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ROCK DENSITY MEASUREMENTS
USING GAMMA RAYS



py

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CONTENTS

					PAGE
SUMMARY					
INTRODUCTION	*	v			. 1
GAMMA RAY ATTENUATION	DENSITY	EVALUATION	# #		2
ACKNOWLEDGEMENTS					8
BIBLIOGRAPHY					9
APPENDIX 1 : Commercia	ally avai	lable gamma	ray density	evaluators	10

FIGURES

- Figure 1 Mass absorption coefficients for silica and calcite for the three main modes of interaction of gamma rays with matter.
- Figure 2 Diagram showing parameters in calculation of optimum beam width.
- Figure 3 Diagram showing dimensions of beam assumed for calculation of strength of source and time constant of ratemeter.

SUMMARY

For gamma rays in the energy range 0.3 to 2.0 MeV, attenuation in a medium is predominantly a function of absorber electron density. Electron density is approximately proportional to bulk density for elements of low or medium atomic number, and as a result rock densities can be determined by measuring the emergent intensity of a gamma ray beam transmitted through a sample.

A simple laboratory or field instrument can be constructed which could measure rock densities to an accuracy of better than one percent. A collimated beam of gamma rays from a 10 millicurie Co or Cs 3 source is directed through a rock sample of known thickness. The intensity of the emergent beam is measured using a scintillometer. A correction is made to the intensity reading for the temporal decay of the source, and an apparent density value for the sample is read from a calibration curve relating emergent intensity and density. Finally, to obtain the true bulk density, corrections are made for the thickness of the sample and the departure from proportionality of bulk density and electron density.

The instrument could also be used to determine the porosity and saturation bulk density of sedimentary rocks of known composition. Porosity is calculated from the measured bulk density and estimated grain density, and saturation bulk density from values for the porosity, dry bulk density, and the density of the saturating fluid.

INTRODUCTION

Quantitative interpretation of gravity data is possible only when the bulk densities of rock units are known. This information is often difficult to acquire as simple measurements on surface samples can give a misleading value for the bulk density of a rock unit as a whole. In sedimentary basins, it may not be possible to estimate formation densities from measurements taken on surface rock samples, because the compaction — and consequently the bulk densities of rocks — increase with depth. In addition, porous surface samples are usually dry; the same rocks below the water table may be saturated and therefore of higher bulk density. In hardrock areas, the main difficulty is in finding surface samples which are representative of the parent rock body as a whole. Igneous bodies generally exhibit magmatic differentiation and metamorphic bodies are rarely homogeneous, the minerals usually being segregated into lenses and bands.

The most reliable density information on sedimentary rocks is obtained by measurements on core samples from boreholes. The porosity of most rocks can be evaluated to an accuracy of better than 3 percent, by measuring the bulk density and estimating the grain density of the sample. Hence the saturation bulk density of the rock can be calculated. For crystalline rocks, an estimate of the bulk density of a rock body can be made by measuring the densities of a large number of carefully selected surface samples.

In both cases a rapid, accurate and preferably non-destructive method of determining the densities of samples is required. The Archimedean method is inadequate in the following respects:-

- (1) It is time-consuming if large numbers of samples are to be measured.
- (2) Density measurements on permeable samples are spurious when water immersion is used. Water permeates through part or all of the sample and the measured dry bulk density is higher than the actual. Applying grease to the surface of the sample could overcome the problem but this is semi-destructive and can also be time-consuming.
- (3) Core samples and surface rock samples vary considerably in size. Some cores are 20 to 30 cm long and 10 to 15 cm in diameter, while others are only 5 to 10 cm long and less than 5 cm in diameter. This causes practical difficulties in the measurement of the volume and mass of the sample. Large samples have to be broken into smaller fragments which again is both time-consuming and destructive.

A simple method of non-destructive density sampling depends on the manner in which a gamma ray beam attenuates in an absorber. In the photon energy window in which Compton scattering predominates, the attenuation of a beam is mainly a function of the thickness and electron density of the absorber. For materials with a constant ratio of atomic number (Z) to atomic mass (A), electron density is directly proportional to bulk density, so that the attenuation of a gamma-ray beam varies in direct proportion to bulk density. Thus if a collimated gamma-ray beam from a source such as Co or Cs is passed through a sample and the intensity of the emergent beam is accurately measured with a scintillometer, the density of the sample can be determined. Corrections are required for variations in thickness of samples, the exponential decay of beam intensity with time, and variations in Z/A for different materials.

This method is discussed in more detail below. An appraisal of commercial instruments which use gamma rays to measure rock densities, is given in Appendix 1.

GAMMA RAY ATTENUATION DENSITY EVALUATION

Law of Absorption

For radiation incident on an absorbing medium, the fractional decrease in intensity (I) per unit thickness (x) of absorber is given by,

$$\frac{dI}{I} = -\mu_{\ell} dx$$

where μ_I , numerically equal to the area per unit volume of absorber causing scattering or absorption, is termed the linear absorption coefficient.

Now
$$\frac{dI}{I} = -\mu_i dx$$

integrating over sample thickness, $\int_{I_{o}}^{I} \frac{dI}{I} = -\int_{o}^{x} \mu_{I} dx \text{ (where I is the incident intensity)}$ $\lim_{I \to \frac{I}{I_{o}}} = -\mu_{I} \times$

The quantity $\mu_{\alpha} = \frac{\mu_{\alpha}}{\sqrt{2}}$ (where ρ is the bulk density of the absorber), is the mass absorption coefficient and is numerically equal to the area per unit mass of absorber causing scattering or absorption.

As
$$\mu_I = \mu_m \rho$$

$$I = I_n e^{-\mu_m \rho x}$$

Interaction of gamma rays with matter

The main types of interaction between gamma rays and matter are pair production, Compton scattering and photoelectric absorption.

Pair production, the conversion of a gamma photon into an electron-positron pair, is of no interest in density determination as $\mu_m \sim 2^{-1}$ very closely. In other words, the attenuation of a gamma ray beam is dependent on the chemical composition of the absorber.

In Compton scattering, gamma photons are scattered out of the primary beam by interaction with electrons in the absorber, and thereby lose some of their energy. The Compton effect is of great interest in bulk density determination because μ_m is almost independent of Z. For absorbers of constant thickness, scattering is a function only of electron density, which is approximately proportional to bulk density.

The photoelectric effect, where photons are absorbed and photoelectrons ejected from the atom, cannot be used for density determination, because attenuation is strongly dependent on the chemistry of the absorber ($\mu_m \approx z^*$).

The variation of μ_m with gamma-ray energy, E_y for the three different types of interaction for calcite and silica is shown in Figure 1. The following points should be noted:-

- Compton scattering predominates in the gamma energy range,
 0.3 2 MeV.
- (2) The mass absorption coefficients for calcite and silica are the same for Compton scattering, but different for pair production and the photoelectric effect.

In most rocks, μ_m for Compton scattering is approximately constant and the intensity of the emergent beam is mainly a function of absorber thickness and bulk density. A small perturbation arises from the Z/A effect which is discussed below.

The Z/A effect

The following analysis shows that attenuation due to Compton scattering is a function of electron density (n_e) rather than bulk density (p). Hence any departure from proportionality of n_e and p for various rock types will cause an error in the measurement of bulk density.

/ = area per unit mass causing scattering or absorption

 (no. of atoms per gram) x (no. of electrons per atom) x (collision cross section per electron)

$$= \frac{N_o}{A} \times Z \times \sigma_e \qquad (1)$$

where No is Avogadros' number, of is the collision cross section of an electron for a given energy, A is atomic mass in atomic mass units.

Electron density,
$$n_e = NZ$$
 where N is the number of atoms Bulk density, $\rho = N(\frac{A}{N_o})$ per unit volume $n_e = N_o(\frac{Z}{A})\rho$(2)

From (1) and (2),

.. Intensity of emergent beam,
$$I = I_o e^{-n_e \sigma_e x}$$
.....(3)

Relations (2) and (3) show that any departure from constancy of $\frac{Z}{A}$ will cause an error in the measured bulk density. For most rocks Z/A is approximately constant and thus $n_* < \rho$ fairly closely. The Z/A values for some common minerals and water are listed below.

Calcite:	0.500
Quartz:	0.500
Orthoclase:	0.496
Olivine:	0.488
Hematite:	0.475
Gypsum:	0.512
Halite:	0.479
Water:	0.555
Magnetite:	0.474
Pyroxene:	0.487

The following points should be noted:-

- (1) Quartz and orthoclase, the principal constituents of acid igneous rocks, both have Z/A values very close to 0.5.

 These two minerals are found in abundance in common sedimentary rocks: for example quartz in sandstone, orthoclase (and other felspars) in clays and shale.
- (2) The Z/A value for water is anomalously high.
- (3) Anhydrous minerals all have Z/A values equal to or less than 0.5. The departure from 0.5 is greatest for minerals containing iron which has a Z/A value of 0.464.
- (4) Assuming that an instrument is calibrated with samples whose Z/A is 0.5, the percentage errors in the measured bulk density for water and magnetite would be respectively +11% and -5%.

For most sedimentary rocks, no correction for the Z/A effect would be required because Z/A values are fairly constant and percentage errors in the measured bulk density would rarely exceed one percent. An assumption in this is that the core samples are dry. Water has a high Z/A and for an accuracy to one percent corrections would be necessary if the water content of the rock exceeded 10 percent.

Measurements on acid igneous rocks such as granite, granodiorite or rhyolite would require only small corrections if at all, because the principal constituents, quartz and feldspar, have a Z/A close to 0.5. Basic igneous rocks however, are largely composed of ferromagnesian minerals such as olivine, pyroxene and hornblende whose Z/A is less than 0.5 by up to 5 percent. Some correction to the measured density may therefore have to be applied. This would require a rough estimate of the mineral composition of the rock so that the average Z/A could be calculated.

If large numbers of rocks of similar composition were being measured, it would be convenient to calibrate the instrument with samples whose Z/A values are close to the mean. Then, provided the standard deviation of Z/A values of all the samples was within a certain limit, no corrections would be necessary.

Decay correction

A monochromatic gamma ray source with photon energies in the range where Compton scattering predominates (0.3 to 2 MeV) and a half-life of not less than five years would be suitable for use in the density evaluator. Two such sources are Co co , with a half-life of 5.3 years, emitting 1.17 and 1.33 MeV photons; and Cs co co with a half-life of 27 years, emitting 0.66 MeV photons.

Radioactive sources decay exponentially according to the law:-

 $I(t) = I_o e^{-\lambda t}$ where the decay constant λ is equal to $\frac{I_{n-2}}{I_{\nu_{\perp}}}$ is half-life, I_o is intensity at time zero, I(t) is intensity after time t.

Allowance must be made for this decay in any measurement of density. There are two alternative methods of removing the effect of source decay from density evaluations:

(1) Multiplying intensity readings by an appropriate timedependent decay factor. Attenuation in a medium is, as seen before, governed by the law:-

$$I = I_{o} e^{-il_{mp} x} \qquad \text{at time zero}$$
After time I , $I(p, \kappa, t) = I(t) e^{-il_{mp} x}$

$$= I_{o} e^{-\lambda t} e^{-il_{mp} x}$$

The time dependence of $I(\rho,x,t)$ can be removed by multiplying by $e^{\lambda t}$ or $I_{\rho}/I(t)$.

Then $I(\mu,x) = I_0 e^{-\lambda m \mu x}$ The decay constant λ is a characteristic of a source and can be found easily. A curve of $e^{\lambda t}$ versus t can be drawn and the decay correction factor determined for any particular time.

(2) Re-calibrating the instrument periodically. This method would be preferable as it would take account of time variations in the efficiency of the scintillometer. If samples were to be measured in batches, it would be advantageous to calibrate the instrument immediately before using it. Samples having a Z/A close to the mean value for all samples to be measured could be used in the calibration, thus avoiding excessive Z/A corrections.

The maximum allowable period between each calibration is calculated as follows:-

The fractional decrease in intensity due to the decay of the source ($\lambda\Delta t$) should not exceed the fractional decrease due to the smallest measurable increase of density ($\mu_m \times \Delta \rho$), i.e. $\lambda\Delta t$ should be less than or equal to $\mu_m \times \Delta \rho$.

Let
$$\lambda \Delta t = \mu_m \times \Delta p$$

Then $\Delta t = \frac{\mu_m \times \Delta p}{\lambda}$ where Δt is the maximum allowable period between calibrations.

For a Cobalt 60 source,

$$N_m = 0.655$$
 cm/gmfor 1.25 MeV photons,
 $\lambda = \frac{10.2}{F_{1/2}} = \frac{0.613}{5.3}$ per year
 $X = 5$ cm (minimum absorber thickness)
 $\Delta p = 0.01$ gm/cm³ (smallest measurable density increment)
 $\Delta t = 0.021$ years
 $\Delta t = 0.021$ years

For a Cesium 137 source, a similar calculation shows that the period between calibrations should be no more than 8 weeks.

In practice, it may prove necessary to calibrate the instrument before and after each use, owing to variation with both time and temperature of the performance of the detector and counter.

Variations in thicknesses of the samples

The calibration curve relating intensity and bulk density will be valid for one thickness of sample only. Call this thickness x_s . A correction must be made for samples of thickness different from x_s . For a standard sample, $x = x_s$, $p = p_s$, say.

For an unknown sample, x = ax, and p is to be measured.

1 - 1 e^{-Mmpx}

The apparent density 90 is read from the calibration curve. The true density is simply $\frac{1}{6}$, 90 apparent density. The required correction factor is therefore $\frac{1}{6} = \frac{x}{2}$.

If the density is to be measured with an accuracy of 0.01 gm/cc, the allowable percentage error in the measurement of sample thickness is 0.5%. For a 5 cm thick sample this error would be 0.25 mm. A precision vernier can measure with this accuracy.

Optimum beam width

Core samples range in diameter from about 5 cm to 12 cm. Most are slightly irregular in section and some are chipped. It is necessary to find an optimum beam width such that the path length through a sample can be considered constant for all the gamma photons in the beam. The required beam width satisfying this condition is calculated as follows:

From Figure 2,
$$l = 2\sqrt{\frac{x^2 - y^2}{4}} = \sqrt{x^2 - y^2}$$

 $dx = x - (1 - x^2 - y^2)$
 $\frac{dx}{x} = 1 - \sqrt{1 - y^2}$
 $= 1 - (1 - \frac{1}{2}y^2)$; $y \ll x$
 $= \frac{1}{2}y^2/2^2$

suppose x = 5 cm (minimum core diameter) and $\frac{dx}{x} = \frac{1}{2cc}$ (error in path length should not exceed error in density)

Then $y = \sqrt{\frac{1}{2cc}} = \frac{1}{2c}$

The core surface is curved so the difference between the path length and the measured width of the core is less than $x-\lambda$ for most photons. The proportional error $\frac{1}{2}$ is therefore considerably less than $\frac{1}{2}$ for a 0.5 cm beam and 5 cm core.

A beam width of 0.5 cm should also be sufficiently small to overcome errors caused by irregularities in the core surface.

Strength of source

Ten millicurie Cobalt 60 sources are available from the Australian Atomic Energy Commission. It will be shown in the following calculation that a source of this strength would be adequate for the purpose of density determination.

Dimensions used in the calculation are as indicated in Figure 3.

> 1 Curie = 3.7×10^{10} disintegrations per second 10 millicurie = 3.7×10^8 d/s

.. 7.4 x 10⁸ gamma photons are emitted per second as each disintegration yields 2 gamma rays. (for Co co).

Number of photons received by detector per second

= 20,000 A

where A, the attenuation factor, equals e-Map x

For a Co source and 12 cm wide core of density 2.5 gm/cm³ $A = e^{-0.65 \times 2.5 \times 12} = 0.1921$

$$A = e^{-0.05 \times 2.5 \times 12} = 0.1921$$

Number of photons counted per second (I) = 3840 (scintillator assumed to be 100% efficient)

The error in density caused by a fractional error in intensity is given by $\Delta \rho = \frac{1}{M_m} \times \frac{\Delta l}{l}$

If $\Delta p = 0.01 \text{ gm/cm}^3$ $\Delta I \leq 0.01 \times 0.055 \times 12 = 0.0066$

The statistical error in a count of N is [N

The fractional error in the count is therefore $\frac{1}{\sqrt{N}}$ = 0.0066 N = 23,000 ·

As only 3840 photons are received at the detector in one second the intensity reading should be made over an interval of about 6 seconds*.

Calibration procedure

Samples used for calibration could be either rock or core samples of accurately known density, or plaster casts uniformly impregnated with varying amounts of fine iron filings, or filings of some other heavy material.

It is unlikely that the core samples or rocks would be of the same thickness. A correction would be required in order that the intensity for each value of density, be valid for one thickness of sample only (call this x_s). From the absorption law, $I = I_s e^{-A_s \rho_s x_s}$, it is obvious that the intensities Y_s , for samples of thicknesses x_s (where $x_s = a_s x_s$) and densities ρ_s , is the same as for samples of thickness x, and densities a, . The quantities a; can be found by measuring the thicknesses of the samples x_i , since $a_i = \frac{x_i}{2}$. The intensity is then plotted against $a_i p_i$ (not just p_i). a_i values should preferably be close to unity. Corrections to the calibration curve for the Z/Aeffect may also be necessary.

The advantages of using plaster casts impregnated with iron filings instead of rock samples for the calibration are that casts can be moulded to the same standard thickness, the densities of the casts can be precisely controlled, and the Z/A correction can be applied with great accuracy. One precaution that should be taken is to measure the emergent intensity of the gamma beam at various places along the length of the plaster cast and average the readings. This will overcome the effect of inhomogeneites in the distribution of iron filings throughout the cast.

Porosity and saturation bulk density determination

If the approximate composition of a sedimentary rock is known, the grain density (ρ_g) of the rock can usually be estimated to better than 0.04 gm/cm. Many common sedimentary rocks are composed predominantly of one mineral only and the grain density is simply the density of the constituent mineral. For instance, sandstone is composed mainly of quartz of mean density 2.66 gm/cm, limestone of calcite of mean density 2.72 gm/cm, and shale of hydrous aluminium silicates of mean density 2.73 gm/cm.

Forosity and saturation bulk density can be calculated as follows:-

If M is the mass of a dry sample and V its volume then dry bulk density $\rho_i = \frac{M}{V}$ If V is the total volume of grains then $\rho_g = \frac{M}{V}$

Porosity (expressed as fraction)
$$\eta = \frac{V - U}{V} = \frac{1 - \frac{\rho_1}{\rho_2}}{1 - \frac{\rho_2}{\rho_2}}$$

Saturation Bulk Density $\rho_1 = \frac{M + (V - U)\rho_1}{V}$ where ρ_1 is the density of the saturating fluid (usually water)

ACKNOWLEDGEMENTS

The author thanks Mr F. Wilkes and Dr. M. Idnurm of the Bureau of Mineral Resources for their constructive criticism of an earlier draft of this report.

The above calculations and those for the optimum beam width serve to illustrate methods for finding a satisfactory balance between beam width, strength of source and the counting time required for obtaining a density measurement of the desired accuracy. Such calculations are intended only as a guide. In the proper design of an instrument, account would have to be taken of the nature and performance of the various component parts available.

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APFFNDIX 1 - Commercially available gamma-ray density evaluators

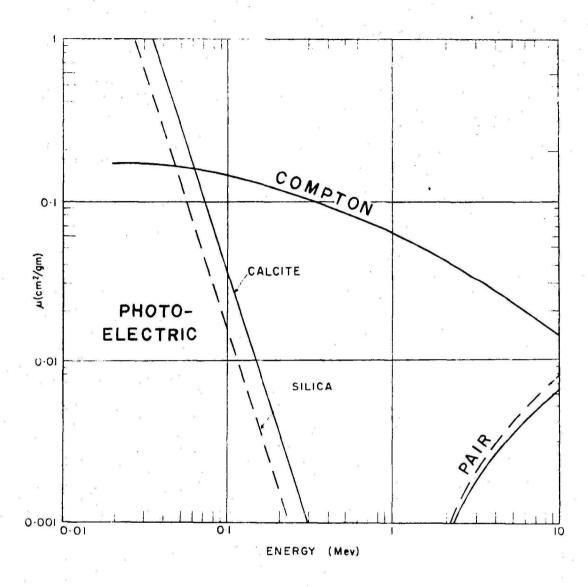
Commercial gamma ray density evaluators are generally of the type which continuously log formation densities when lowered down a borehole. They consist essentially of a sonde and recorder. From a source at one end of the sonde, a collimated beam of gamma-rays is directed into the walls of the borehole. A proportion of those gamma photons which undergo Compton scattering, are reflected back towards a detector at the other end of the sonde. The intensity of the backscatter is a measure of the formation density.

To the knowledge of the author, only one density measuring device based on attenuation of a directly transmitted gamma ray beam has been developed commercially. This instrument, developed by Marathon Oil Company for Varian, did not reach the production stage before it was discarded as being uneconomical. For interest's sake a brief description of the instrument is included here.

A variable speed drive system moves the core material between a shielded gamma ray source and a shielded detector; a precision caliper measures the sample thickness, and a computer calculates density and porosity from the measured parameters. Measured thickness and computed density and porosity are recorded on a strip chart. The sample and recorder chart are driven at the same speed, making possible a direct comparison of the record and the core sample. Cores up to 3 feet in length and 1 to $4\frac{1}{2}$ inches in diameter can be measured. The measuring rate is 3 inches per minute. The accuracy of porosity measurements is about 1% (density accuracy not given but would be within 1%). The estimated cost of the instrument if commercially available would be between \$10000 and \$15000.

Comment: This instrument would find its main application in detailed porosity logging of cores in oil exploration. It would be more sophisticated and costly than necessary for determining bulk densities for gravity interpretation. The continuous recording apparatus would be superfluous as many core samples accessible to BMR, are taken from widely-separated points along the length of a core. Even if the full length of core was available, it would not be necessary to make continuous measurements of the bulk density.





Mass absorption coefficients for silica and calcite for the three main modes of interaction of gamma rays with matter (after Tittman & Wahl, 1965)

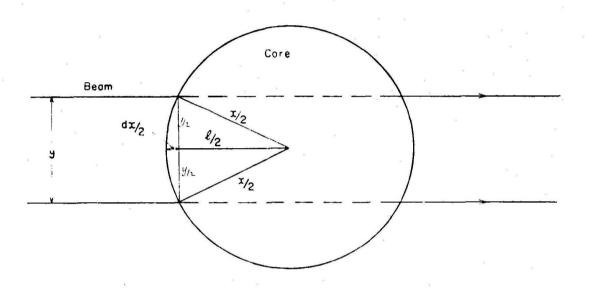


Diagram showing parameters

in calculation of optimum

beam width

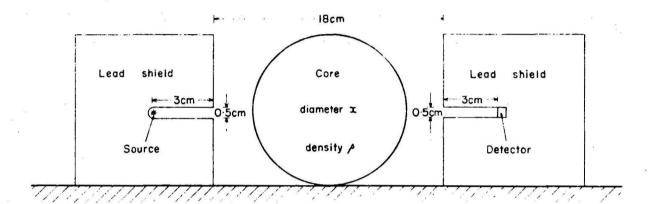


Diagram showing dimensions of beam assumed for calculation of strength of source and time constant of ratemeter.