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Abstract

Pressure Pulse testing of oil wells can often provide more information about the oil reservoir than that obtainable from other test procedures. The theory was developed and published in 1970 in the US Journal of Petroleum Technology. The explanation of the theory did not provide the necessary details for completely understanding the procedure by engineers who might use it. This record amplifies the theory and shows detailed derivations of the equations used. The example in the 1970 Paper is also provided with detailed calculations shown.

Contents

INTRODUCTION

Planning and then analysing the pressure pulse tests of an injection (or producing) well and its offsetting well is described in the paper "Planning and Analysis of Pulse Tests" by W.E. Brigham, Journal of Petroleum Technology, May 1970, pages 618 to 624. The mathematics appear to be quite complicated and cumbersome; however, in actual practice the results can be quite useful. The purpose of this report is to explain the derivation of the formulae given in the JPT so that the user of the figures will understand the formulae and hence use it with confidence, yet with reservations in interpretation of the results due to certain assumptions made in the development of the analyses.

The pressure pulse is created by injecting or producing a well at a given rate for a certain period, then closing the well in. The pressure pulse thus created is observed in an offset well. The magnitude of this pulse and the time delay observed can provide useful information on the reservoir characteristics provided certain other data are available from other sources. The pressure pulse can be created repeatedly - preferably with the duration of each pulse and the time between pulses being equal. As will be seen in the following sections the mathematical treatment (and hence the test analyses) are greatly simplified if the test is conducted so that the injection/production times are equal to the shut-in times and the rate of injection/production is maintained from one pulse to the next pulse.

The mathematical description of pressure behaviour of a well in an infinite reservoir is described by the line source well solutions for the familiar radial flow equations with small and constant compressibility. This solution is described in "Pressure Build-up and Flow Tests in Wells" by C.S. Matthews and D.G. Russell, Monograph Volume 1, Society of Petroleum Engineers of AIME, New York (equation 2.31).

References are given in this monograph, Volume 1, for those wishing to understand the line source solution. Probably the best reference is that of Van Everdingen and Hurst "The Application of the Laplace transformation to Flow Problems in Reservoirs" trans. AIME (1949) 186, p.305-324.

This explanation does not use the group of terms called "transmissibility" and "storage" since this development is not required to understand or use the methods described in the paper by Brigham.

It is with the line source solution of the radial flow equation that the following discussion starts.

Pulse Testing theory

The pressure change due to production or injection in a well at time t and a distance r from the well is given by the formulae in "Darcy" units.

$$\Delta P = -\frac{q\mu}{2\pi kh}\left\{-\frac{1}{2}Ei\left(-\frac{\phi c\mu r^2}{4kt}\right)\right\}$$

where ΔP is in atmospheres of pressure change

q is production rate cm^3/sec at reservoir conditions

μ is viscosity in cps

k is the permeability in darcies

h is the formation thickness in cms

ϕ is the effective porosity (fraction)

c is the compressibility in vol/vol/atmosphere

r is the distance from the well in cms

t is the time in seconds

and $-Ei(-x) = \int_x^{\infty} \frac{e^{-u}}{u}\,du$

Thus the pressure disturbance in a given well can be observed in a nearby well (a distance r between the wells) at some later time, t, with the beginning of the disturbance in the given well occurring at time $t = 0$.

The sign convention used here is:

$$\Delta p = P - P_i \qquad (P_i = \text{initial pressure})$$

so that ΔP is positive for injection pulses and negative for production pulses.

Rather than make the conversion from "Darcy" units for each quantity in the above formula the formula itself can be restated in "oil field" units as follows:

$$\Delta P = - \frac{70.6\, q \mu B}{kh} \left\{ -Ei\left(- \frac{56{,}900\, \phi c \mu r^2}{kt} \right) \right\}$$

or

$$\Delta P = - \alpha q \left\{ -Ei\left(- \beta \frac{1}{t} \right) \right\}$$

where $\alpha = \dfrac{70.6\, \mu B}{kh}$

$$\beta = \frac{56{,}900\, \phi c \mu r^2}{k}$$

Note: $$70.6 = \frac{159{,}000 \frac{cm^3}{bbl} \quad 14.696 \frac{psi}{atmos} \quad 1}{(3600)(24)\frac{sec}{day} \quad 4\pi (10^{-3} \frac{mds}{darcy}) \quad 30.48 \frac{cm}{ft}}$$

$$56{,}900 = \frac{(30.48 \frac{cm}{ft})^2 \quad 14.696 \frac{psi}{atmos}}{4 \times 60 \frac{sec}{min} \quad \times \quad 10^{-3} \frac{mds}{darcy}}$$

Hence, in "oil field" units

ΔP	is in p.s.i.
q	is in barrels per day (B/D) stock tank oil
μ	is unchanged in units (cps)
k	is in millidarcies
h	is formation thickness in ft.
ϕ	is unchanged
c	is expressed in psi^{-1}
r	is distance in feet
t	is in minutes
B	is the oil volume factor reservoir barrel/barrel of stock tank oil

If a well is produced for a time Δt, and then shut in for a similar period , Δt , the pressure drop in an adjacent well can be calculated from the following formula:

$$\Delta P = \alpha q \left\{ + Ei\left(-\frac{\beta}{t}\right) - Ei\left(-\frac{\beta}{t+\Delta t}\right)\right\}$$

If a well is produced and shut in repetitively and each flow and each shut-in time is equal to the same Δt then the following formula applies:

$$\Delta P = \alpha q \left\{ \sum_{n=0}^{N-1} (-1)^n \; Ei\left(-\frac{\beta}{t - n\Delta t}\right)\right\}$$

The above equation is the result of applying the theory of superposition developed and reported in analyses of multiple rate and pressure build-up data.

Assuming a pulse length of time Δt_1 , a shut-in time Δt_2, then a second pulse length Δt_3 , and again a shut-in time Δt_4 , etc, then the pressure response is:

for Δt_1:

$$\Delta P = \alpha q \, Ei\left(-\frac{\beta}{t}\right)$$

for Δt_2:

$$\Delta P = \alpha q \left\{ Ei\left(-\frac{\beta}{t}\right) - Ei\left(-\frac{\beta}{t - \Delta t_1}\right)\right\}$$

for Δt_3:

$$\Delta P = \alpha q \left\{ Ei\left(-\frac{\beta}{t}\right) - Ei\left(-\frac{\beta}{t - \Delta t_1}\right) + Ei\left(\frac{\beta}{t - \Delta t_1 - \Delta t_2}\right)\right\}$$

where $q_1 = q_3 = q_5$ etc and for shut-in time
$q_2 = q_4 \ldots = 0$

If the pulse test is planned and carried out such that all of the Δt's are equal in time the mathematics are of course simplified. The following assumes all Δt's are equal. The pulse repetition rate and pressure response in the offsetting well is shown in Figure 1.

The time t_{Lo} is defined as the lag time for start of the detection of the first pressure pulse, P_o :

$$P_o = P_i + \alpha q \, Ei\left(-\frac{\beta}{t_{Lo}}\right)$$

The peak of the first pulse is reached at a time t_{L1} after the first pulse is discontinued, Δt. The pressure at this time is given by:

$$P_1 = P_i + \frac{\alpha q}{\delta}\left\{ Ei\left(-\frac{\beta}{t_{L1}+\Delta t}\right) - Ei\left(-\frac{\beta}{t_{L1}}\right)\right\}$$

The end of the first pulse is detected at t_{L2} after $2\Delta t$ or the time the second pulse is started. The pressure, P_2, at this time is given by:

$$P_2 = P_i + \frac{\alpha q}{\delta}\left\{ Ei\left(-\frac{\beta}{t_{L2}+2\Delta t}\right) - Ei\left(-\frac{\beta}{t_{L2}+\Delta t}\right) + Ei\left(-\frac{\beta}{t_{L2}}\right)\right\}$$

Referring again to Figure 1 a tangent line can be drawn from P_0 to P_2 which corresponds to times t_{L0} and $t_{L2} + 2\Delta t$.

As an approximation only and for ease of mathematical treatment t_{L0} and t_{L2} are assumed to be equal. Then the tangent line can be described mathematically as follows:

$$\frac{dP}{dt} = \frac{P_2 - P_0}{t_{L2} - 2\Delta t - t_{L0}} = \frac{P_2 - P_0}{2\Delta t}$$

$$= \frac{\alpha q}{\delta\, 2\Delta t}\left\{ Ei\left(-\frac{\beta}{t_{L2}+2\Delta t}\right) - Ei\left(-\frac{\beta}{t_{L2}+\Delta t}\right) + Ei\left(-\frac{\beta}{t_{L2}}\right) - Ei\left(-\frac{\beta}{t_{L0}}\right)\right\}$$

The last two terms cancel out when $t_{L2} = t_{LO}$

hence:

$$\frac{dP}{dt} = \frac{\alpha q}{2\Delta t}\left\{ Ei\left(-\frac{\beta}{t_{L2}+2\Delta t}\right) - Ei\left(-\frac{\beta}{t_{L2}+\Delta t}\right)\right\}$$

For purposes of analysis another tangent line is also drawn at the pulse peak given by P_1 which is parallel to the tangent line connecting P_0 to P_2 described above. This is stated mathematically:

$$\frac{dP}{dt} = \frac{d}{dt}\left[P_i + \alpha q \left\{ Ei\left(-\frac{\beta}{t_{L1}+\Delta t}\right) - Ei\left(-\frac{\beta}{t_{L1}}\right)\right\}\right]$$

$$= \alpha q \left\{ -\frac{e^{-\frac{\beta}{t_{L1}+\Delta t}}}{t_{L1}+\Delta t} + \frac{e^{-\frac{\beta}{t_{L1}}}}{t_{L1}} \right\}$$

this is true since:

$$\frac{dP}{dt} = \frac{d}{dX}\left[\int_x^{\infty} \frac{e^{-u}}{u}\, du\right]\frac{dx}{dt}$$

where

$$x = \frac{\beta}{t}$$

$$dx = -\frac{\beta}{t^2}\, dt$$

$$\frac{dP}{dt} = \left[\frac{e^{-u}}{u} \Bigg|_{\frac{\beta}{t}}^{\infty} \right]\frac{dx}{dt}$$

$$= -\frac{e^{-\frac{\beta}{t}}}{\frac{\beta}{t}}\left(-\frac{\beta}{t^2}\right)$$

$$= +\frac{e^{-\frac{\beta}{t}}}{t}$$

Since the upper and lower tangent lines have the same slope, the equations for the two lines are equal. Thus:

$$\frac{\alpha q}{2\Delta t}\left\{Ei\left(-\frac{\beta}{t_{L_2}+2\Delta t}\right)-Ei\left(-\frac{\beta}{t_{L_2}+\Delta t}\right)\right\} =$$

$$= \alpha q\left[-\frac{e^{-\frac{\beta}{t_{L_1}+\Delta t}}}{t_{L_1}+\Delta t}+\frac{e^{-\frac{\beta}{t_{L_1}}}}{t_{L_1}}\right]$$

This equation can be restated in dimensionless form by making the following substitutions:

$$\Delta t_D = \frac{\Delta t}{\beta} = \frac{K\,\Delta t}{56,900\,\phi c \mu r^2}$$

$$t_{D_L} = \frac{t_{L_1}}{\Delta t}$$

$$Ei\left\{\left(-\frac{1}{\frac{1}{\beta}\Delta t\left(\frac{t_{L_2}}{\Delta t}+2\right)}\right)-Ei\left(-\frac{1}{\frac{1}{\beta}\Delta t\left(\frac{T_{L_2}}{\Delta t}+1\right)}\right)\right\}=$$

$$= -2\cancel{\Delta t}\left[+\frac{e^{-\frac{1}{\frac{1}{\beta}\Delta t\left(\frac{t_{L_1}}{\Delta t}+1\right)}}}{\cancel{\Delta t}\left(\frac{t_{L_1}}{\Delta t}+1\right)}-\frac{e^{-\frac{1}{\frac{1}{\beta}\Delta t\left(\frac{t_{L_1}}{\Delta t}+1\right)}}}{\cancel{\Delta t}\left(\frac{t_{L_1}}{\Delta t}\right)}\right]$$

and therefore:

$$Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}+2\right)}\right)-Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}+1\right)}\right)=$$

$$=-2\left[\frac{e^{-\frac{1}{\Delta t_D\left(t_{DL}+1\right)}}}{t_{DL}+1}-\frac{e^{-\frac{1}{\Delta t_D\left(t_{DL}\right)}}}{t_{DL}}\right]$$

The above equation gives the relationship of lag time in dimensionless form, t_{DL}, to pulse time also in dimensionless form. This relationship was calculated and presented in the form:

$$(\Delta t_D)\, t_{DL} \quad vs. \quad t_{DL}$$

Using the same mathematical type of analysis the response time for later cycles can be obtained. (See figure 5 of Brigham's paper).

In order to utilise the magnitude of the pressure pulse for obtaining additional information we consider the following:

The magnitude of the pulse is defined as the pressure difference between the pulse peak (P_1) and the "base line". The "base line" is the tangent line which was drawn from P_0 to P_2 as described above. Mathematically this is the pressure at t_{L1}, which is P_1, less the pressure at the mid-point of the tangent line between P_0 and P_2. This is expressed as:

$$\delta P = P_1 - \frac{P_0 + P_2}{2}$$

where δP is this pressure difference, and the pressures are defined as above:

$$P_1 = P_i + \alpha q \left\{ Ei\left(- \frac{1}{\frac{\Delta t}{\beta}\left(\frac{t_{L1}}{\Delta t} + 1\right)} \right) - Ei\left(- \frac{1}{\frac{\Delta t}{\beta}\,\frac{t_{L1}}{\Delta t}} \right) \right\}$$

and:

$$P_0 = P_i + \alpha q \left\{ Ei\left(- \frac{1}{\frac{\Delta t}{\beta}\left(\frac{t_{L0}}{\Delta t}\right)} \right) \right\}$$

$$P_2 = P_i + \alpha q \left\{ Ei\left(- \frac{1}{\frac{\Delta t}{\beta}\,\frac{t_{L2}}{\Delta t} + 2} \right) + \right.$$

$$\left. - Ei\left(- \frac{1}{\frac{\Delta t}{\beta}\left(\frac{t_{L2}}{\Delta t} + 1\right)} \right) + Ei\left(- \frac{1}{\frac{\Delta t}{\beta}\left(\frac{t_{L2}}{\Delta t}\right)} \right) \right.$$

These equations can be restated in the dimensionless terms so that we may write:

$$\delta P = P_i + \alpha q \left\{ Ei\left(-\frac{1}{\Delta t_D\,(t_{DL}+1)}\right) - Ei\left(-\frac{1}{\Delta t_D\,(t_{DL})}\right)\right\} +$$

$$- \frac{P_i}{2} - \frac{\alpha q}{2}\left\{ Ei\left(-\frac{1}{\Delta t_D\,(t_D)}\right)\right\} +$$

$$- \frac{P_i}{2} - \frac{\alpha q}{2}\left\{ Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}+2\right)}\right) - Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}+1\right)}\right)\right.$$

$$\left. + \; Ei\left(-\frac{1}{\Delta t_D\,(t_{DL})}\right)\right\}$$

The subscripts to the dimensionless time lag, t_{DL}, have been dropped since we assume that

$$t_{DL1} = t_{DL2} = t_{DL3} = \cdots$$

and the equation for δP simplifies to (the P_i terms cancel out):

$$\delta P_D = \frac{\delta P}{\alpha q} = -\frac{1}{2}\,Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}+2\right)}\right)$$

$$+ \frac{3}{2}\,Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}+1\right)}\right)$$

$$- 2\,Ei\left(-\frac{1}{\Delta t_D\left(t_{DL}\right)}\right)$$

The above equation gives the relationship of dimensionless pressure, δP_D, to dimensionless pulse time, Δt_D, and dimensionless time lag, t_{DL}. Since Δt_D and t_{DL} are interrelated we express δP_D as a function of t_{DL} only. In fact for purposes of presenting the function the correlation data are presented as follows:

$$\delta P \left(t_{DL} \right)^2 \text{ vs. } t_{DL}$$

Using the same mathematical type of analysis the magnitude of the pressure pulses detected in the offsetting well for later cycles can be obtained. (See Figure 6 of Brigham's paper).

Example of

Design and Applications of the test

In order to design a pressure pulse test properly the following parameters must be estimated (or guessed at):

		Example
k	is permeability (mds)	100
h	is thickness (feet)	20
μ	is viscosity of the flowing fluid (cps)	2
c	is compressibility (vol/vol/psi)	1×10^{-5}
ϕ	is porosity (fraction)	0.20

The distance between the "pulsed" well and the "detector" well, r (in feet), is also required but is usually measured, not assumed or estimated. Likewise the rate of production, q, and the volume factor, β, are also known quantities which are required for the analysis.

First we want to establish the optimum time for the duration of the pressure pulse. This is obtained from Figure 5 in Brigham's paper.

The maximum value of $\Delta t_D(t_{DL})$ appears to be 0.385 and the dimensionless time lag is in the range of 0.35 to 0.50. In Brigham's paper the value of t_{DL} = 0.35 was used, that is the lowest value obtainable from the curve. It is not clear why this was used but we have no experience to verify or refute this assumption.

$$\Delta t_D (t_{DL}) = 0.385 \text{ for } t_{DL} = 0.35$$

$$\Delta t_D = \frac{0.385}{0.35} = 1.10$$

but

$$\Delta t_D = \frac{\Delta t}{\beta} \text{ (by definition)}$$

$$\Delta t_D = \frac{K}{56\,900\,\mu\phi c r^2} \Delta t$$

then:

$$\Delta t = \frac{56\,900\,\mu\phi c r^2}{K} \Delta t_D$$

and for Brigham's example:

$$\Delta t = \frac{56900(2)(0.20)(1\times10^{-5})(500)^2}{100}$$

$$\Delta t = 626 \ min \quad = 10.4 \ hrs.$$

(used: 10 hrs.)

The magnitude of the pulse to be expected we fine (see Figure 6 of Brigham's paper):

$$\delta P_D (t_{DL})^2 = 0.022 \ \text{for} \ t_{DL} = 0.35$$

$$\delta P_D = \frac{0.022}{(0.35)^2} = 0.18$$

but $$\delta P_D = \frac{\delta P}{\alpha q} \ (\text{by definition})$$

$$\delta P_D = \frac{kh}{70.6 \mu B q} \delta P$$

then: $$\delta P = \frac{70.6 \mu B q}{Kh} \delta P_D$$

And for Brigham's example

$$\delta P = \frac{70.6(2)}{1000(20)} B q (0.18)$$

$$= 1.27 \times 10^{-2} B q$$

(this is a different value from the example since the dimensionless response used was the average odd pulse and the first pulse response was used in the above which is 10% larger than the example in Brigham's paper).

This completes the design of the pulse test and the results can be fed back through these same equations to determine any two of the five parameters which were estimated for the test design - but of course only if the other three parameters are known from other sources.

If four of the parameters are known from other sources then pulse testing can verify or cause to be modified, the distance, r, between wells. An impermeable boundary separating the two wells can also be identified.

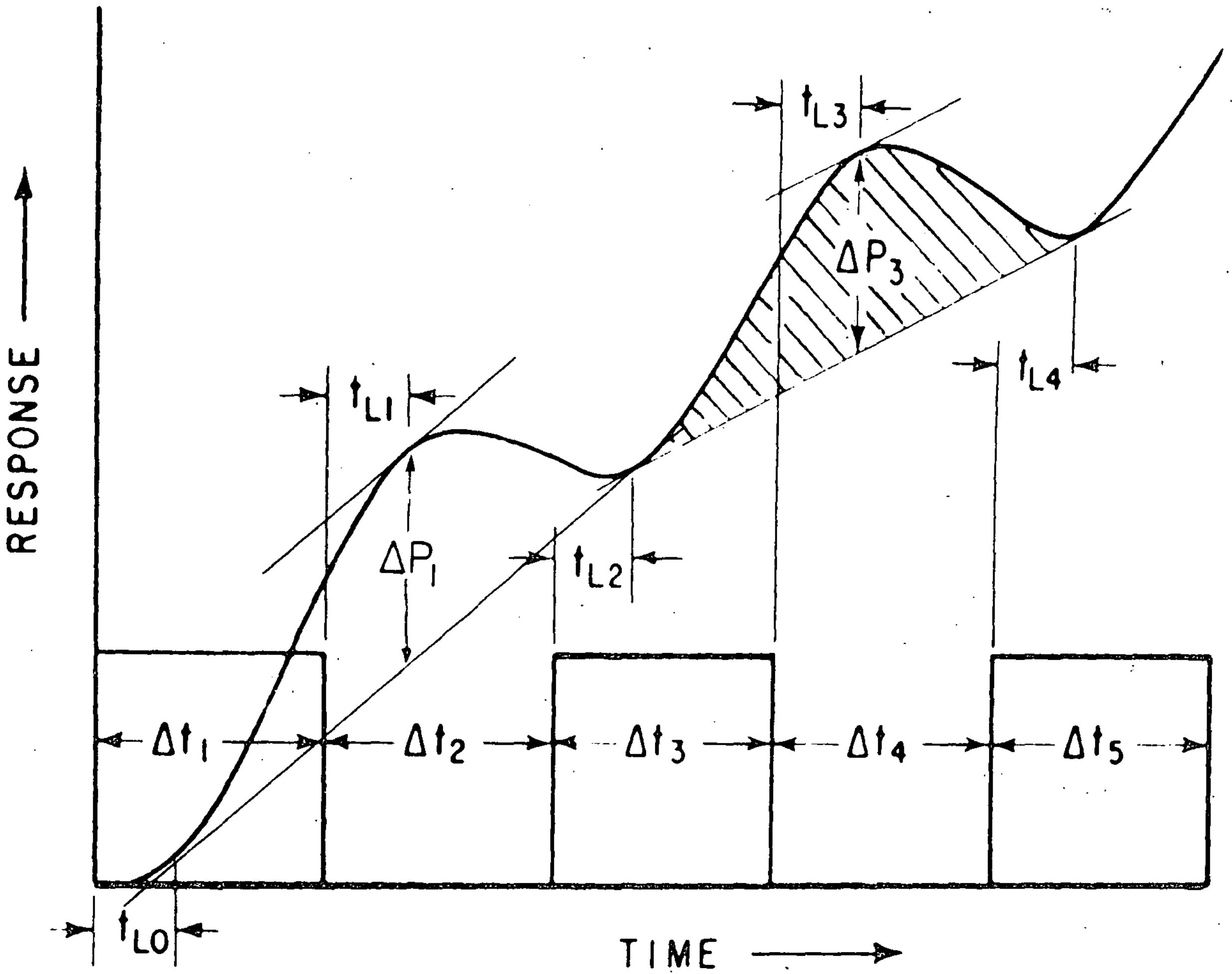

Fig. 1—Pulse-test pressure response in shut-in well.

After W. E. Brigham "Planning and Analysis of Pulse-Tests"
J. of Petroleum Technology (AIME) May 1970

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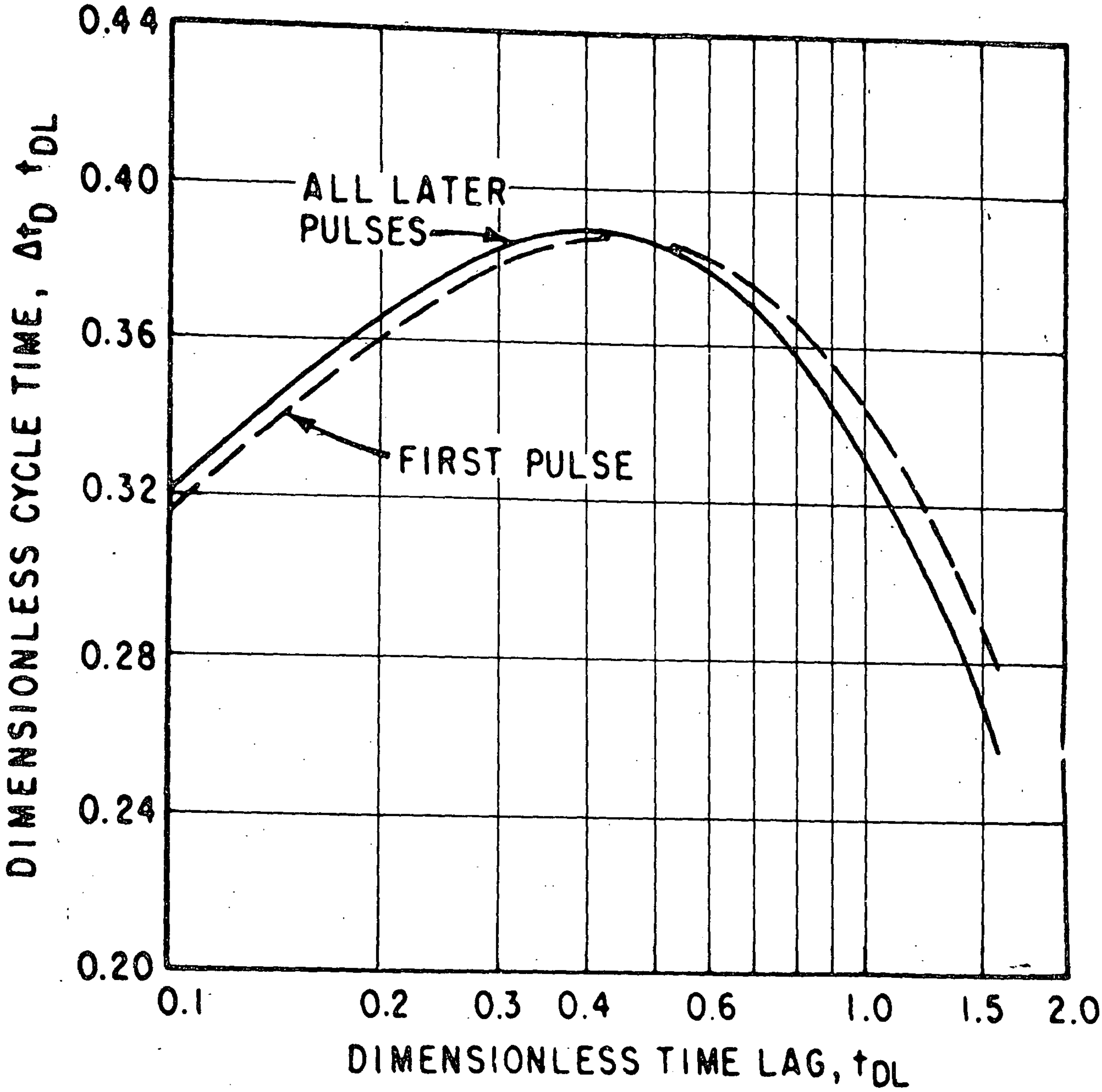

Fig. 5—Pulse-test time-lag response (all pulses).

After W. E. Brigham "Planning and Analysis of Pulse – Tests"
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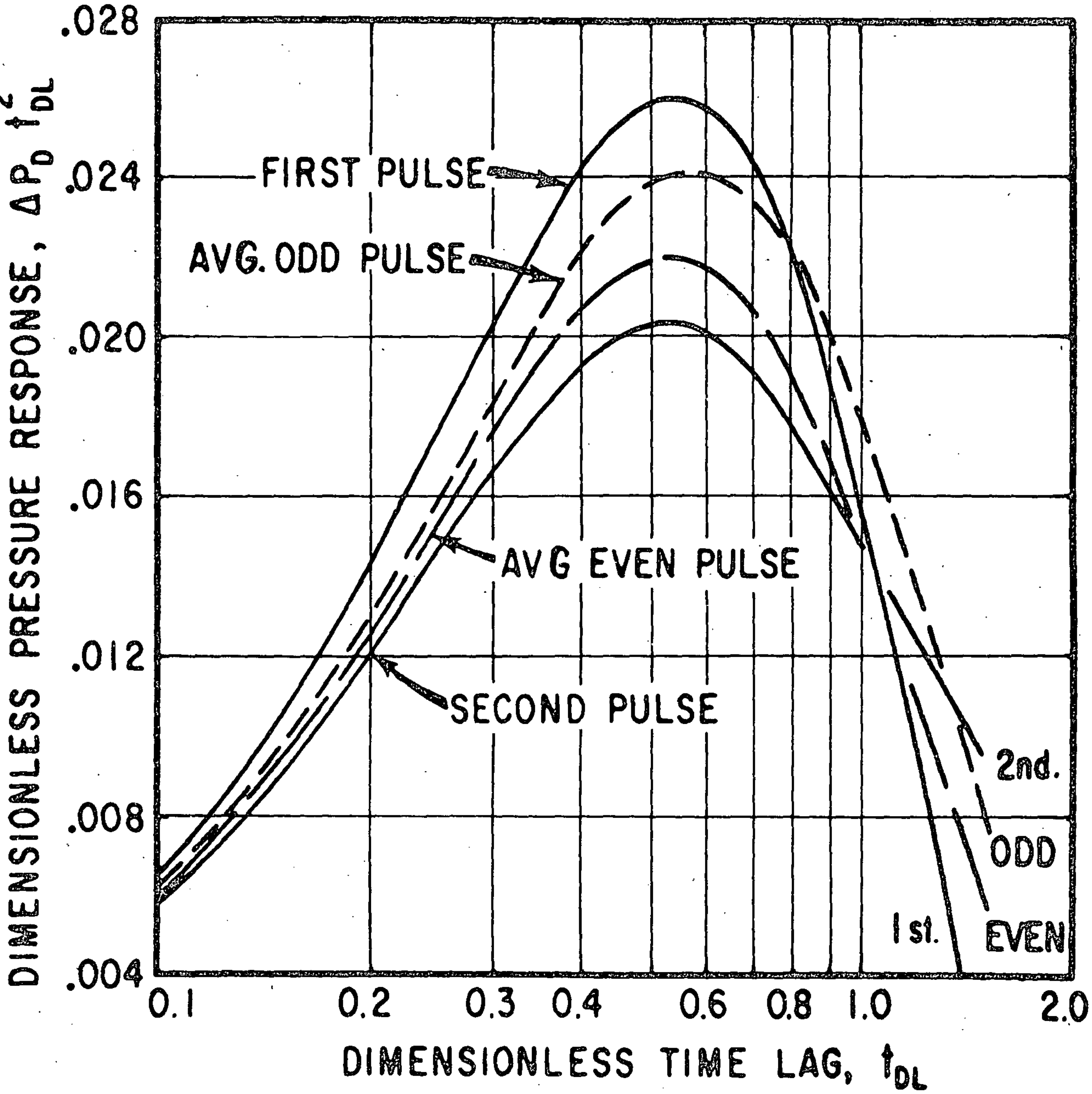

Fig. 6—Pulse-test pressure response.

After W E Brigham "Planning and Analysis of Pulse-Tests" J. of Petroleum Technology (AIME) May 1970