Applying UBC-GIF potential field inversions in greenfields or brownfields exploration

(MAG3D and GRAV3D)

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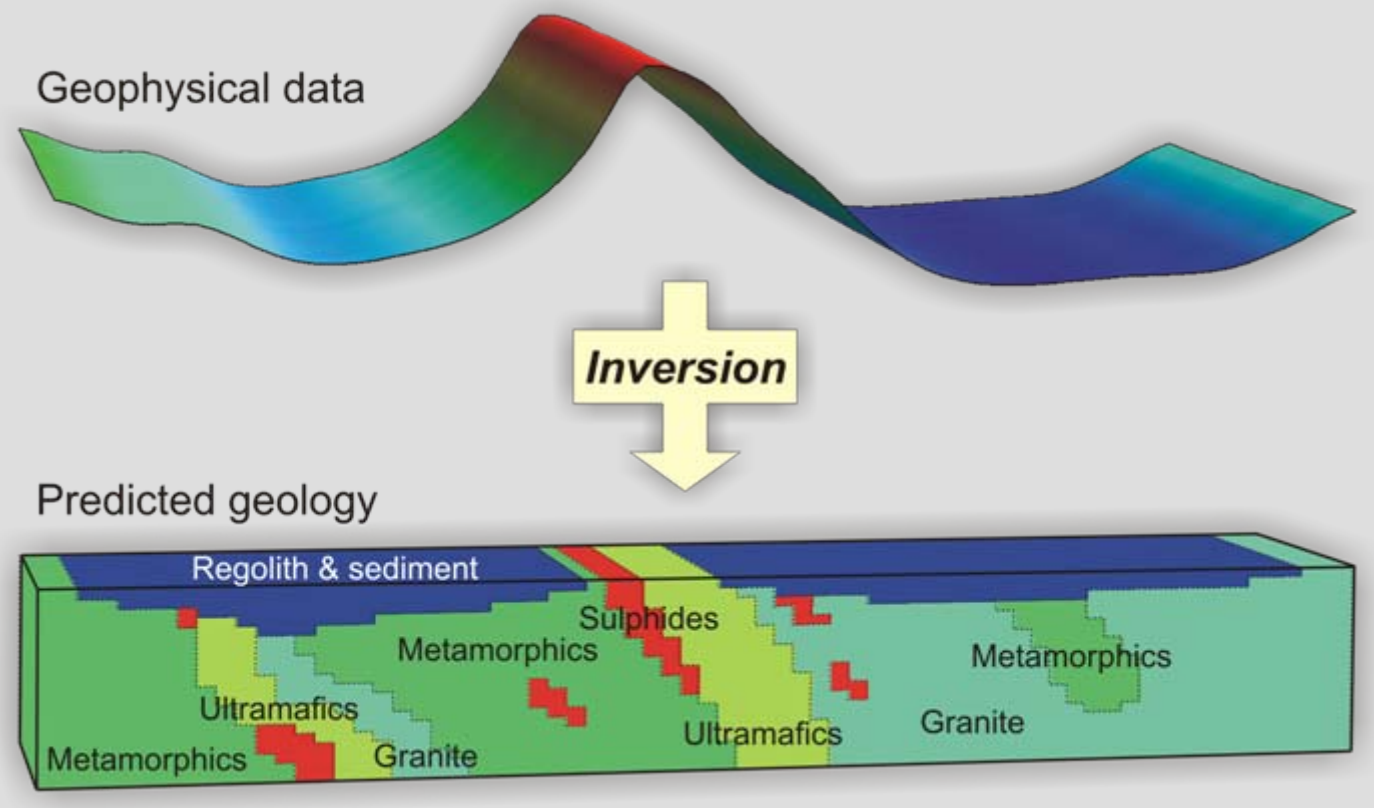
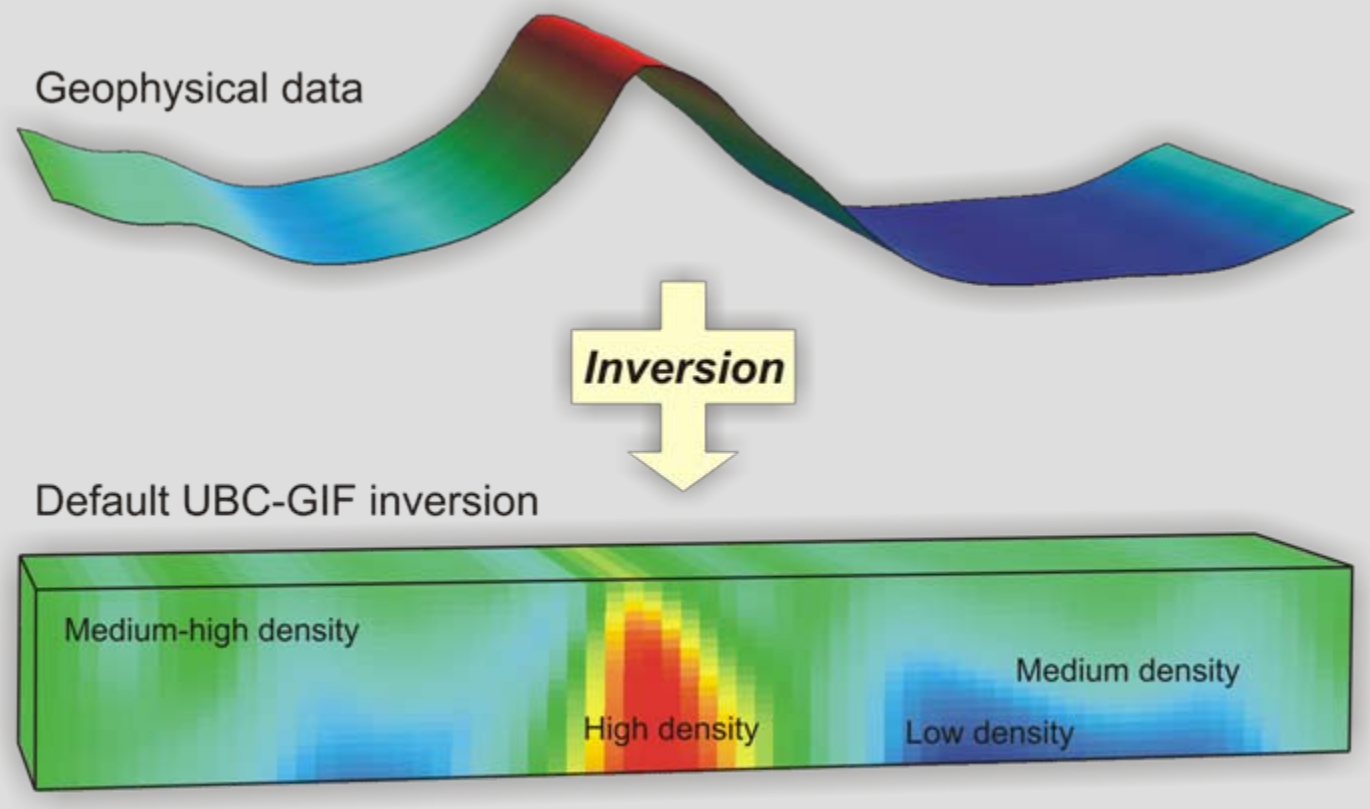
***pmd**CRC**

Geologically realistic inversion of gravity and magnetic data
Melbourne – July 1, 2006

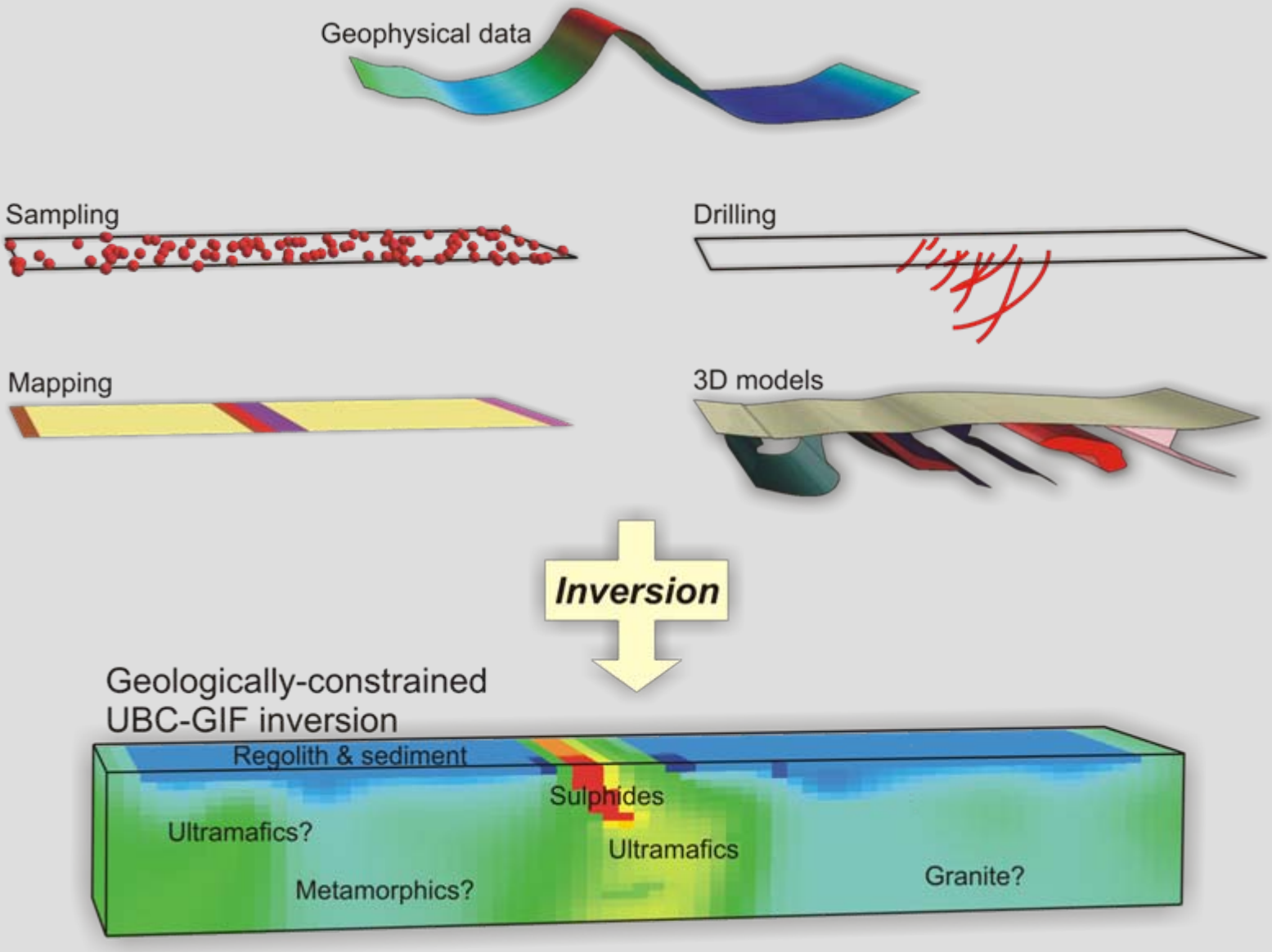
We wish we could…



But inverting the data alone gives…

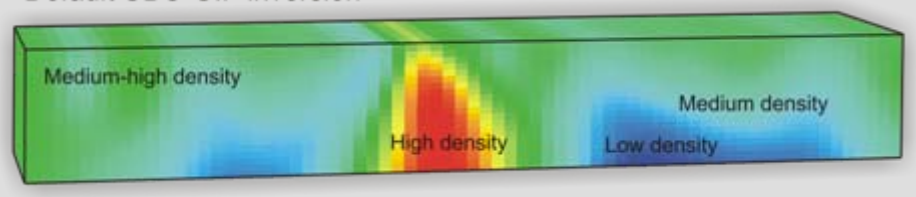


Adding geological constraints gives…



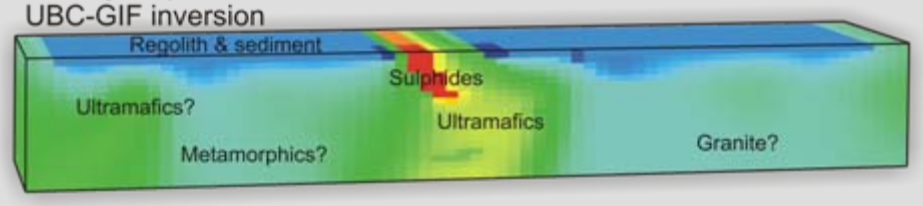
What's the difference?

Default UBC-GIF inversion



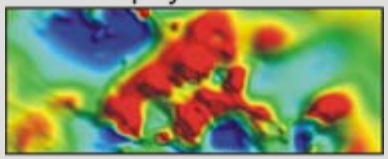
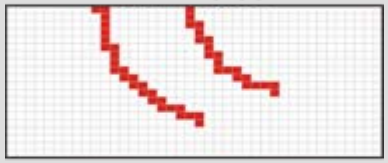
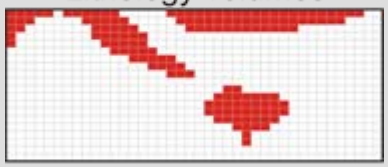
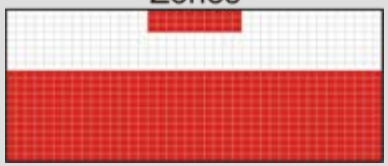
- Infinite number of solutions
- Geophysical data and mathematics are the *only* constraints
- Model does not have to be geological
- Could be a decent first guess if nothing else is known
- If you don't agree with the model you MUST have some other information

Geologically-constrained UBC-GIF inversion



- Infinite number of solutions
- Geophysical data, mathematics, *and expected geology* are constraints
- May start to be predictive of geology, because model is based on some geological expectations
- If you don't agree with the model you MUST have some other information

Types of geological constraints

Greenfields → brownfields	Geophysical data 	Framework
Greenfields → brownfields	Surface information	Physical properties Orientations
Greenfields → brownfields	Drill holes 	Physical properties Orientations
Brownfields	Lithology volumes 	Physical properties Orientations Positions & shapes
Brownfields	Cross sections	Physical properties Orientations Positions & shapes
Greenfields → brownfields	Zones 	Physical properties Orientations Positions & shapes

Inversion

- **A geophysical survey, measures:**

True response

Model:
Physical properties

$$\mathbf{d}^{\mathrm{obs}} = \boldsymbol{\varepsilon} + \mathbf{d}^{\mathrm{true}}$$

$$\mathbf{d}^{\mathrm{true}} = \mathbf{G}\mathbf{m}$$

Observed data

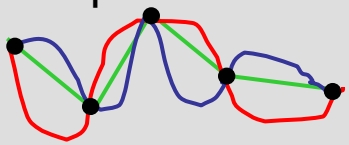
Noise

Kernel, or Forward Operator:
Specifies the physics

- **Inversion seeks to find <u>m</u> given <u>d^{obs}</u>**

Inversion problems

- **Several problems arise in obtaining a solution to the inverse problem:**
  - ***Underdetermined*: More unknowns (m) than data (d^{obs})**
  - ***Non-existence*: Noise may mean that there is NO model that can fit the data**
  - ***Instability*: Small changes in the data, *especially noise*, can cause large changes in the recovered model**
  - ***Non-uniqueness*:**
    - Even a finite number of noise-free data can be reproduced in an infinite number of ways



    - Potential field data has no inherent depth information – single layer at surface could fit the data as well as a detailed 3D model

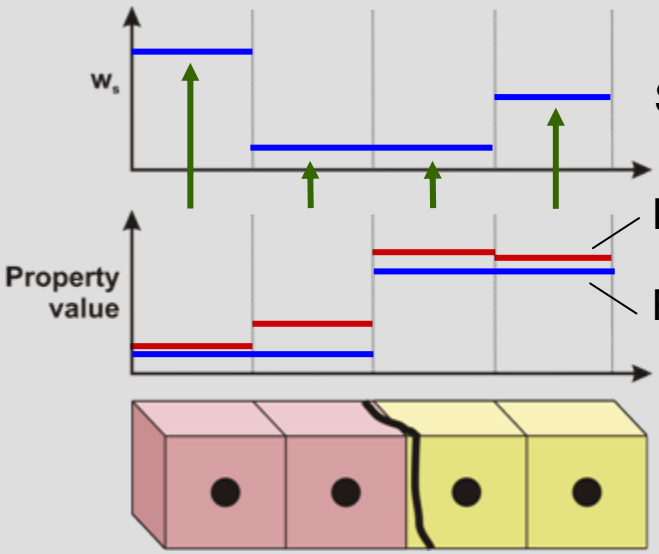
- **Calculate the inverse model solution using optimisation**
  - **Optimal model and optimal data misfit**

Model objective function: Smallness

$$\phi_m(m) = \alpha_s \int_V w_s \left[w_r(z)(m - m_{ref}) \right]^2 dV + \ldots$$

- **Attempts to match the recovered model to a reference model (small differences)**
  - **Specific cells can be matched more closely than others**
- ***m*: Recovered model – our result**
- **m_{ref}: Reference model – expected physical properties**
- **$w_r(z)$: Depth weighting function**
  - **Balances decay of potential field response with increasing depth in the model**
- **w_s: Cell smallness – confidence in reference model for each cell**
- **α_s: Model smallness – confidence in whole reference model**

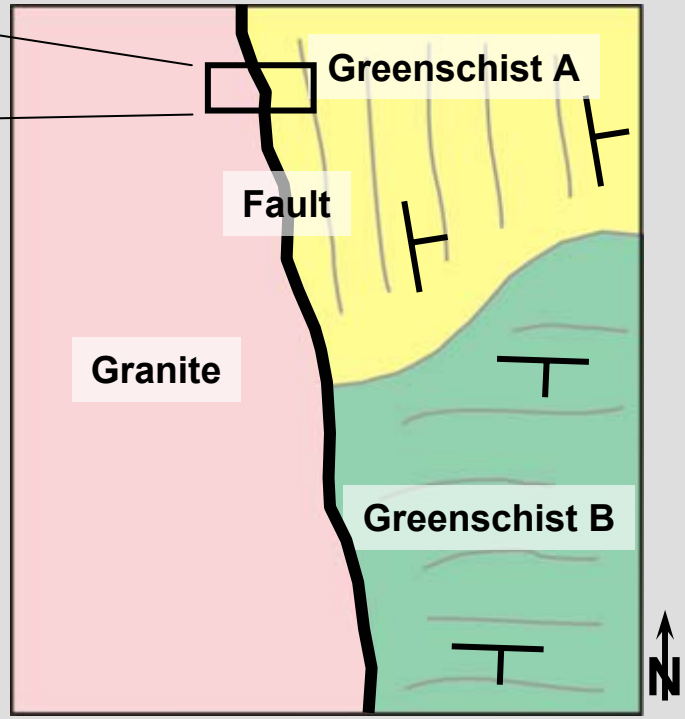
Property-based constraints



Smallness or "confidence" (w_s)

Recovered model (m)

Reference model (m_{ref})



- **Smallness weights (w_s)**
  - **Default is 1: "low confidence"**
  - **Weights are relative**
    - w_s = 10 is 10 × more confident than w_s = 1
  - **Higher smallness weights → closer match**
    - May compensate elsewhere in the model

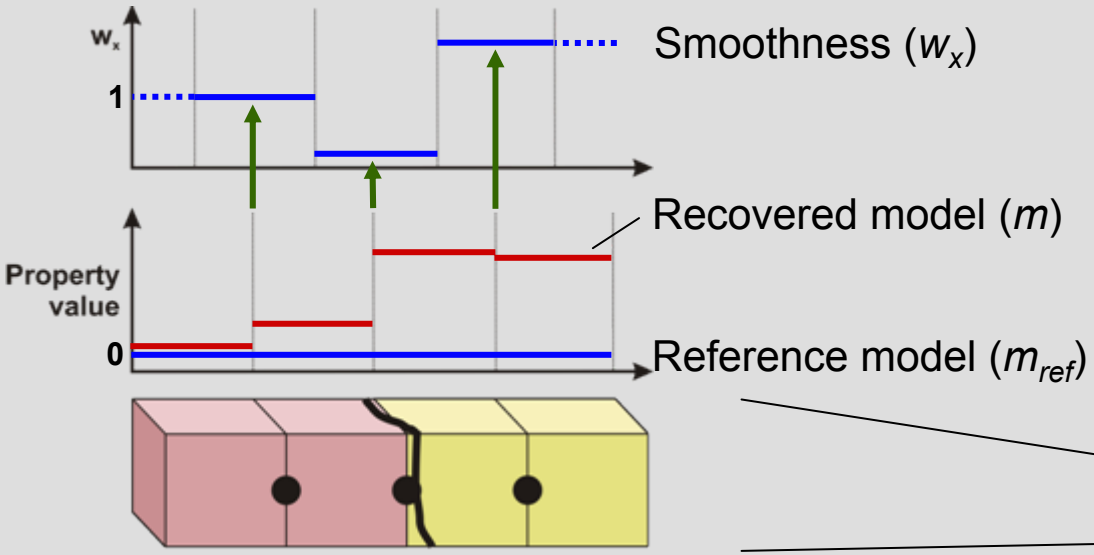
Model objective function: Smoothness

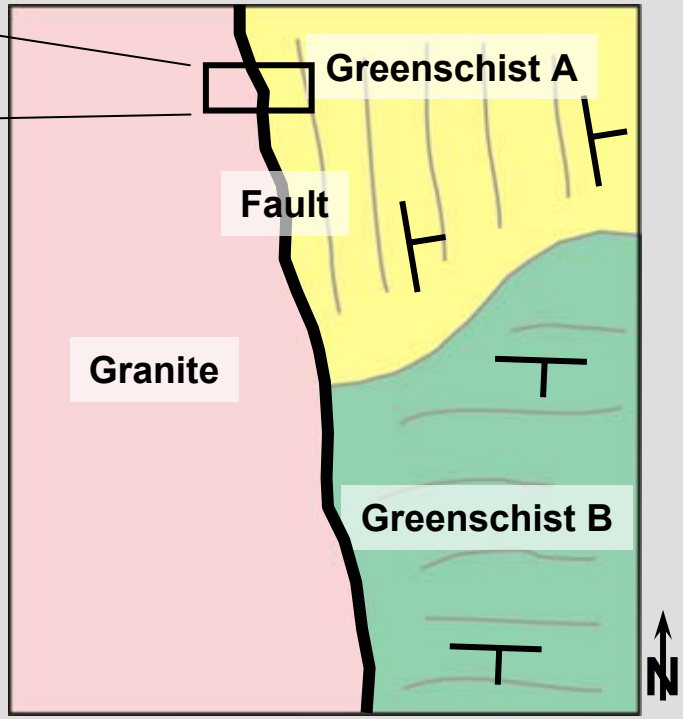
$$\phi_m(m) = \alpha_s \int_V w_s \left[w_r(z)(m - m_{ref}) \right]^2 dV + \ldots$$

$$\alpha_x \int_V w_x \left[\frac{\partial}{\partial x} w_r(z)(m - m_{ref}) \right]^2 dV + \ldots$$

- **Spreads differences between the recovered model and reference model over several cells in the *x*-direction**
  - ► **Allows specific pairs of cells to vary more or less smoothly**
- w_x: **Smoothness across each cell boundary in the x-direction**
- α_x: **Model smoothness in the x-direction**

Position-based constraints





- **Default reference model is zero everywhere**
- **Smoothness weights (w_x, w_y, w_z)**
  - **Default is 1: "moderately smooth"**
  - **Weights are relative**
    - w_x = 10 means smoothness is 10 × more important than w_x = 1
  - **Values < 1 indicate promote roughness**

Model objective function: Smoothness

$$\phi_m(m) = \alpha_s \int_V w_s \left[w_r(z)(m - m_{ref}) \right]^2 dV + \ldots$$

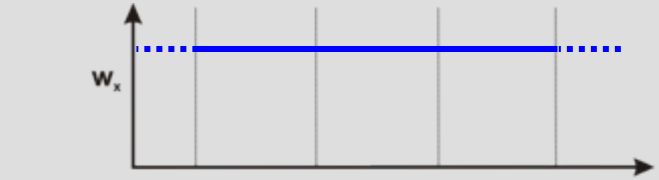
$$\alpha_x \int_V w_x \left[\frac{\partial}{\partial x} w_r(z)(m - m_{ref}) \right]^2 dV + \ldots$$

$$\alpha_y \int_V w_y \left[\frac{\partial}{\partial y} w_r(z)(m - m_{ref}) \right]^2 dV + \ldots$$

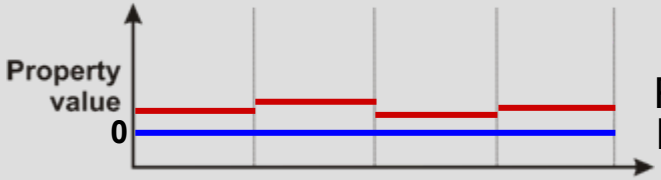
$$\alpha_z \int_V w_z \left[\frac{\partial}{\partial z} w_r(z)(m - m_{ref}) \right]^2 dV$$

- **Smoothes model differences in the *y*- and *z*-directions**
- α_y, α_z, w_y, w_z

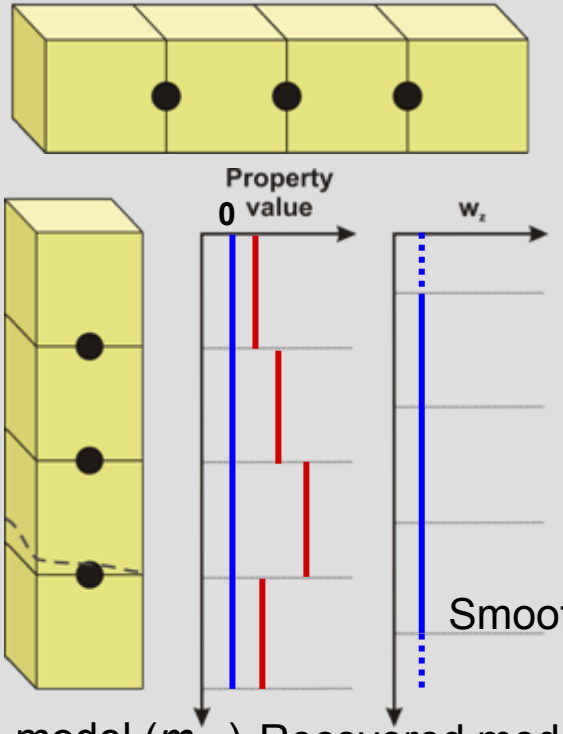
Orientation-based constraints



Smoothness (w_x)



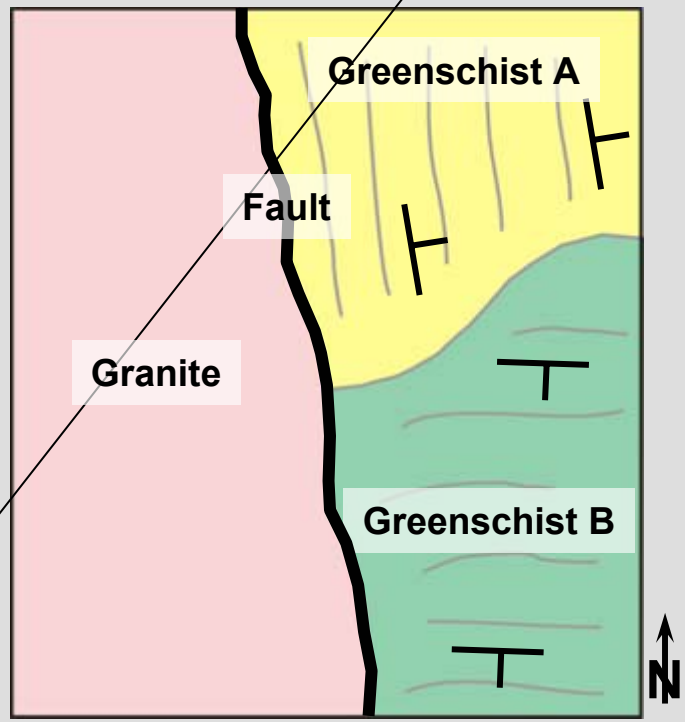
Recovered model (m)
Reference model (m_{ref})



Smoothness (w_z)

Ref. model (m_{ref}) Recovered model (m)

Cross-section



Balancing model smoothness and smallness

- **Balance is defined for the whole model by 4 user-defined α parameters**
  - **Fundamentally controlled by cell size-squared:**

$$\alpha_x = c_x \cdot \alpha_s \left(\Delta x\right)^2$$

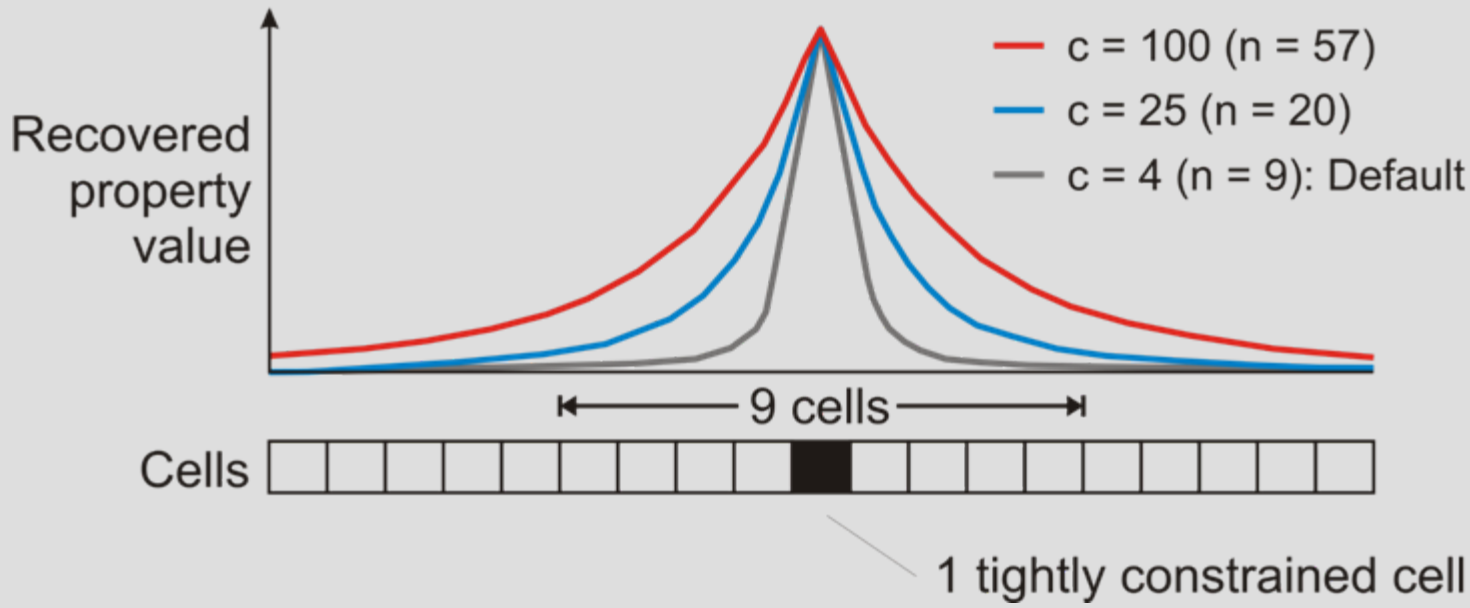
$$\alpha_y = c_y \cdot \alpha_s \left(\Delta y\right)^2$$

$$\alpha_z = c_z \cdot \alpha_s \left(\Delta z\right)^2$$

- **Δx, Δy, and Δz: cell dimensions in each direction**
- **α_s: Proportion of smallness: default = 0.0001**
- **c_x, c_y, and c_z: Model smoothness parameters in each direction**
  - **Values of 1 should equally balance smallness versus smoothness**

Model smoothness parameters: c_x, c_y, c_z

- **Typically use values between 4 and 25**
  - **Equivalent to old length scales:** $L_x = \Delta x \cdot \sqrt{c_x} = 2\Delta x$
- **Geological strike or continuity can be reproduced with high *c* values**
  - **Empirical rule of thumb:** $c_x \geq 2 \cdot (n_x - 7)$ $\quad n_x \geq 9$
    (*n* is number of cells influenced)



Data misfit

$$\phi_d = \left\| \mathbf{W_d} \left(\mathbf{Gm} - \mathbf{d}^{\mathbf{obs}} \right) \right\|^2$$

$$\mathbf{W_d} = \mathrm{diag}\left(\frac{1}{\sigma_i} \right)$$

- **Ensures the response of the recovered model matches the observed data, to within uncertainty**
- **W_d: Weights based on expected standard deviations (σ_i) for each data point**
  - **Large uncertainty → fit that data point less closely**
- **Gm: Predicted response of the recovered model**
- **d^{obs}: Observed data**

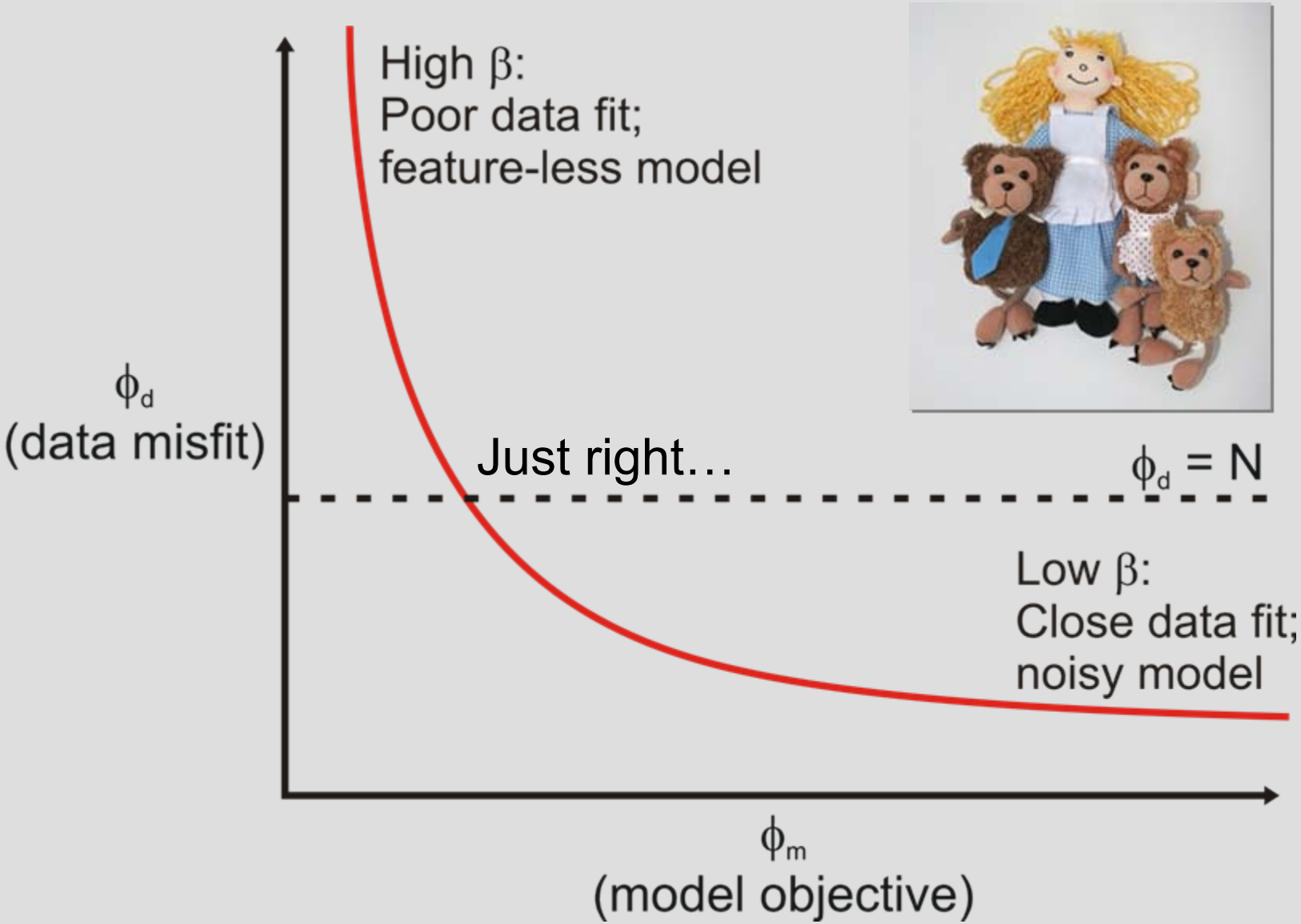
Obtaining a solution: Optimisation

$$\text{minimize } \phi = \phi_d + \beta\phi_m$$

$$\text{such that } \phi_d = \phi_d^*$$

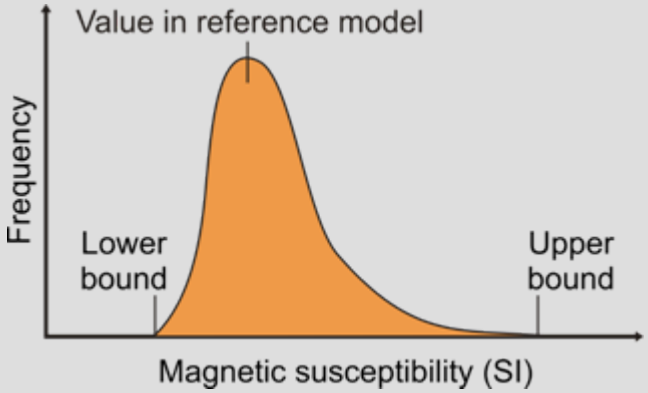
- ϕ: **Total objective**
- ϕ_m: **Model objective function**
- ϕ_d: **Data misfit**
- ϕ_d^*: **Target data misfit**
  - ► **Usually equals the number of data points**
- β: **Trade-off parameter**
  - ► **Balances data misfit and model-objective function**

Trade-off parameter



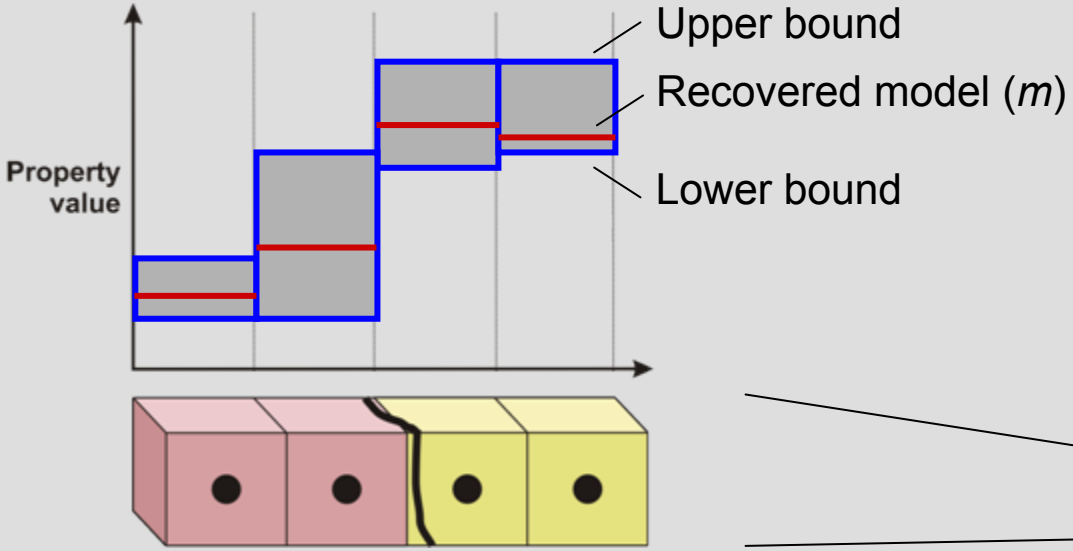
Additional non-linear constraints

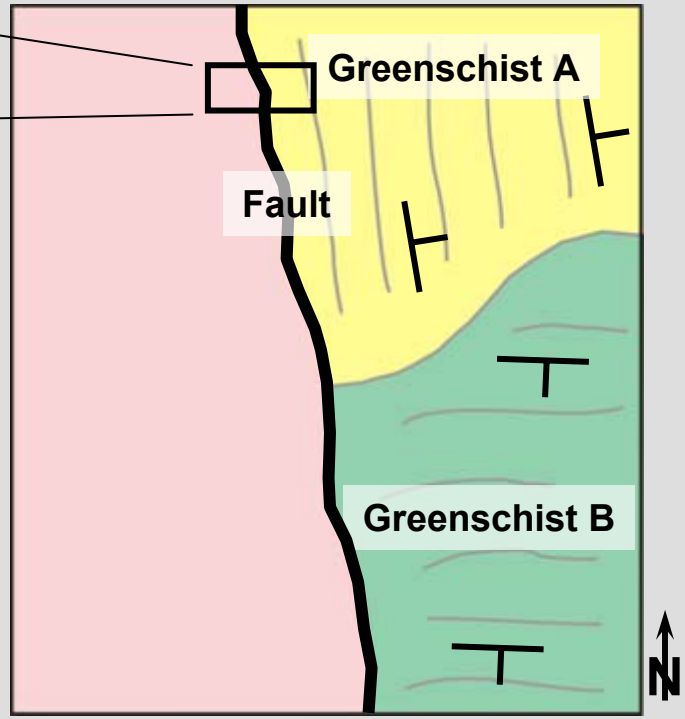
- **Physical property bounds (lower bound and upper bound)**
  - **Assigned for individual cells**
  - **Can be used with or without a reference model and smallness weights**
  - **Recovered model *MUST* lie between the lower and upper bounds**
    - Extremely powerful for restricting possible model space
    - Can lead to convergence problems if bounds are too narrow over too much of the model



- **Positivity (only for magnetics)**
  - **Enforces a lower bound of 0 SI**
    - Only positive magnetic susceptibilities are recovered

Property-based constraints: another way





- **Use wider bounds if less confident**
- **Use narrower bounds if more confident**

Parameter summary

- **Must set:**
  - **Cell sizes (Δx, Δy, Δz)**
    - Default model smallness and smoothness (α_s, α_x, α_y, α_z)
  - **Standard deviations of the data (σ_i)**

- **If geological information is available, set:**
  - **Model smoothness parameters (c_x, c_y, c_z) $\rightarrow$ (α_x, α_y, α_z)**
  - **Cell reference model (m_{ref}) & smallness weights (w_s)**
  - **Cell property bounds (*lower, upper*)**
  - **Cell boundary smoothness weights (w_x, w_y, w_z)**

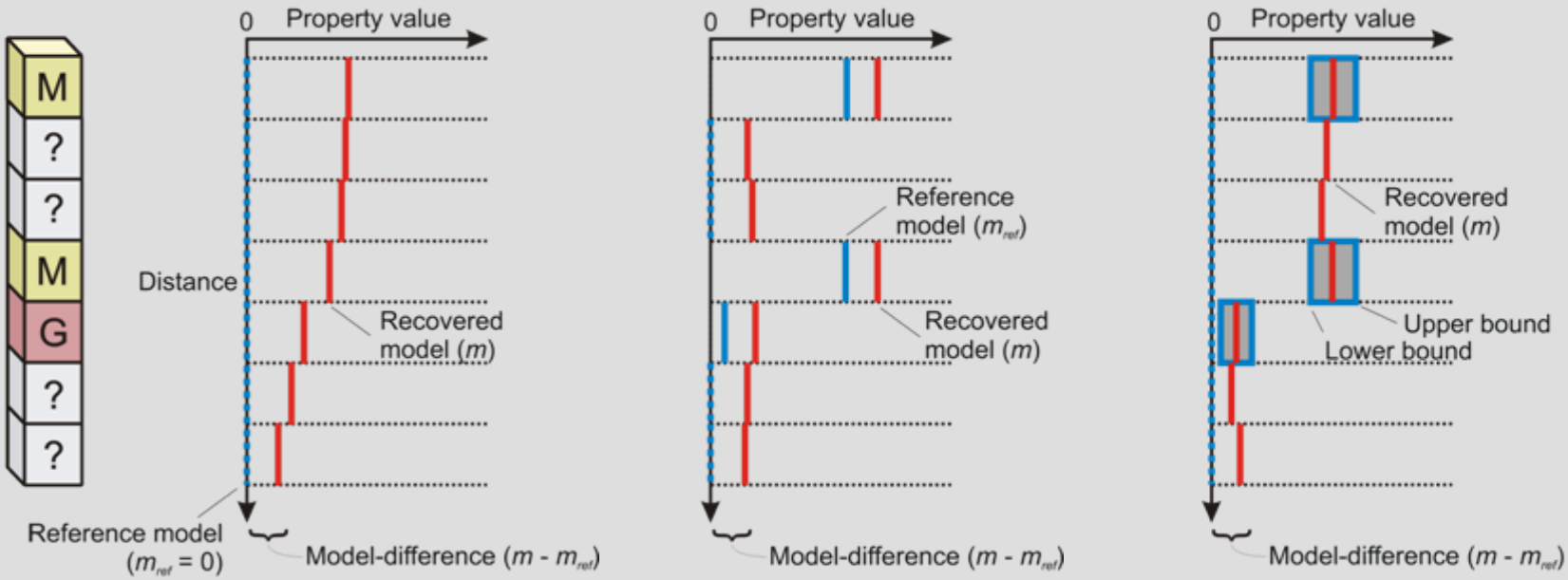
- **No need to change (but is possible):**
  - **Depth weighting**
  - **Trade-off parameter (β)**

UBC-GIF inversion summary

- **Minimum input required is geophysical data and a 3D mesh**
- **Resulting default recovered model will reproduce the data to within uncertainty, with a model that appears "small" and "smooth"**
- **3 ways to include detailed geological information**
  - **Property-based:**
    - Reference model with smallness weights
    - Bounds
  - **Position-based:**
    - Smoothness weights
  - **Orientation-based:**
    - Smoothness weights
- **Use which ever methods are most appropriate given the available geological information**

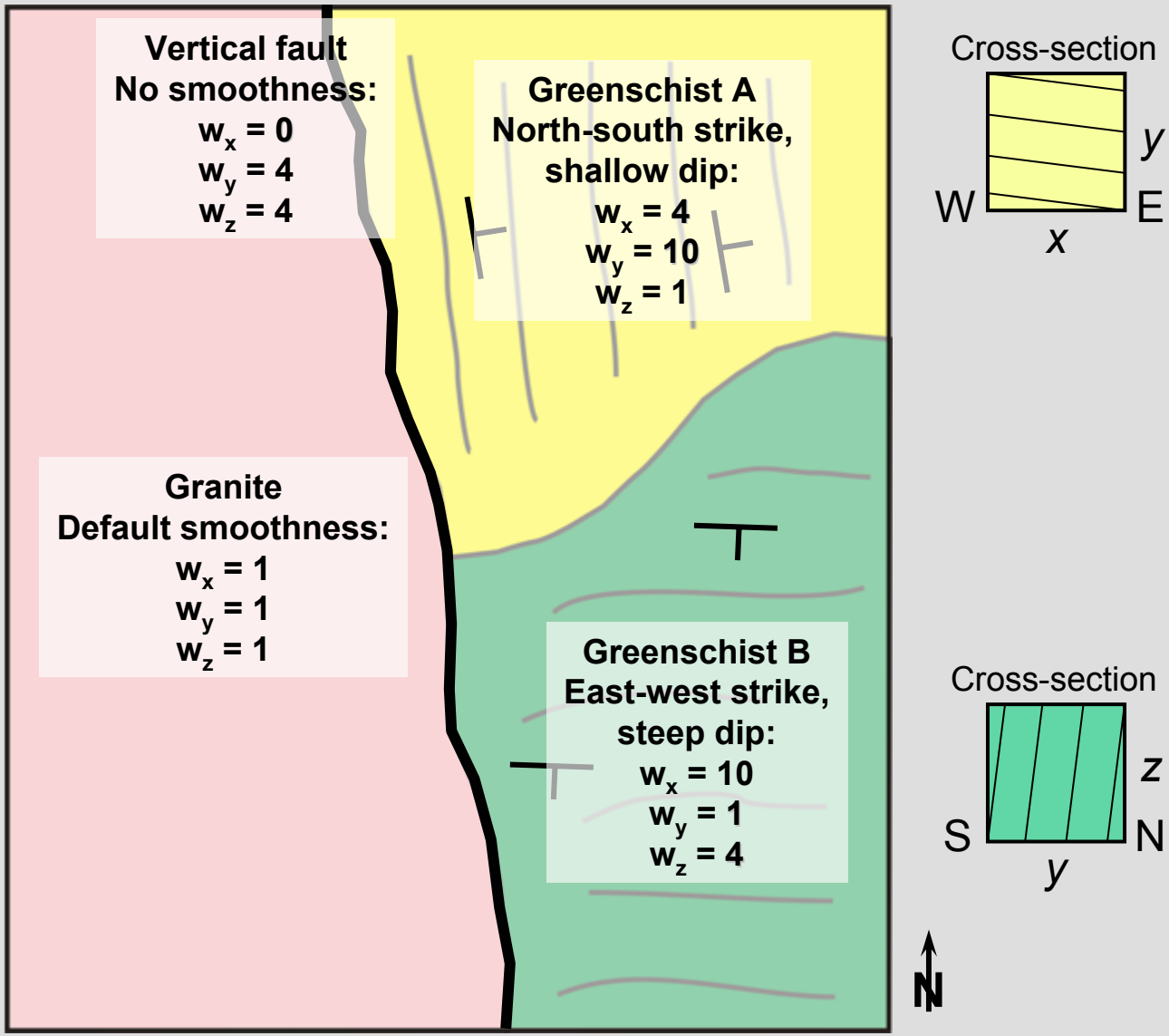
Comparison of property-based constraints

STYLE:	**Default** (Smooth model)	**Geologically-constrained** (Smooth model-difference)	**Geologically-constrained** (Smooth model)
CONSTRAINTS:	Default zero reference model Default smallness weights Default (wide) bounds	**Non-zero reference model** **Variable smallness weights** Any bounds	Default zero reference model Default smallness weights **Narrow bounds**



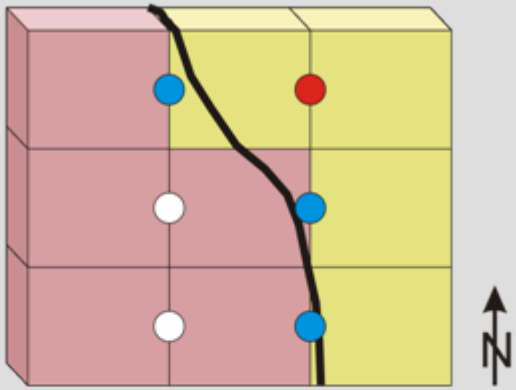
RESULT:	Model is small and smooth	Model has breaks where reference model changes	Model has breaks where bounds change sharply
WHEN TO USE:	No geological constraints are available	Expected properties and full extent of bodies is known **(full 3D model)**	Range of properties is known at scattered locations **(surface, drill holes, cross-sections)**

Position- and orientation-based constraints

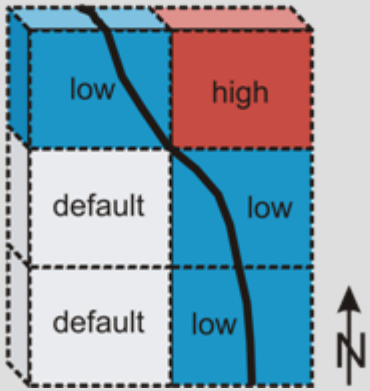


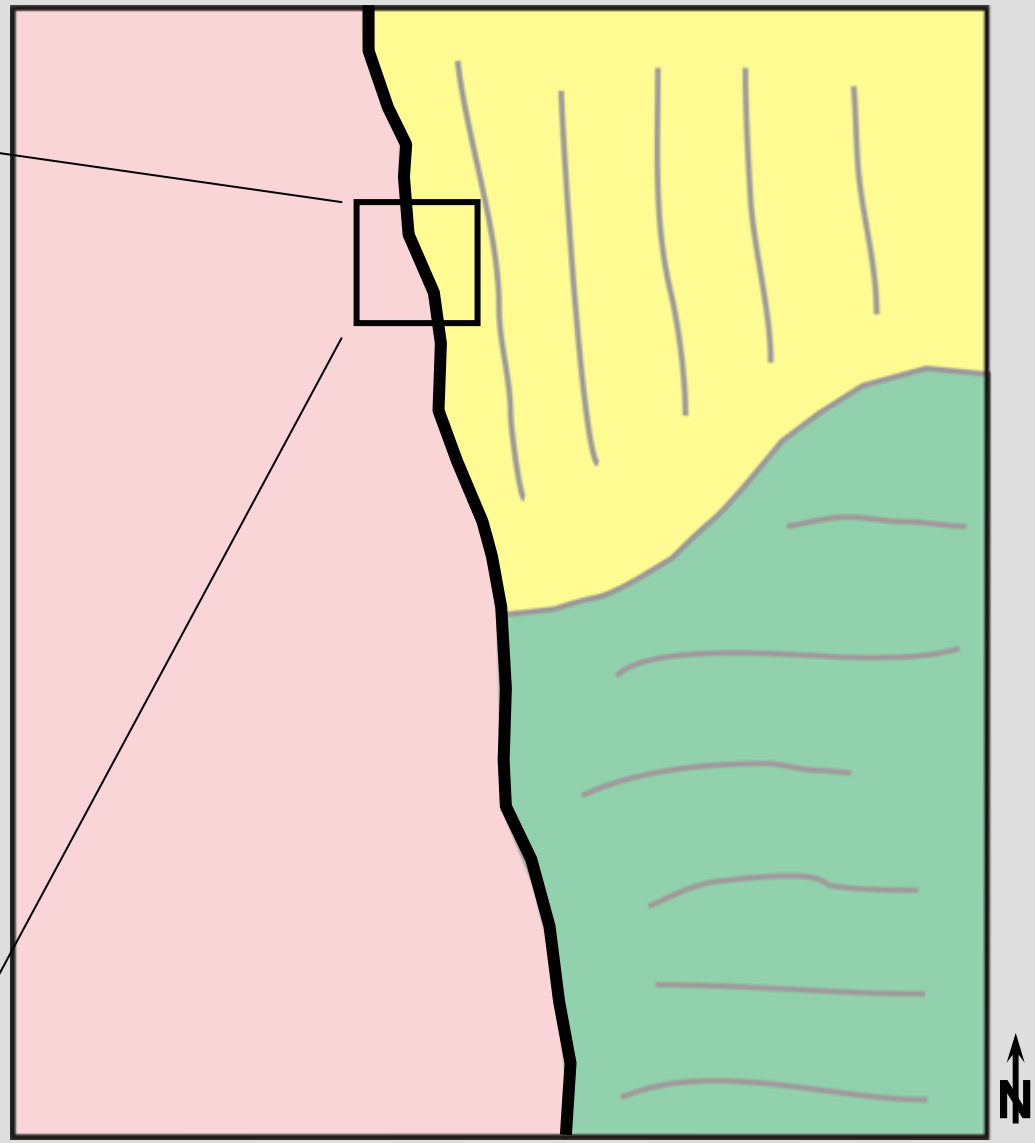
Options for geological constraints

Property information:
m_{ref} & w_s, and/or bounds



Position/orientation information: w_x



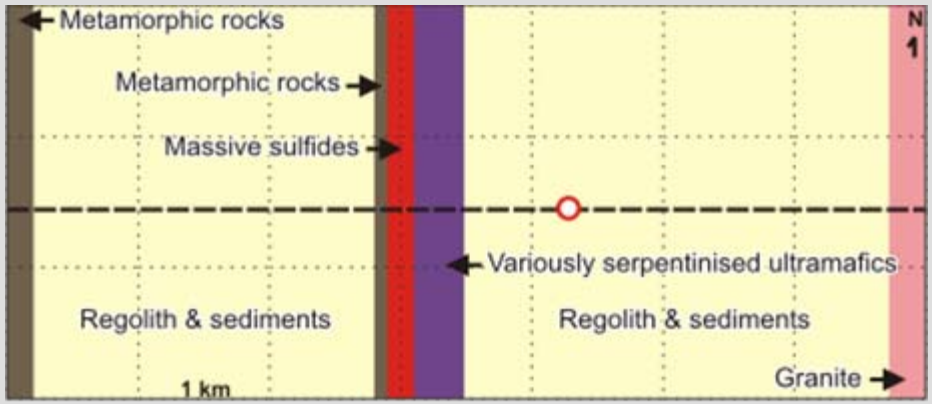


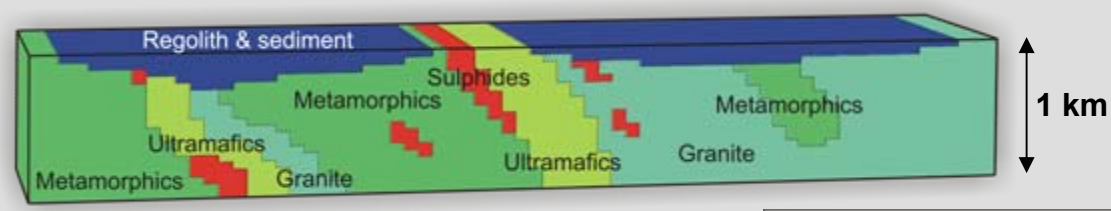
Brief inversion preparation checklist

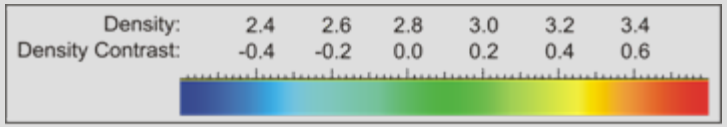
1. Define problem to be addressed
2. Define volume of interest (depth, width and length of desired mesh)
3. Define data area
4. Define cell sizes
5. Pad the mesh to prevent boundary effects
6. Upward continue the potential field data to the width of the cells
   - Removes high frequencies that can only be attributed to smaller cells
7. Calculate and remove any regional data trend (Li and Oldenburg, 1998)

A synthetic geological example

- Simple geological model
  - ► North-south-striking granite-greenstone belt
  - ► Some outcropping ultramafic units and sulphides, but more expected under extensive cover







Densities and density contrasts

Rock type	Actual (g/cm^3)	Basic sampling Minimum (g/cm^3)	Basic sampling Maximum (g/cm^3)	Detailed sampling Minimum (g/cm^3)	Detailed sampling Maximum (g/cm^3)
Regolith and sediments	2.0	1.7	2.4	1.85	2.15
Granite	2.7	2.65	2.85	2.55	2.85
Sulfides	3.8	3.4	4.2	3.65	3.95
Ultramafics	3.1	2.75	3.2	2.95	3.25
Metamorphics	2.8	2.6	3.0	2.65	2.95

- **Converting between density and density contrast → *approximate***
  - **Estimate average density expected within inversion volume**
    - Perhaps the average density of most abundant rock (→ median)
  - **Subtract average density to get density contrast:**

$$\Delta\rho = \rho - \bar{\rho}, \qquad \bar{\rho} \cong 2.8 \text{ g/cm}^3$$

- **No conversion required for magnetic susceptibility**

Densities and density contrasts

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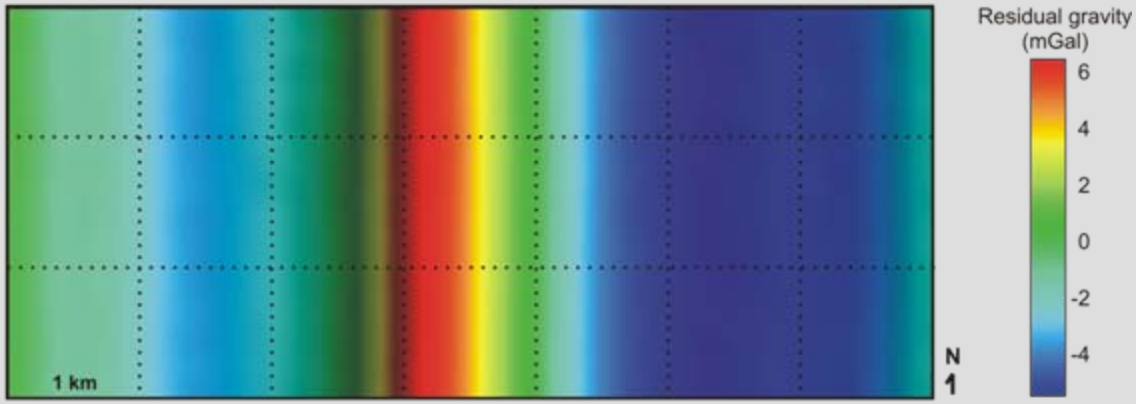
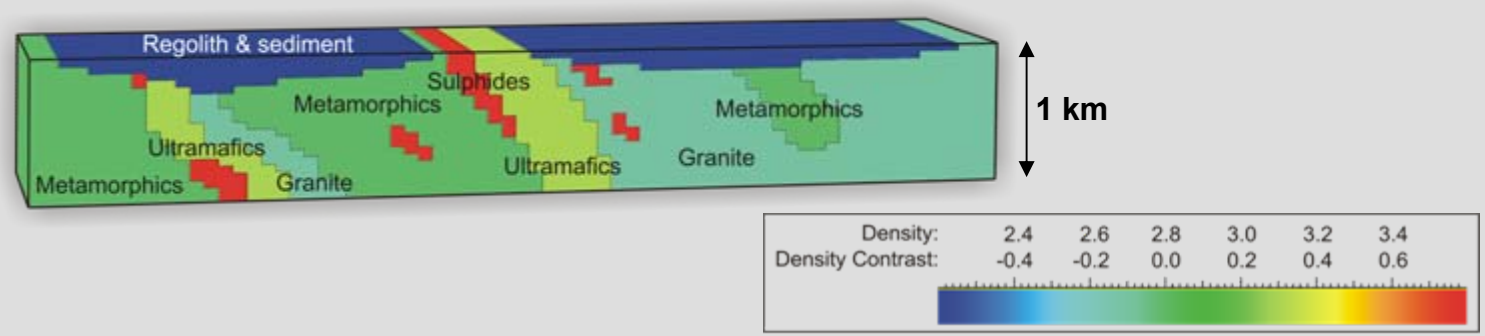
Metamorphics	2.8	2.6	3.0	2.65	2.95

Regolith and sediments	–0.8	–1.1	–0.4	–0.95	–0.65
Granite	–0.1	–0.15	+0.05	–0.25	+0.05
Sulfides	+1.0	+0.6	+1.4	+0.85	+1.15
Ultramafics	+0.3	–0.05	+0.4	+0.15	+0.45
Metamorphics	0.0	–0.2	+0.2	–0.15	+0.15

A synthetic geological example

- Synthetic gravity data, with noise added, upward continued to width of the cells
- Series of 3D gravity inversions incrementally adding geological constraints as they become available during exploration and development





Inversion setup

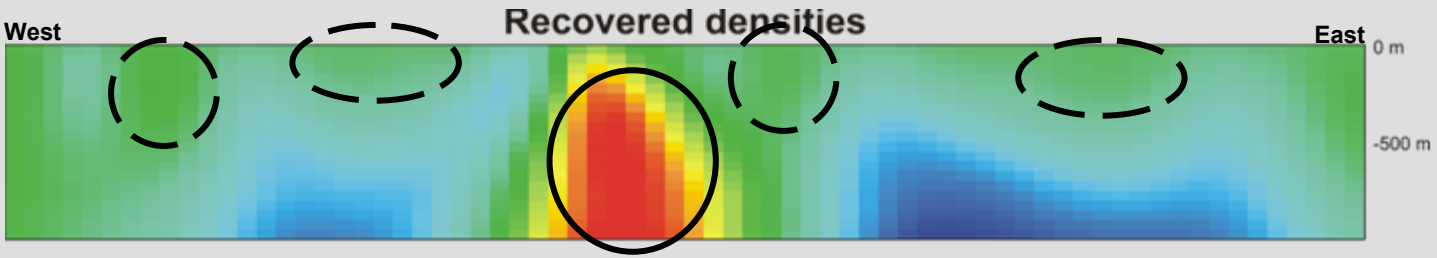
- **Mesh (volume of interest): 42,000 cells (+ extra padding)**
  - **East-west: 70 × 100 m cells (7 km)**
  - **North-south: 30 × 100 m cells (3 km)**
  - **Vertical: 20 × 50 m cells (1 km)**

- **Assigning α's**
  - **Use default model smoothness parameters: $c_x = c_z = 4$**
  - **Know north-south strike:**
    - **$c_y \geq 2 \cdot (n_y - 7)$:** n_y = 20 cells (2 km) → **$c_y = 26$**
    - $\alpha_s = 0.0001$, $\alpha_x = 4$, $\alpha_y = 26$, $\alpha_z = 1$

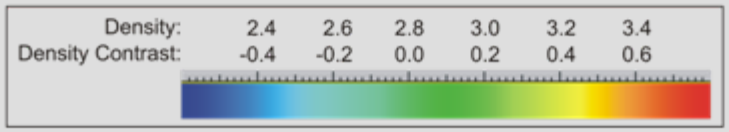
Default inversion

- No geological constraints

- Zero reference model
- Default bounds

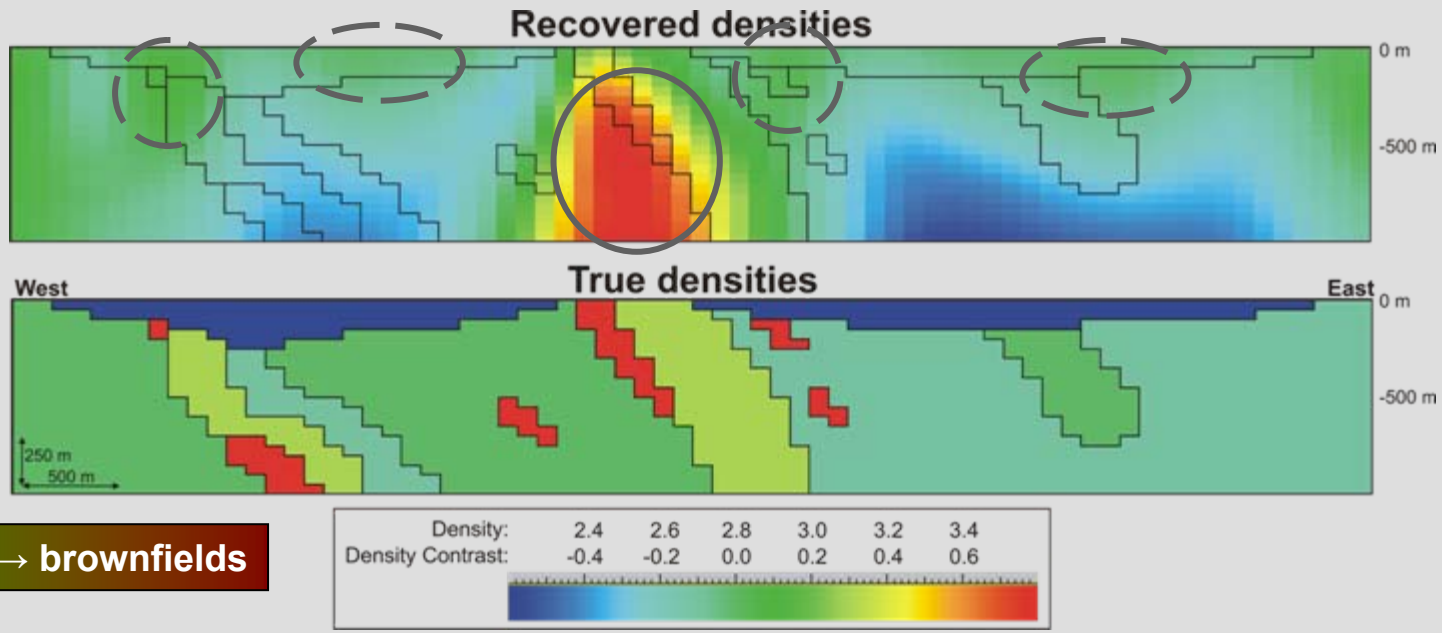


Greenfields → brownfields



Default inversion

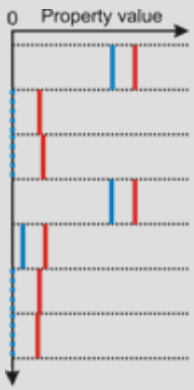
- No geological constraints
- Zero reference model
- Default bounds

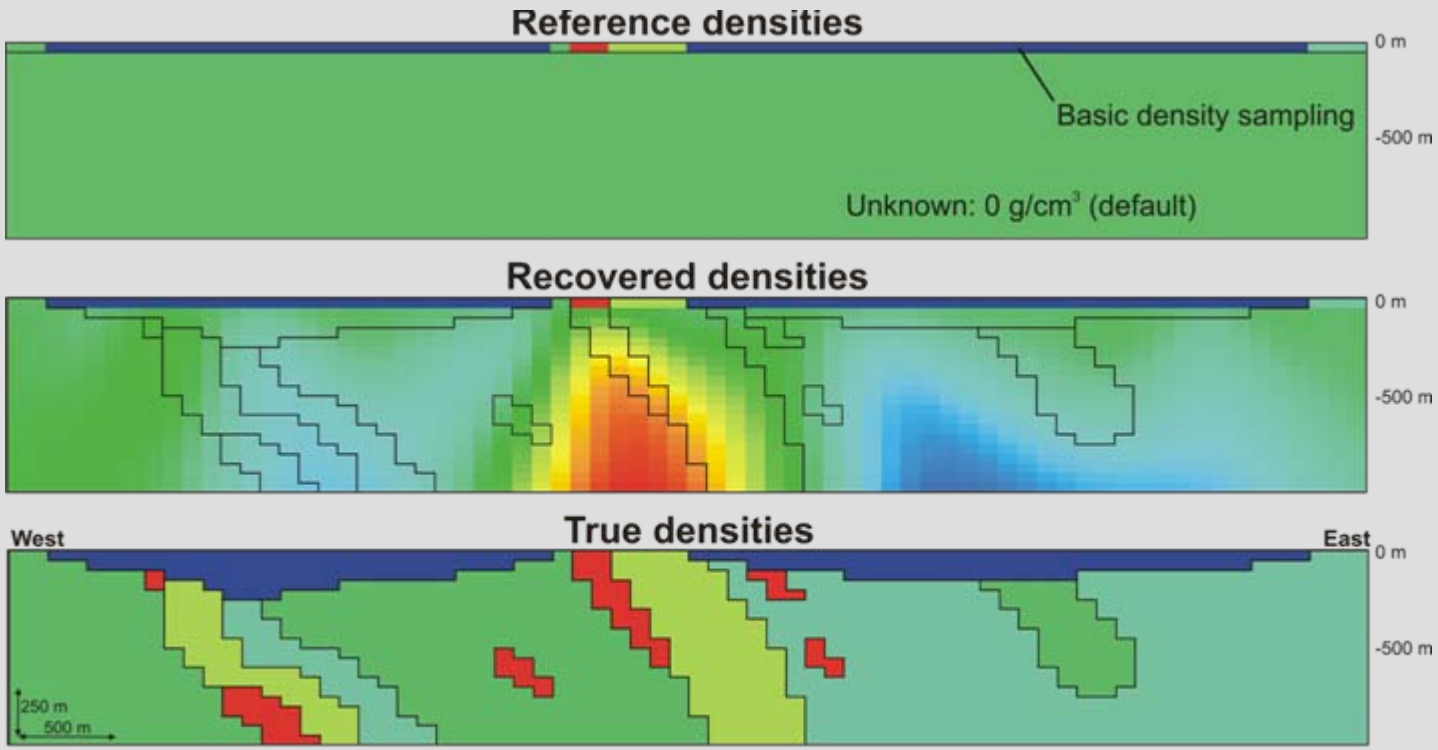


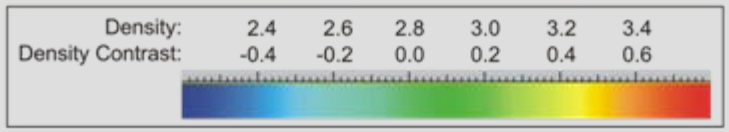
Greenfields → brownfields

Add outcrop (smooth $m - m_{ref}$)

- Surface mapping
- Some surface density measurements
- Estimate background density = 2.8 g/cm^3
- Reference model for surface cells based on mapping and mean densities
- Default bounds



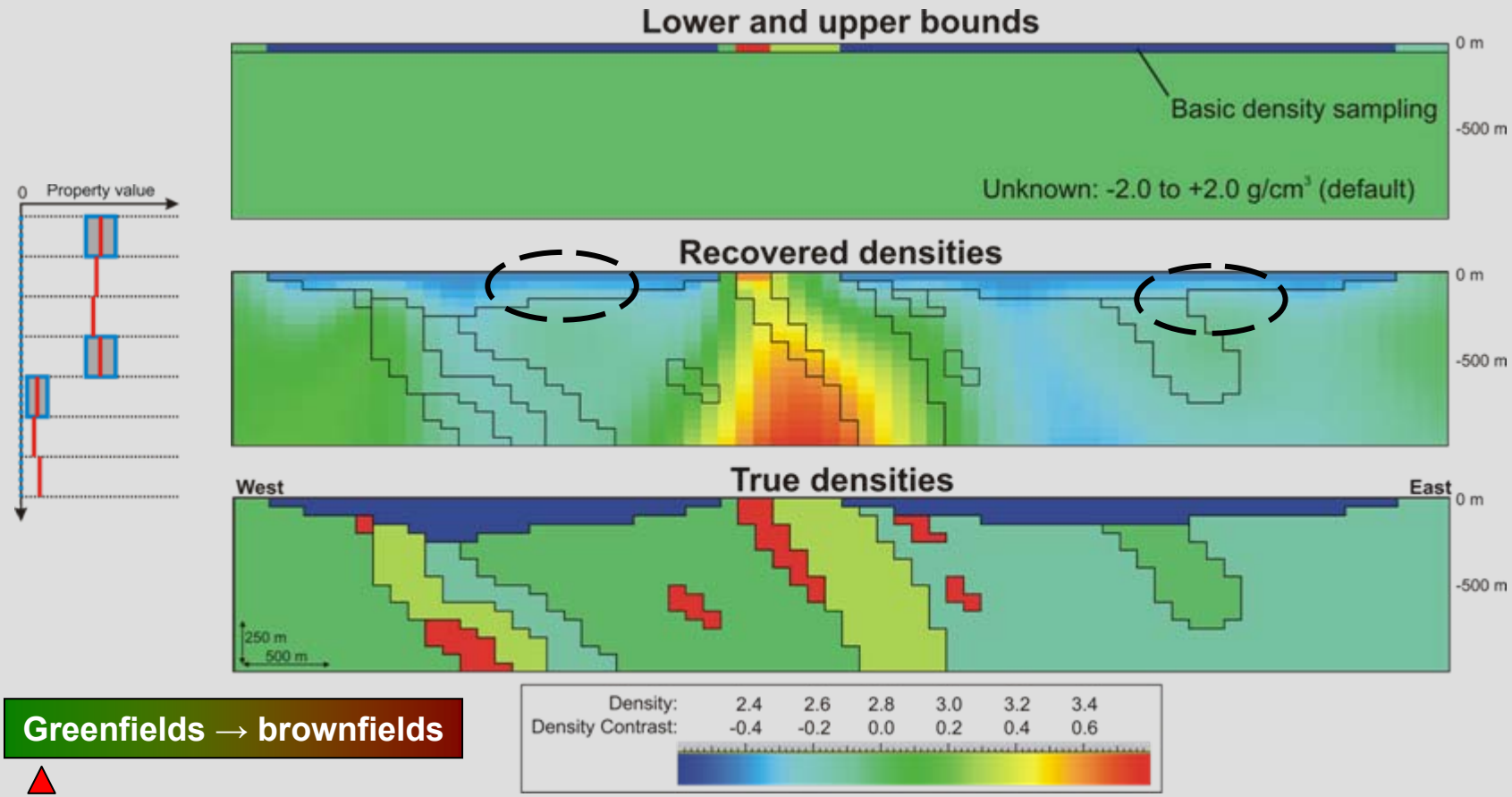




Greenfields → brownfields

Add outcrop (smooth *m*)

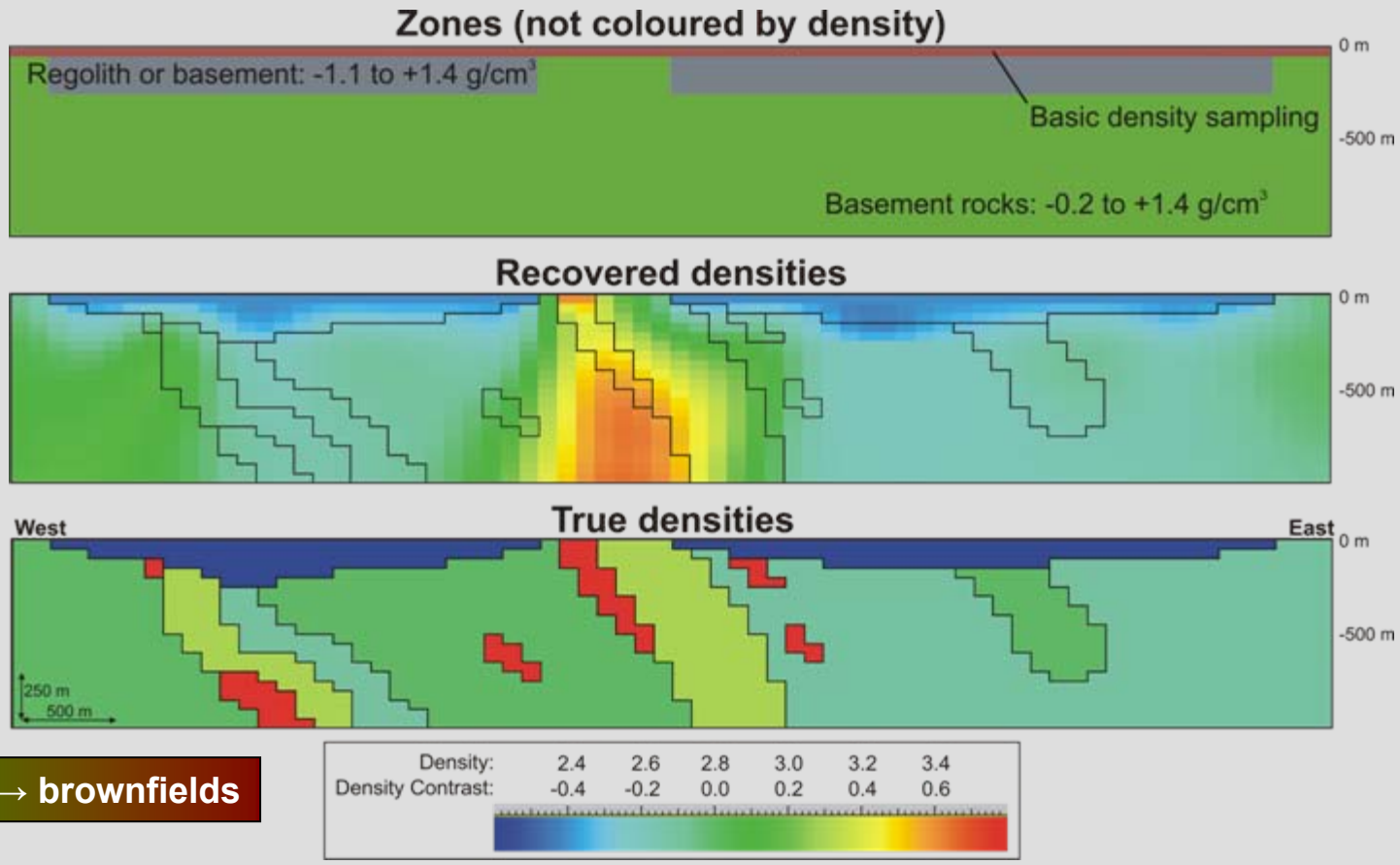
- Surface mapping
- Some surface density measurements
- Zero reference model
- Narrow bounds for surface cells based on measured densities



Greenfields → brownfields

Add geological zones (smooth *m*)

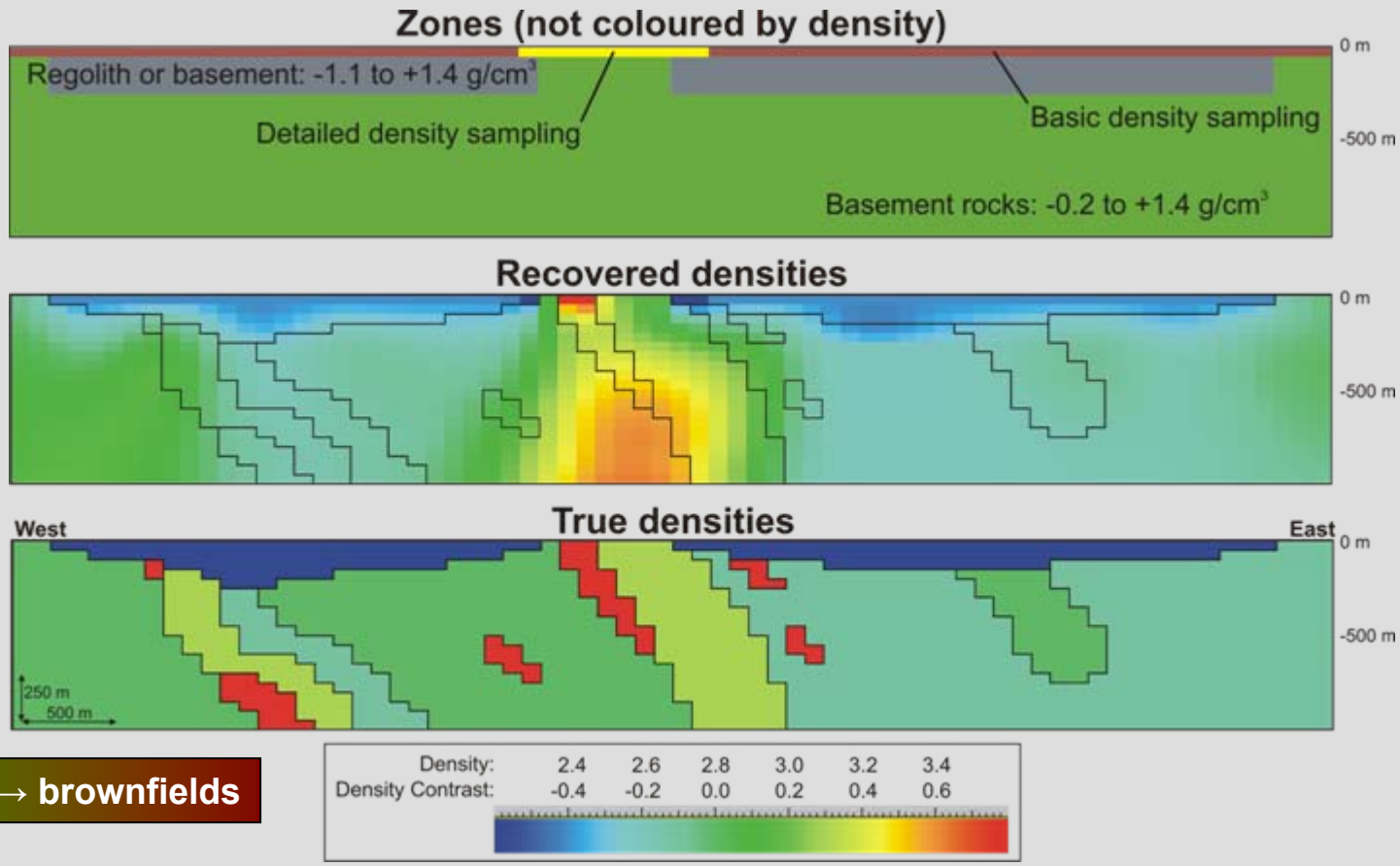
- General geological concepts based on expected variability
- Narrow bounds for surface cells based on measured densities
- Broad bounds based on concepts



Greenfields → brownfields

Add detailed surface sampling (smooth *m*)

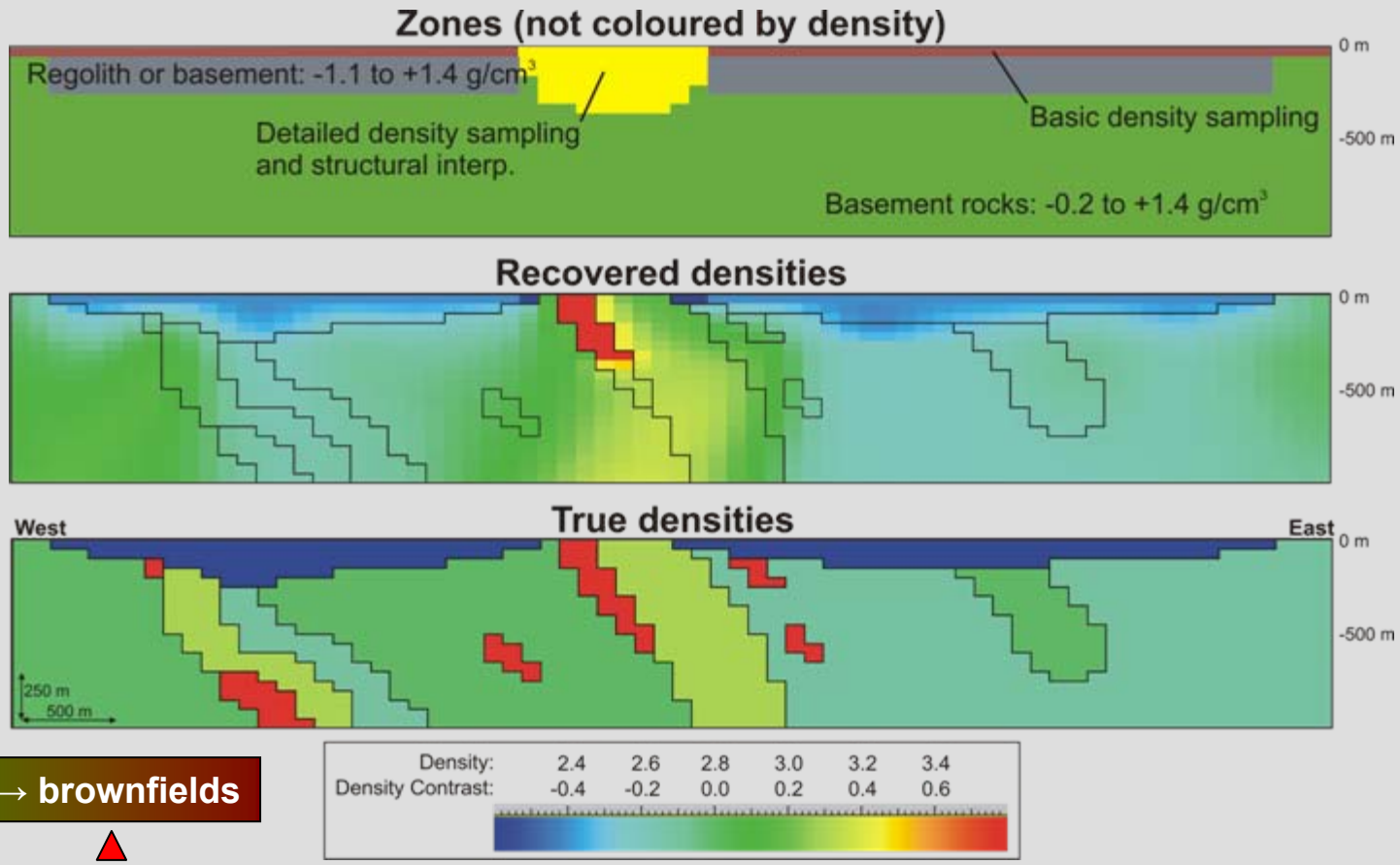
- **Detailed surface sampling over small area**
- **Tight (± 0.15 g/cm^3) bounds over 1 km × 1 km area at surface**



Greenfields → brownfields

Add partial 3D model (smooth *m*)

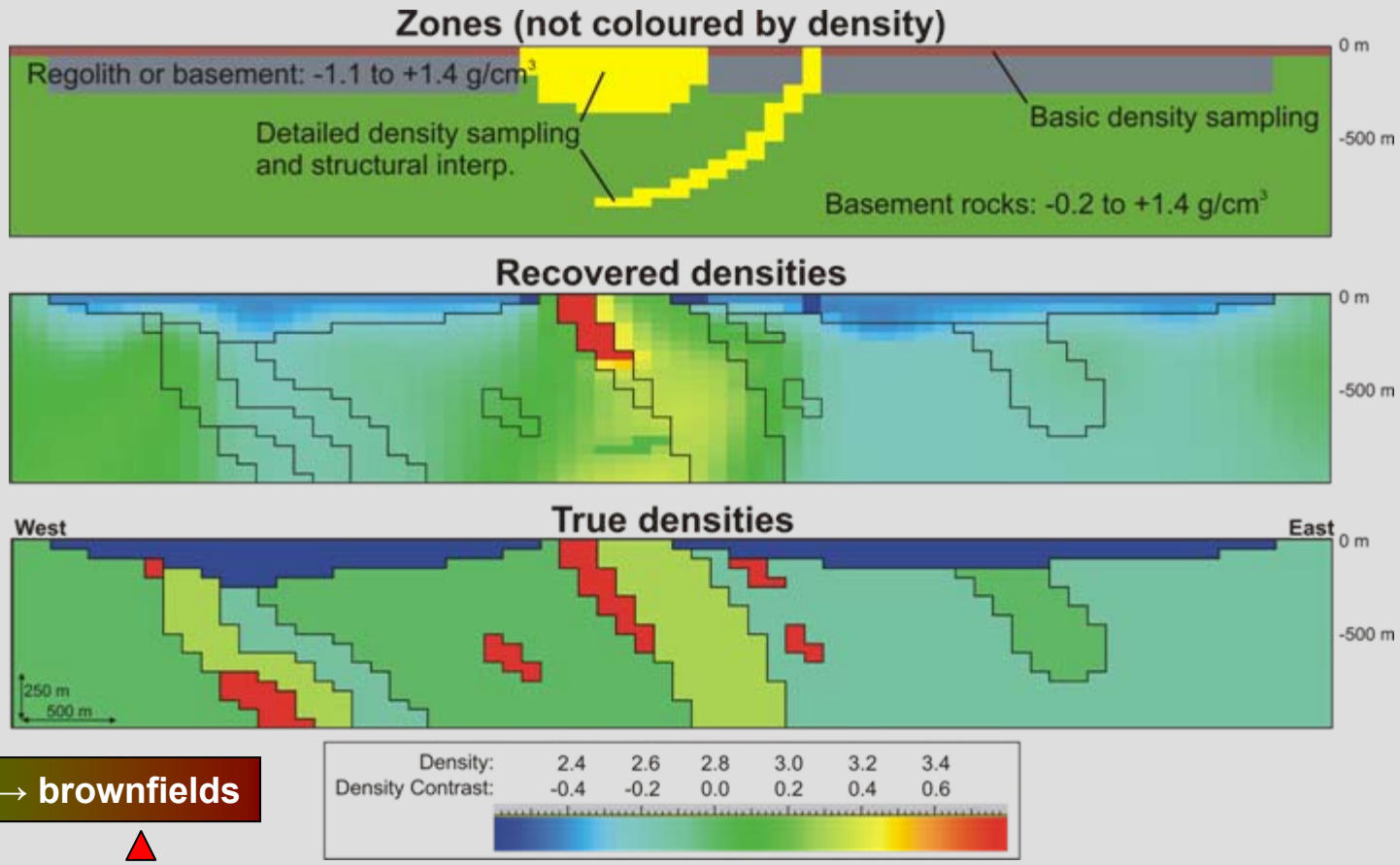
- **3D model based on shallow drilling, sampling and structural interpretation**
- **Tight (± 0.15 g/cm^3) bounds over 3D model that is 1 km × 1 km to a depth of 350 m**



Greenfields → brownfields

Add 1 deep drill hole (smooth *m*)…

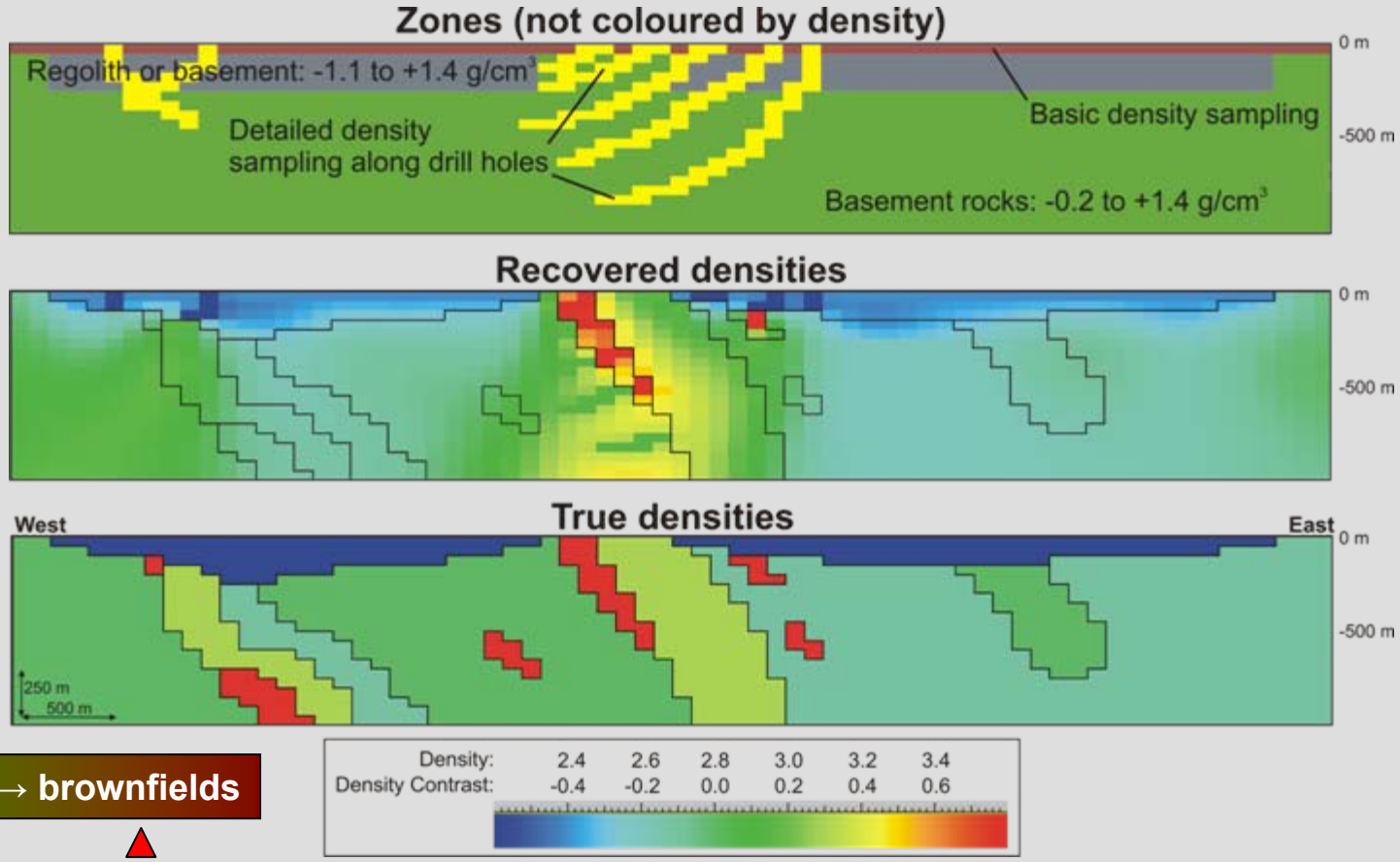
- **Deep drilling with sampling**
- **1 drill hole, 2 km long**



Greenfields → brownfields

…Or many drill holes (smooth *m*)…

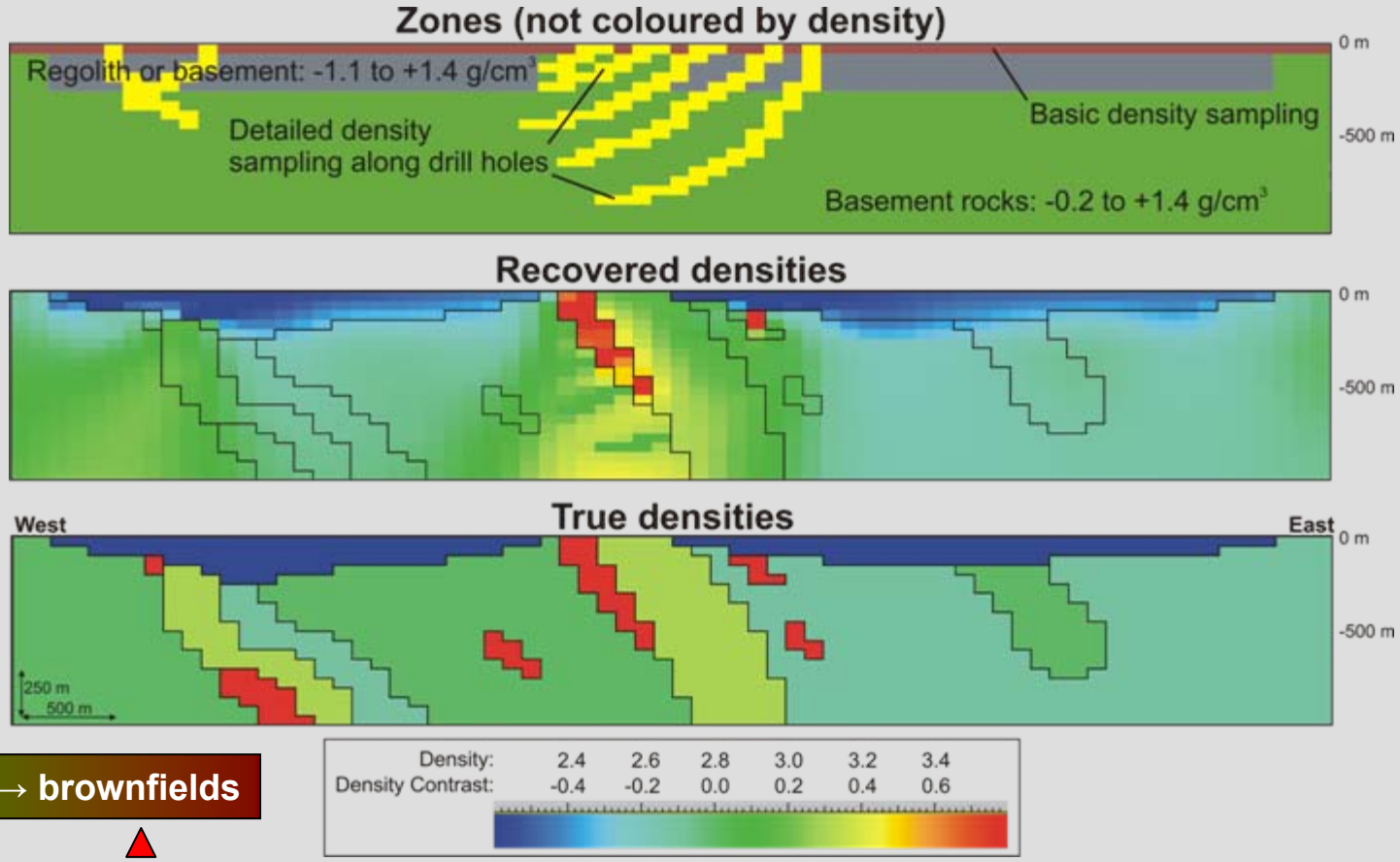
- **Drilling: 7 holes in section, 14 out of section (up to 1.7 km away)**
- **Tight (± 0.15 g/cm^3) bounds along drill holes**
- **No detailed surface sampling or 3D model**



Greenfields → brownfields

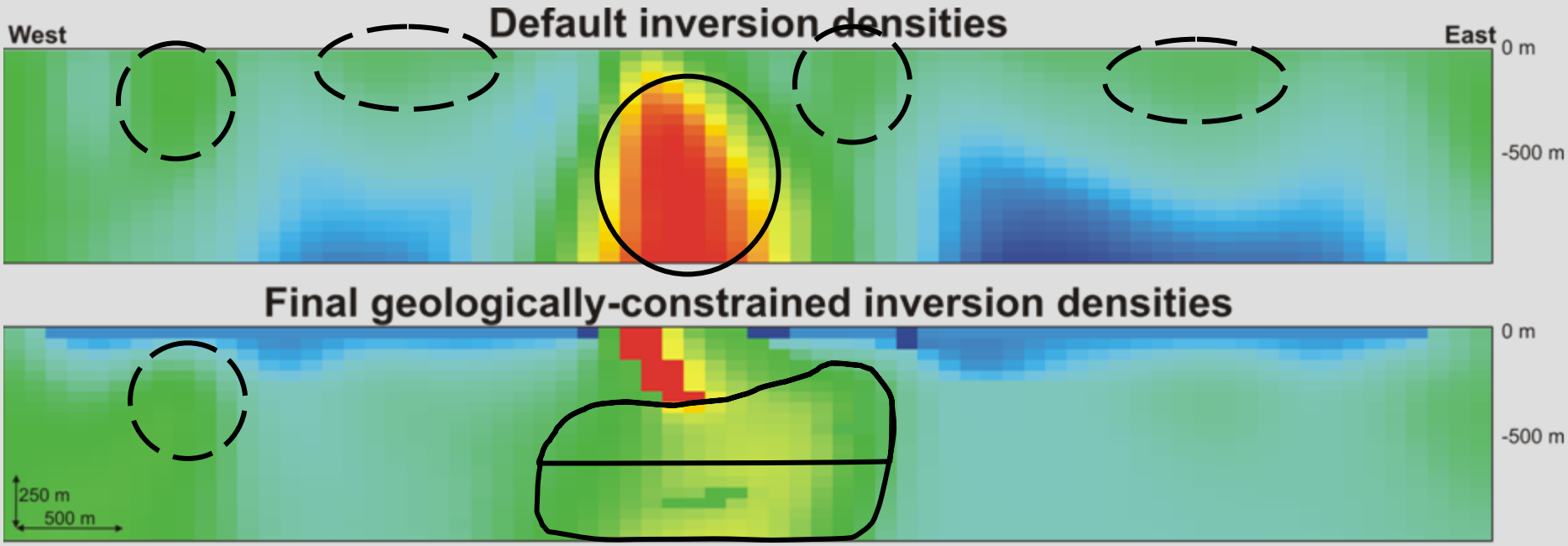
…and structural trends (smooth *m*)

- **Drilling: 7 holes in section, 14 out of section (up to 1.7 km away)**
- **Tight (± 0.15 g/cm^3) bounds along drill holes**
- **Smoothness weights based on structural trends and contacts**

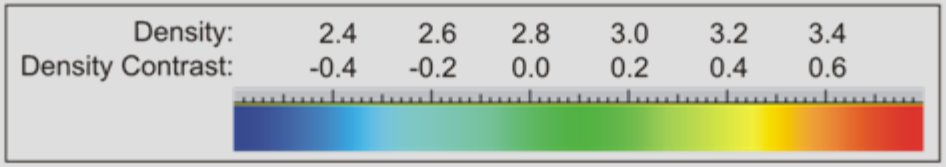


Greenfields → brownfields

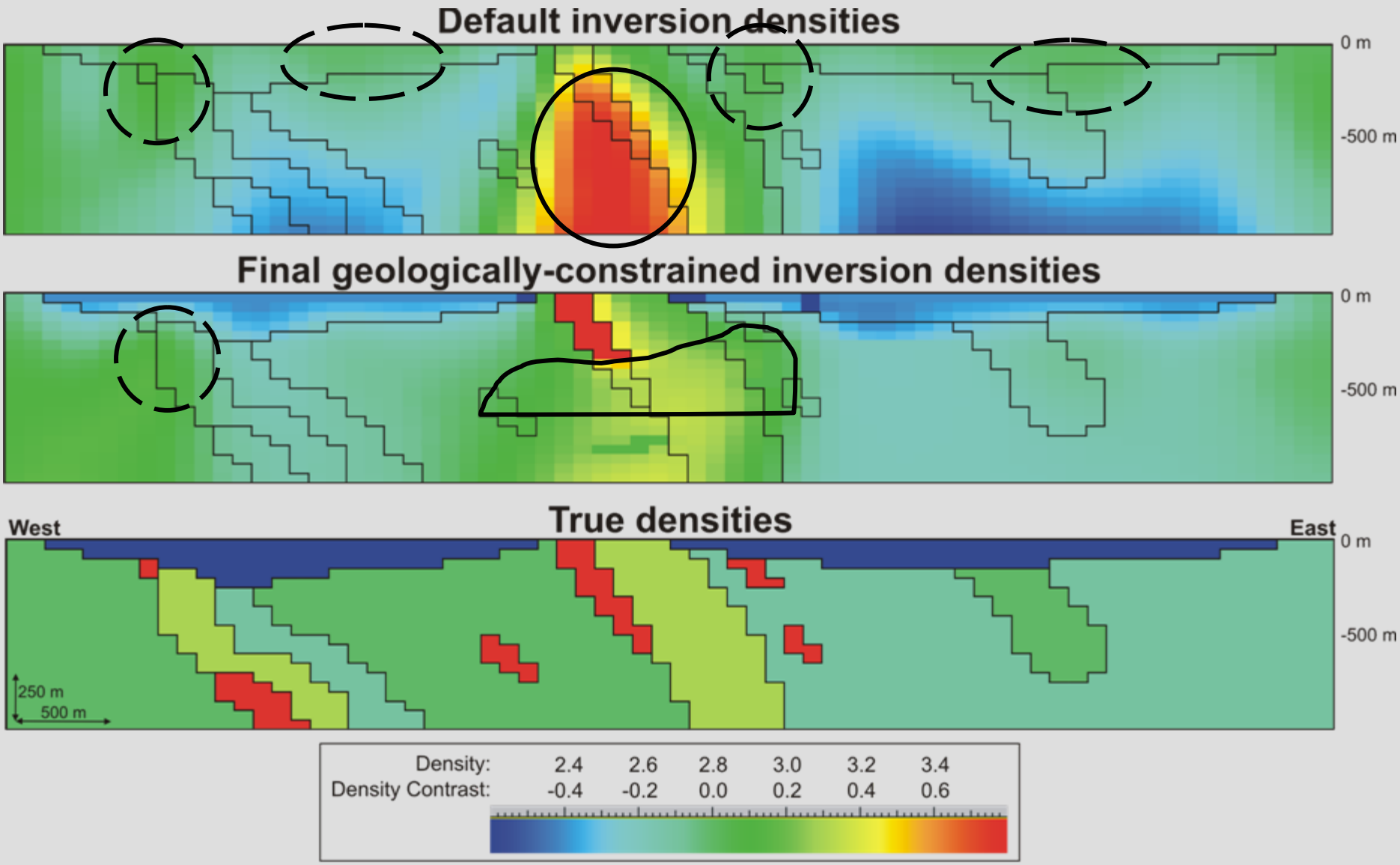
Inversion example summary



- ***Only*** **difference is that geological information *routinely* collected in exploration and development was included as inversion constraints *as it became available***

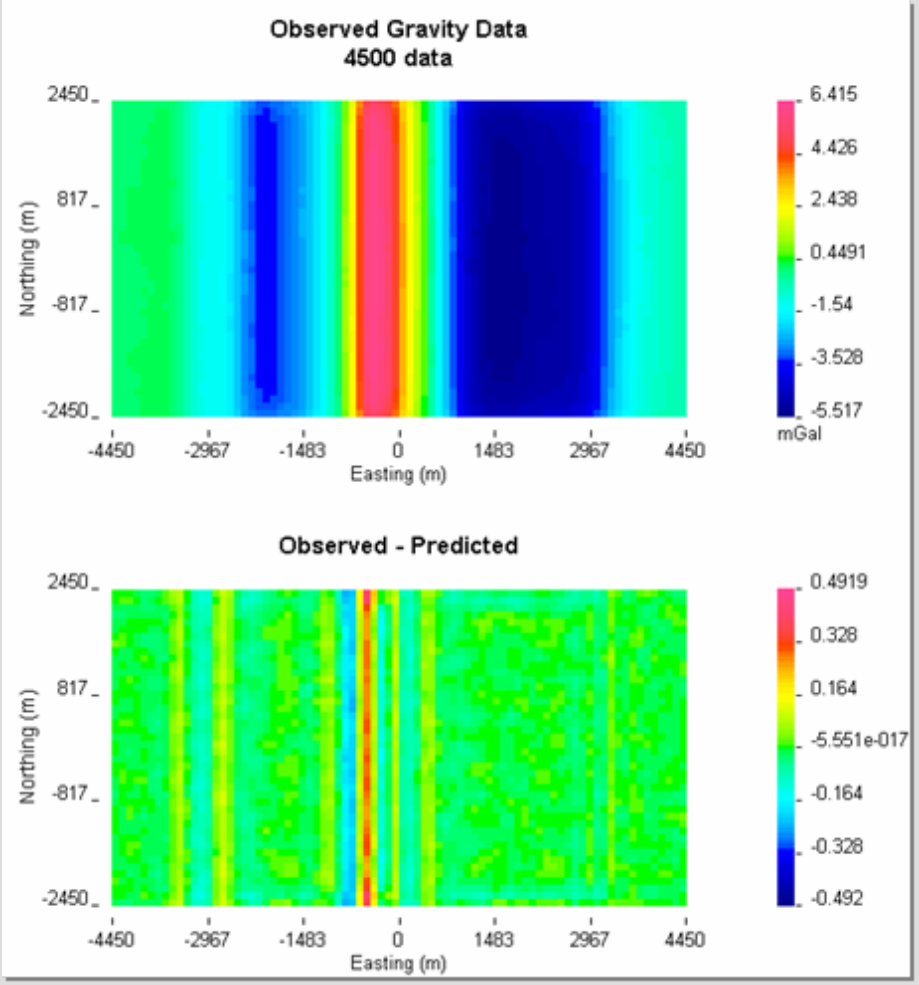


Inversion example summary

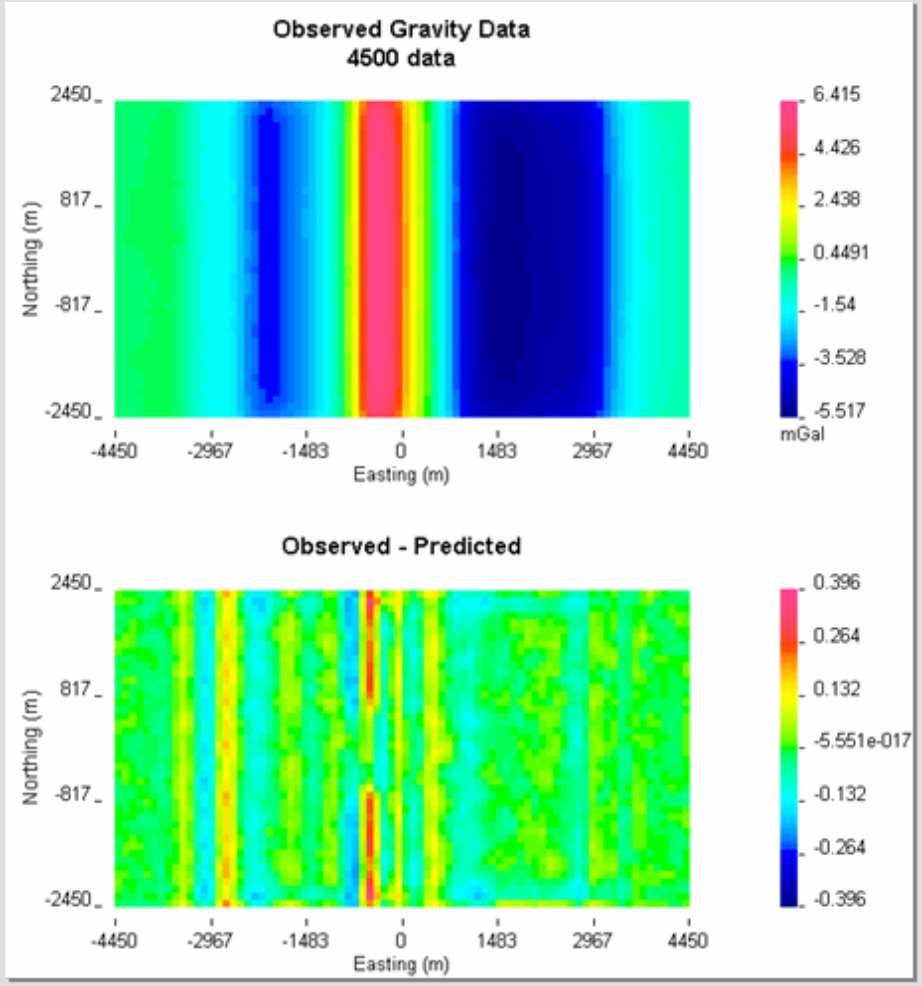


Both fit the data equally well

Default inversion



Final geologically-constrained inversion



Inversion example summary

- **Constraints used (in approximate order of usefulness):**
  - **Partial 3D model**
    - Only if trustworthy and good property measurements are available
  - **Many drill holes**
    - Drill holes remain only way to get actual geological information from depth
  - **Surface maps with basic density estimates**
    - Identifying zones of weathered material is especially important
  - **Geological concepts**
    - Wider bounds where more geological variety is expected
    - Narrower bounds where geology is expected to be more homogeneous
  - **Smoothness weights based on observed structural trends and contact positions**
  - **Detailed surface property measurements**

Geological constraints summary

- **A basic model can be obtained using no geological information**
  - **Identify lateral positions of anomalies**
  - **Identify relative magnitudes and sizes of anomalies**
  - **Basic depth estimates**
  - **Low confidence in targets**
- **A large range of geological information can be readily included to improve recovered models**
  - **Mapping, density measurements, drilling, 3D models**
  - **No "special" data requirements**
  - **Include new geological data as it becomes available to refine model**
    - ***Don't need to create a full 3D model!***
- **Where constraints are sparse, use bounds instead of a reference model**
  - **Extrapolate the constraints out into the model**

Some continuing research at UBC-GIF

- Other types of geological constraints
  - How might structural style be imparted on a model?
  - Supplying smoothness weights in non-orthogonal directions
- Other forms of regularisation
  - "Blocky" models, sharp contacts
- How to ease the inclusion of constraint information?
- Volume-scaling of physical property measurements
- Interpreting and classifying physical property models in terms of geology
  - How can we get geology instead of physical properties?
  - Mapping mineralogy in 3D

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