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THE MODEL ALGORITHM FOR THE GABHYD MODEL OF THE GREAT ARTESIAN BASIN

by

G.E. SEIDEL

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CONTENTS

FIGURES

ABSTRACT

The basic equations are described for the GABHYD model, a digital computer model based on the finite-difference approach which simulates the hydraulics of the artesian groundwater flow in the Great Artesian Basin.

An outline is given of the application of the equations to the geometry, time steps, and solution sequence of the model.

1. INTRODUCTION

1.1 Purpose of the model

In 1971, BMR commenced a hydrogeological study of the Great Artesian Basin (GAB). The object of this study was to obtain from the co-operating State authorities copies of their hydrogeological data from the GAB, combine them with data available at BMR, and to assess the basin's artesian water resources. It was decided that the best way of combining the collected data was to use them in a computer-based groundwater model. The defined purpose of the model is to provide from all available data an assessment of available water resources and, by simulating the actual flow conditions, to provide a tool suitable for predicting the effects of water extraction, in particular for management purposes.

1.2 Choice of model type

In the interest of versatility and transferability it was decided to use a numerical model based on a digital computer rather than an analogue model. Two different approaches to numerical modelling were considered: finite difference and finite element. At the time when a decision on the model type had to be made, finite-element models for transient three dimensional problems were in the early stages of development, whereas finite-difference models were reasonably well proven for problems as complex as the modelling of the GAB.

In the end a finite-difference model was chosen with data defined on a regular square grid, simulating the aquifers as two dimensional layers and the aquitards by their resistance to vertical leakage (quasi-three-dimensional), and employing a point successive iterative method of solving an implicitly formulated model equation.

1.3 Program design

The computer available for developing the model was the CYBER 76 operated by the Division of Computing Research, CSIRO. Although fast in actual computation this computer had the disadvantage of relatively small available core storage and slow (i.e., expensive) data input and output. Also, being operated by a computing research organisation it was subject to relatively frequent changes to hardware and operating system, often requiring corresponding changes to programs. To best meet these conditions the model was designed in molecular form, so that groups of independent programs performed the tasks of data preparation, parameter adjustment (calibration), coding the running instructions for the model, running the model, and processing the model results. These program units can be modified individually without affecting other units and are linked together by the common data base.

2. MODEL ALGORITHM

2.1 Model equation

The basic operating equation for the GABHYD model for most numerical models of groundwater flow is the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial \theta}{\partial t}$$

where u, v, w are the flow rates in the x, y, z direction of an elemental volume and θ is the water content.

As long as laminar (non-turbulent) flow is considered, the Darcy equation describing seepage flow through a porous medium may be substituted into the continuity equation. The resulting equation may then be rewritten into a finite-difference form.

Alternatively the derivation of the model equation may be based on formulating the physical process expressed by the Darcy equation and the continuity equation directly in finite-difference

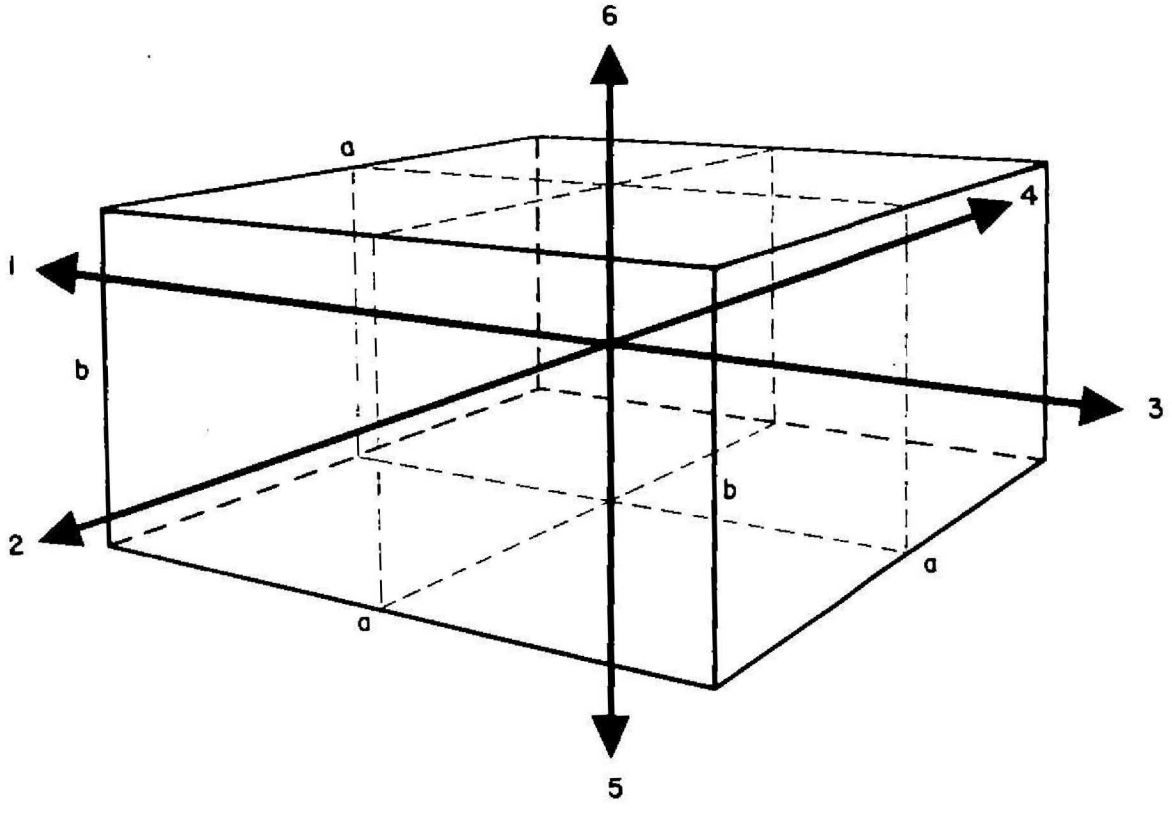


Fig. 1 Elemental volume of model based on square grid a x a for aquifer of thickness b, with six directions of flow

form. The result is the same as when using the former method; however, the second is more easily followed and makes the limitations posed by finite difference approximations more apparent.

Based on an elemental volume as in Figure 1 the horizontal flows F_1, F_2, F_3, F_4 may be written as:

$$F_i = k_i \times \frac{h_i - h}{a} \times b \times a \quad \text{for } i = 1 \text{ to } 4, \tag{1}$$

where k_i is the hydraulic permeability between the elemental volume considered and the element adjoining in direction i, and h is the hydraulic heads (potentials) defined accordingly. The directional transmissivity $T_i = k_i \times b$ may be substituted and the equation simplifies to

$$F_i = T_i (h_i - h). \tag{2}$$

Similarly for the vertical flows F_5, F_6:

$$F_i = k_i \times \frac{h_i - h}{b} \times a \times a \quad \text{for } i = 5 \text{ to } 6, \tag{3}$$

which simplifies to equation (2) by defining a vertical-leakage factor

$$T_i = k_i \times \frac{a^2}{b} \tag{4}$$

These finite-difference forms of the Darcy equation substituted into the continuity equation yield:

$$\sum_{i=1}^{6} T_i (h_i - h) + Q + \frac{\Delta w}{\Delta t} = 0, \tag{5}$$

where Q is the term for any discharge or recharge within the volume - i.e., well discharge, $\frac{\Delta w}{\Delta t}$, represents the rate of change in water content during time interval Δt, and the summation term is the sum of the individual flows through the six sides of the elemental volume or cell.

The water content ψ can be related to the hydraulic head or potential by:

$$\psi = S \times h \times a^2 \text{ or } \Delta\psi = S \times \Delta h \times a^2, \qquad (6)$$

where S is the storativity (= specific storativity x aquifer thickness).

To represent Δh, the heads at the beginning and at the end of time interval Δt are considered:

$$\Delta h = h_{t + \Delta t} - h_t. \qquad (7)$$

Substituting this into (6), and (6) into the continuity equation (5) yields the model equation for the GABHYD model:

$$\sum_{i=1}^{6} T_i \, (h_i - h) + Q + \frac{(h_{t + \Delta t} - h_t) \times S \times a^2}{\Delta t} = 0 \qquad (8)$$

The heads h_i and h can be used either at the time t or $t + \Delta t$. For time t, the resulting continuity equation is explicitly formulated; for time $t + \Delta t$, the continuity equation is implicitly formulated. The latter has been used because it is computationally stable regardless of the length of the time interval Δt. However, in the practical implementation of the model equation some potential instability is introduced because the calculation of Q as a function of h_t for a free flowing artesian well is effectively explicit (see also 2.3).

The model equation is implemented in subroutine TRANSI in the following steps:

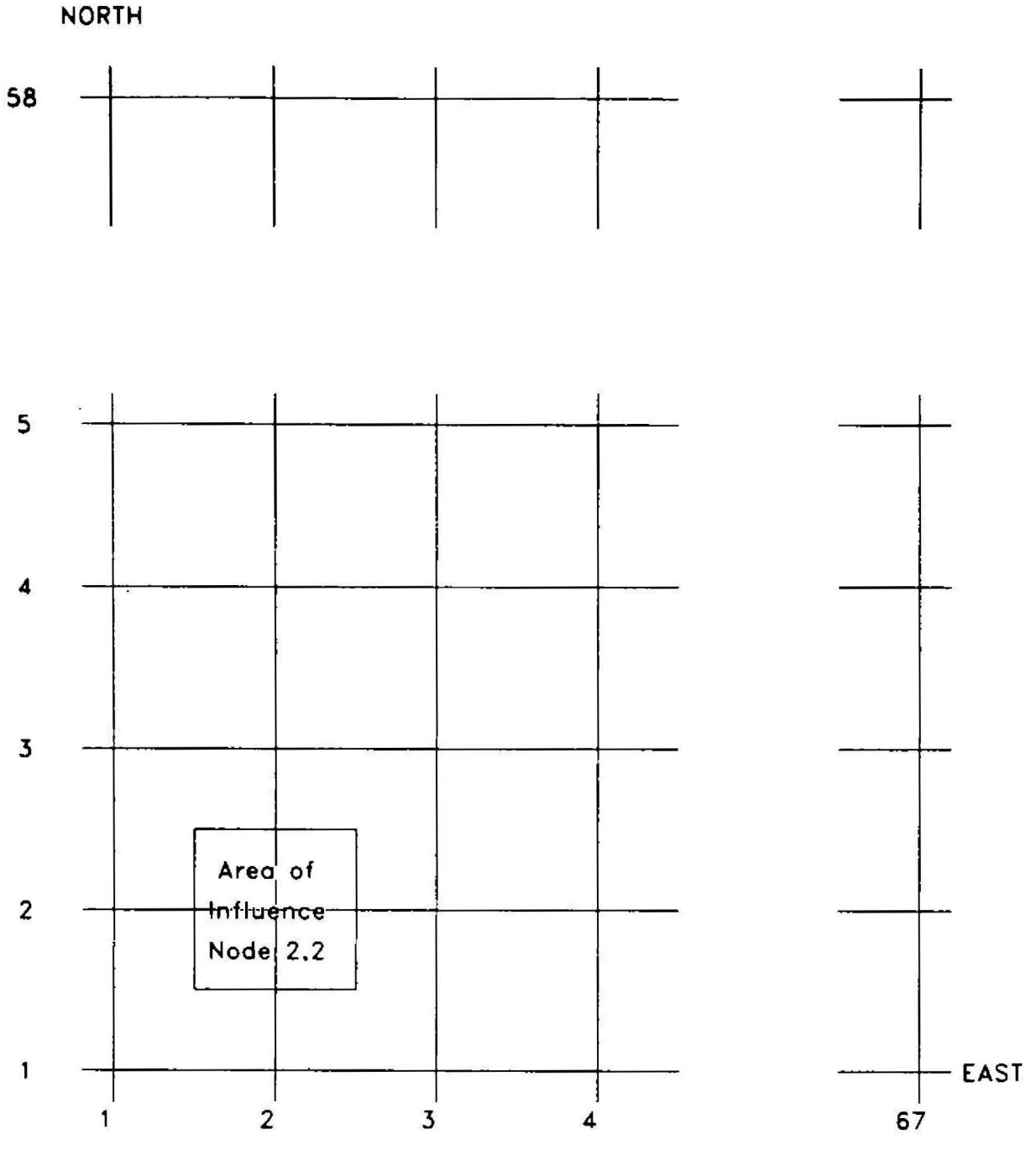
$$SH = \sum_{1}^{6} h_i \times T_i$$

$$ST = \sum_{1}^{6} T_i$$

$$SD = \frac{S \times a^2}{\Delta t}$$

$$DEL = \frac{(SH + h_t \times SD - Q}{SD + ST} - h_{t + \Delta t}$$

$$h^*_{t + \Delta t} = h_{t + \Delta t} + ORF \times DEL$$



Point 1,1 Longitude 133.30
Latitude 32.

N-S Gridlines parallel to Longitude 144.

100 km

Fig. 2. Definition of Grid

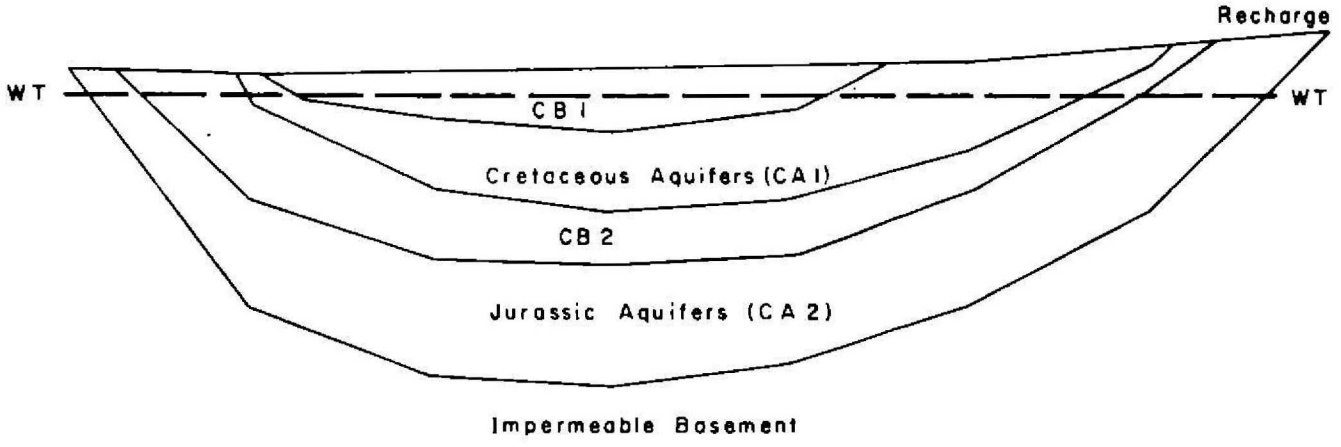


Fig. 3 Vertical Cross-section of Model Prototype

DEL is the correction term for calculating the new approximation $h^*_{t + \Delta t}$ from the old approximation $h_{t + \Delta t}$. ORF is the over-relaxation factor which was set to one for the GABHYD model, i.e. practically no over-relaxation was used making the method a plain point successive iteration.

2.2 Model geometry

The Great Artesian Basin is large (1.6 million square km) and its geometry complex. A large number of aquifers can be distinguished geologically. Hydraulically the geometry is a little simpler because some aquifers are connected and can be considered one hydraulic unit. For a more detailed description of the aquifers and their hydraulic prototype equivalent see Audibert (1976) and Habermehl (1980).

The result of the simplification of the real basin for modelling purposes has been named the model prototype of the basin. In this prototype the basin is represented in the horizontal plane by a network of nodes formed by 67 gridlines in the east-west direction and 58 in the north-south direction. The distance between the gridlines and hence the limit of resolution of the model is 25 km. Figure 2 shows the definition of the grid, including the position of its origin (location of node 1, 1). The area of influence of each node is the square 25 km by 25 km surrounding the node.

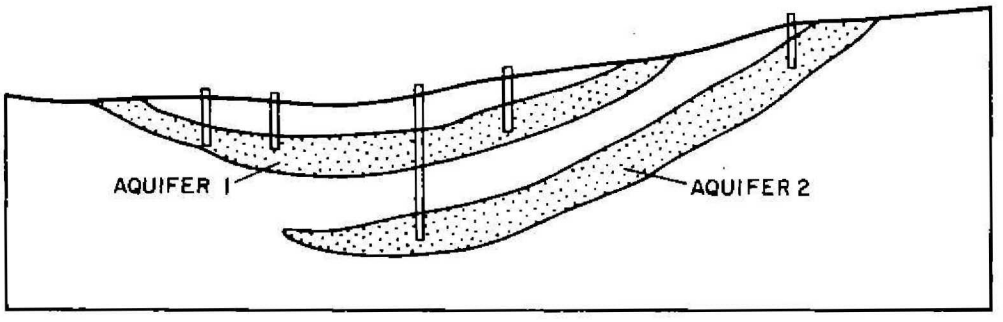
Discretisation in the vertical direction is by aquifer group. Two groups are modelled and named in the prototype 'Cretaceous' and 'Jurassic' (Fig. 3). In some areas of the basin it would have been desirable to distinguish between more aquifers, in particular within the 'Jurassic' sequence; however, a lack of adequate data prevented this, and even with only two aquifer groups included in the model, there were enough data for a reliable calibration only for the 'Jurassic' group. The 'Cretaceous' group aquifer is calculated by the model but was not calibrated, and it is not recommended that predictions be attempted for it.

Each aquifer is represented by one layer of gridnodes. This type of discretisation is called quasi-three-dimensional since it allows only for direct vertical leakage without storage in the aquitard. The aquitard is modelled only through the resistance of flow it offers to vertical leakage, which is represented numerically by a 'vertical transmissivity' as defined in 2.1 and is assigned to the node of the lower aquifer.

Data items for the aquifers are assigned to arrays indexed I, J, K, where I is the east-west index in the range 1-67, J is the north-south index 1-58, and K is a data layer index which is defined differently according to the situation. Table 1 lists the 16 basic data layers of the model, each having 67 x 58 elements, and shows some alternative data names for the various layers - e.g., PA (63, 15, 5) refers to the current potential of aquifer 2 (Jurassic group) for the node at 63 east, 15 north; TE (63, 15, 1) refers to the directional east-west transmissivity on the same node but for aquifer 1 (Cretaceous group). The latter might alternatively be referred to as HY (63, 15, 3).

TABLE 1. INTERNAL DATA STRUCTURE OF GABHYD

Data layer	Contents	Array detailed	Names general
1	Watertable potentials (metres)	PA	PA
2	Potentials aquifer 1 at time t - Δt		
3	Potentials aquifer 1 at time t		
4	Potentials aquifer 2 at time t - Δt		
5	Potentials aquifer 2 at time t		
6	Vertical leakage factor up from aquifer 1	VP	HY
7	Vertical leakage factor up from aquifer 2		
8	Transmissivity east-west aquifer 1	TE	
9	Transmissivity east-west aquifer 2		
10	Transmissivity north-south aquifer 1	TS	
11	Transmissivity north-south aquifer 2		
12	Storativity aquifer 1	SC	
13	Storativity aquifer 2		
14	Well discharge year t aquifer 1	DQ	DQ
15	Well discharge year t aquifer 2		
16	Geometry codes combined	IG	IG



Aquifer Geometry Code

0	2	4	4	5	5	4	5	2	0	0	0	0	Digit 1
0	0	0	1	5	4	5	5	5	5	4	2	0	Digit 2

Fig. 4 CORRESPONDENCE BETWEEN PROTOTYPE GEOMETRY AND GEOMETRY CODE

Layer 16 is an array of 10 integers. This contains the geometric information for the node. Digit 1 refers to aquifer 1, digit 2 to aquifer 2, etc.

The meaning of each integer is:

0 aquifer does not occur on this location (node)
1 this node is part of an impermeable boundary
2 this node is part of a prescribed potential boundary
4 node is inside aquifer, recorded potentials are available
5 node is inside aquifer, recorded potentials are not available.

Figure 4 shows how this multidigit code is constructed for the two-layer case considered by this model. During execution the model traverses the grid systematically and from the geometry code it obtains the information on how to treat a particular node.

2.3 Time discretisation

The model operates on two nested time loops. The outer loop or major time step is directly user-controlled and stated in full years. It controls the frequency of reading input for controlling the model and of writing output. For example the major time step specification

FROM = 1960, TO = 1985, STEP = 1

causes the model to (i) read control information for 1960, (ii) advance until the end of 1961 is reached, (iii) write output for 1961, (iv) then read control information for 1961, (v) advance, etc., until the end of 1985, when output for 1986 is written and the run terminates.

The control information consists of instructions on how to calculate well flow for the particular year by specifying controlled flow rates directly, or by specifying well-flow coefficients for calculation of free flow from potentials.

The minor time step is the internal computing interval corresponding to Δt in the model equation. It is dynamically adjusted by the model within limits and using parameters set by the user. These parameters are:

YMIN	1/YMIN = minimum minor time step (years)
YMAX	1/YMAX = maximum minor time step (years)
MAXI	maximum number of iterations in time step
CON	convergence limit in metres
TMF	time-step multiplication factor

The actually used time interval, DT, is adjusted according to the speed of convergence. To begin, DT is set to the minimum value = 1/YMIM. During the first time step iterations continue until the average change to potentials is less than the value specified by CON or until the maximum number of iterations MAXI have been reached. The number of iterations required for this is noted. The next time step is:

DT X TMF if the number of iterations was less than ½ of MAXI
DT / TMF if the number of iterations was more than 3/4 of MAXI
unaltered if the number of iterations was between ½ to 3/4 of MAXI

Consequently if TMF is chosen greater than 1 (e.g., 1.5) the time steps DT are increased geometrically as long as relatively few iterations are required for convergence, and reduced if the number of iterations approaches the specified maximum. At the beginning of each time step the discharges by free flow are calculated from the net pressure at each node (potential minus ground elevation, corrected for temperature) multiplied by a free flow coefficient specified as data for the major time step. This flow rate is thus valid for the entire time step - i.e., the time discretisation of the flow rate is explicit, whereas the recalculation of potentials by iterations is implicit. This neccesitates some extra consideration of the maximum time step to prevent instability

of the model operation due to the free flow term in it. By experience it was found that a maximum time interval of 0.25 years (i.e. YMAX = 4) was safe for the data used in the GABHYD model.

Within these limits the method of controlling the minor time step proved economical and free of observable problems.

Using a major time step of more than one year is not recommended because the loss of detail in the output is proportional to the length of the time step, whereas the saving in model processing time is relatively small. Instead it is recommended that the value of 1 year which is assigned as default be retained.

2.4 Solution sequence

The terms major time step and minor time step as applicable to GABHYD have been discussed already in 2.3. Figure 5 is a macroflow chart illustrating the nesting of the corresponding time-step loops. This section deals in particular with the logical implementation of the iterative solution to the continuity equation, i.e., it describes in detail the operation of the minor time-step loop.

For an implicit solution it is necessary to hold in memory storage the potentials for each node at the beginning and at the end of the time step. These are termed the initial and the current potential respectively.

At the beginning of the solution sequence the initial potentials are set equal to the current potentials and free flows are recalculated in accordance with current net pressures and free-flow coefficients. Then subroutine TRANSI is called, which carries out the actual iterative solution for the transient case.

There are four nested loops in TRANSI. The outermost loop of the routine increments the iteration number IT from 1 up to the maximum MAXI unless convergence is reached before. The second loop increments the aquifer number NA from 1 to 2, the third the I index from west to east, the fourth and innermost loop the J index from south to north. Within the innermost loop the geometry code for the node I, J, NA is decoded, and if the node is within the aquifer the current potential is recalculated using the current potentials of the up to six surrounding nodes (but practically no more than five because only a two layer case is considered). Half of these will either be on boundaries or will just have been recalculated themselves in a previous step of the same iteration (e.g. the nodes with indices I-1, J, NA; I, J-1, NA; I, J, NA-1). Recalculation proceeds by first calculating the change in potential DEL. The new current potential is then the old plus DEL x ORF. If ORF is > 1, over-relaxation results and the iteration method is called point successive over-relaxation (PSOR); if ORF = 1 the method is known by the name 'Gauss-Seidel iteration'.

The changes in potential are accumulated in variable SUM, which at the end is divided by the number of changes NSU to yield an average change which is compared to convergence criterion CON to determine whether the solution has converged sufficiently. It was found that only a relatively small number of iterations was required for each time step, because the time steps themselves were relatively small and in this situation a plain iteration with ORF = 1 is preferable. Hence over-relaxation was not used for the GABHYD model. This would have to be reconsidered if longer time intervals were used. If the average change is found to be less than CON the minor time step is completed by assigning the recently calculated current potentials as the new initial potentials. The new elapsed time is calculated by adding the time step, and if the end of the major time step has not been reached a new minor step is determined as described in 2.3.

Calibration and operation of the model are described in Seidel (1978a, b). An application example is given in Seidel (1980).

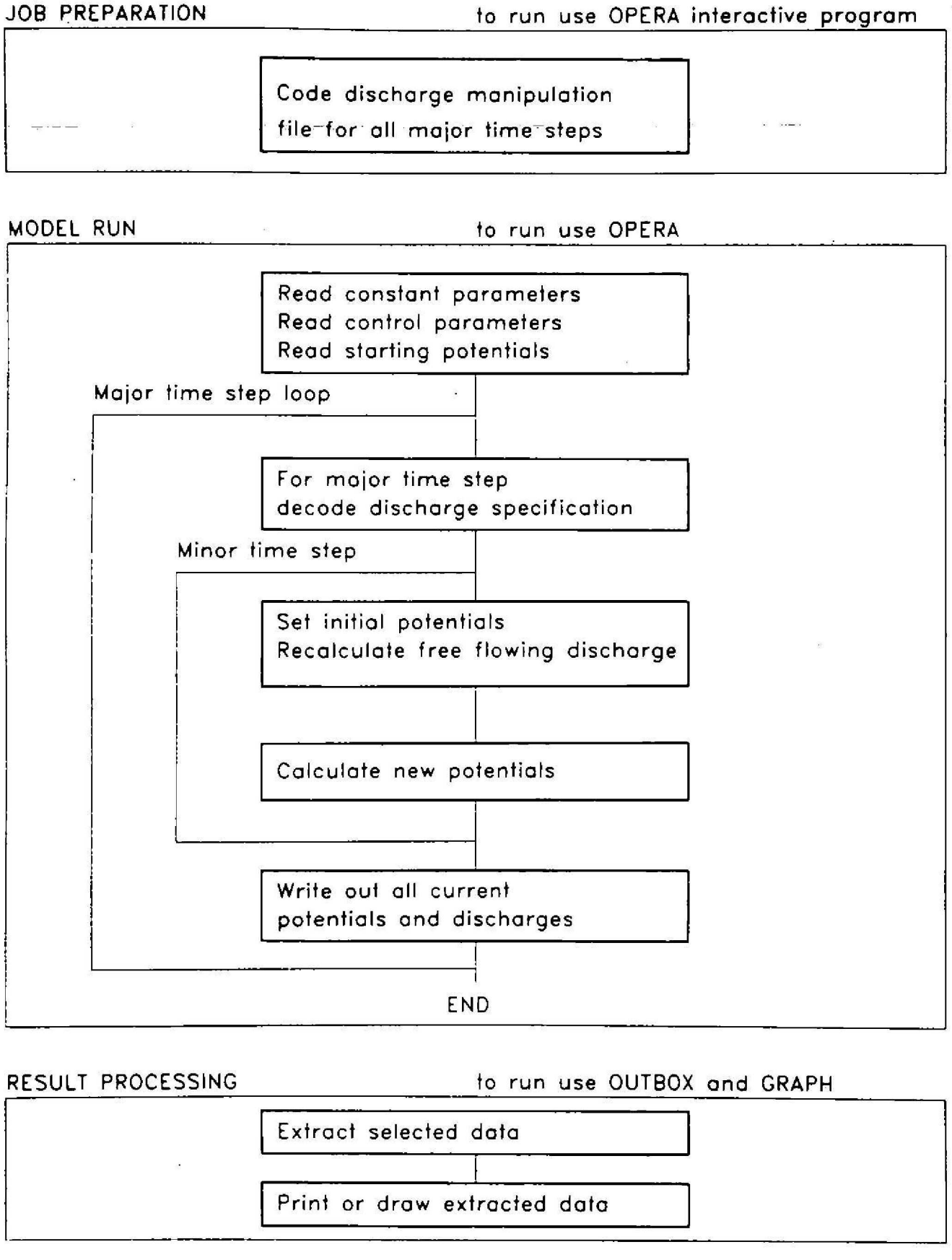


Fig. 5 Model Processing Sequence

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