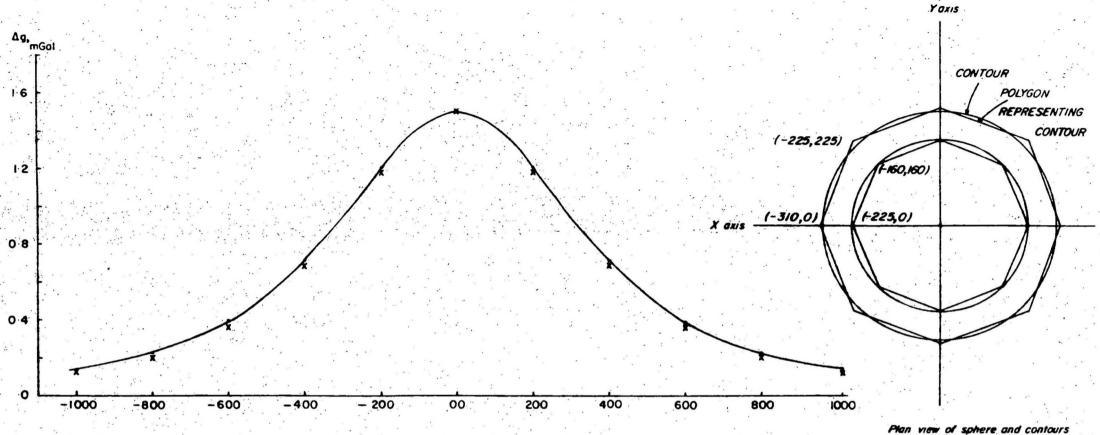
IF(XIPI:EQ.0.AND, YIPI:EQ.0.) XIPI=-DPX/10000.

GO TO 3001

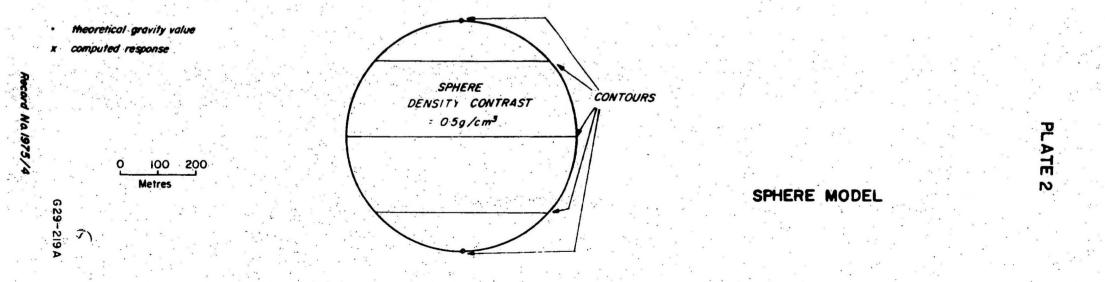
3000 XIPI=XLIST(NC, LVP)
```

PROGRAM LISTING









(NO YLIST OR YLIST FOR CONTOURS I AND 5 AS THESE ARE SINGLE POINTS)

0.0 - 160.0 - 2250 - 150.0 - Y LIST (4)

-3 1 a0 - 2 2 0.0 - YLIST (3)

0.0 - 160.0 - X LIST (4)

dito

ditto

d'.to

ditto

220.0

160.0

- 160.0

0.0

0.0

- 2 2 5.0

31 0.0

0.0

2250

2 2 0.0

160.0

1 60.01

0.0

2 2 5.0

-220.0

1.60.0