Centre for Advanced Data Inference
Research School of Earth Sciences

Australian Government
Geoscience Australia

Holistic inversion of time-domain AEM data

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Outline

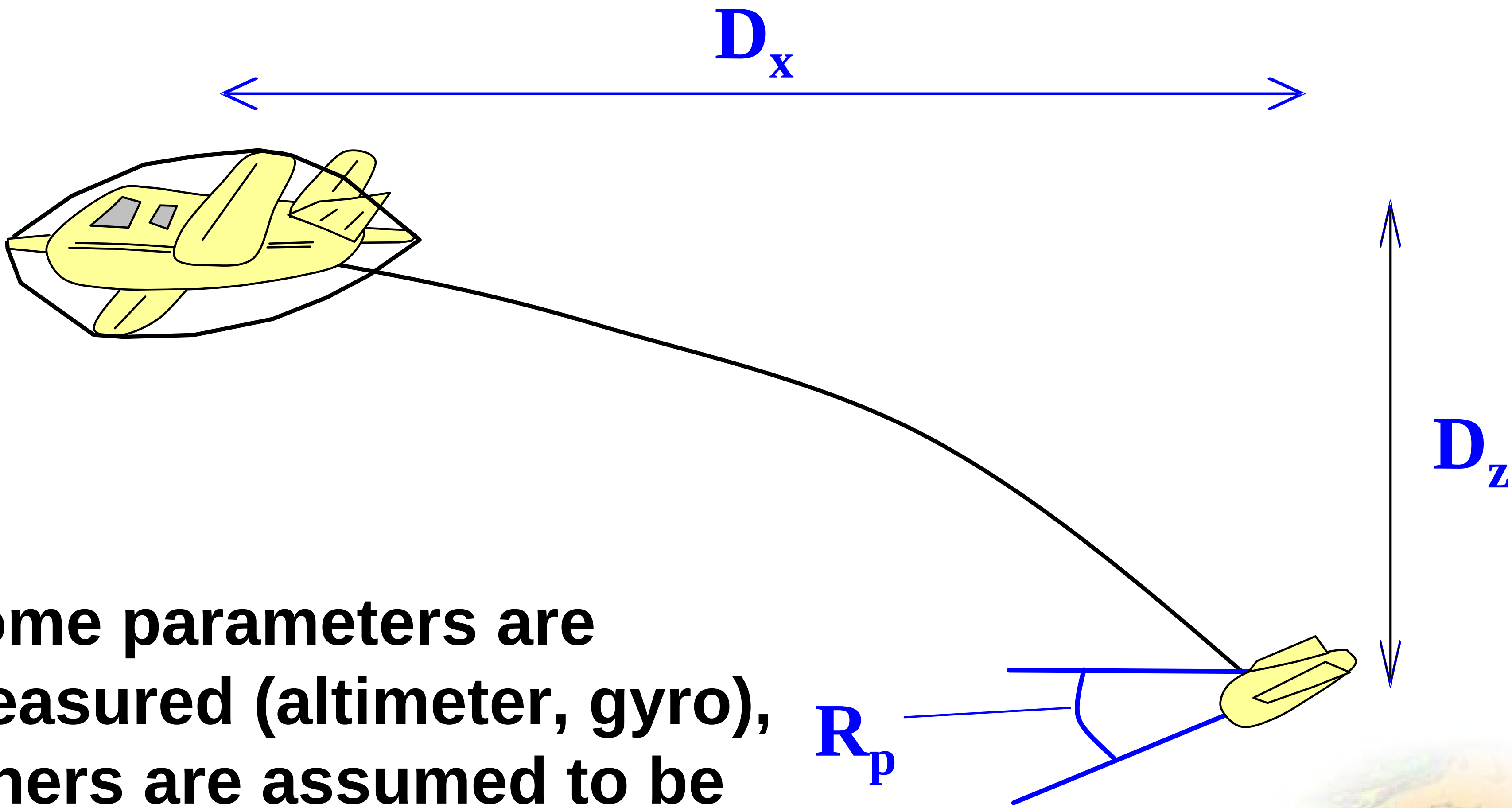
- Motivation
- Methodology
- Example
- Future Possibilities
- Conclusions

Motivation

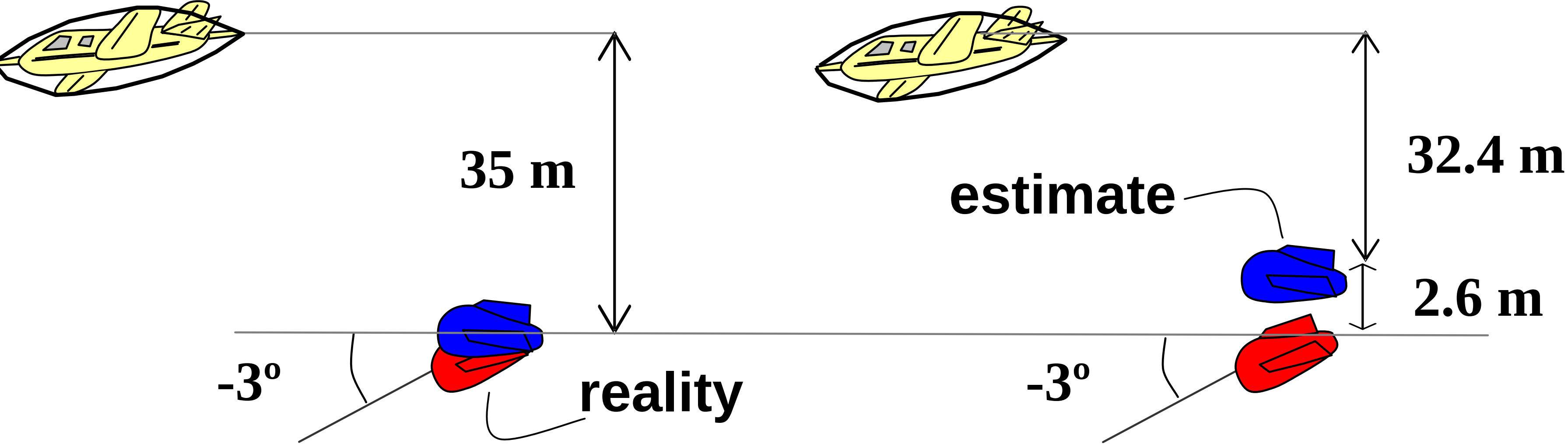
- **Bringing together 3 established ideas**

1. **Solving for geometry – improves results by simulating the real system & allowing data to be fitted** Lane et al (2004), Sattel et al (2004).
2. **Laterally/spatially constrained - exploits spatial coherency via inversion of whole lines or datasets, instead of inverting single samples and then stitching them together.** Auken & Christiansen (2004),, Brodie & Sambridge (2006), Vieozzli et al (2008), Vallee & Smith (2008).
3. **Simultaneous inversion for conductivity and calibration parameters ensures consistency.** Brodie & Sambridge (2006).

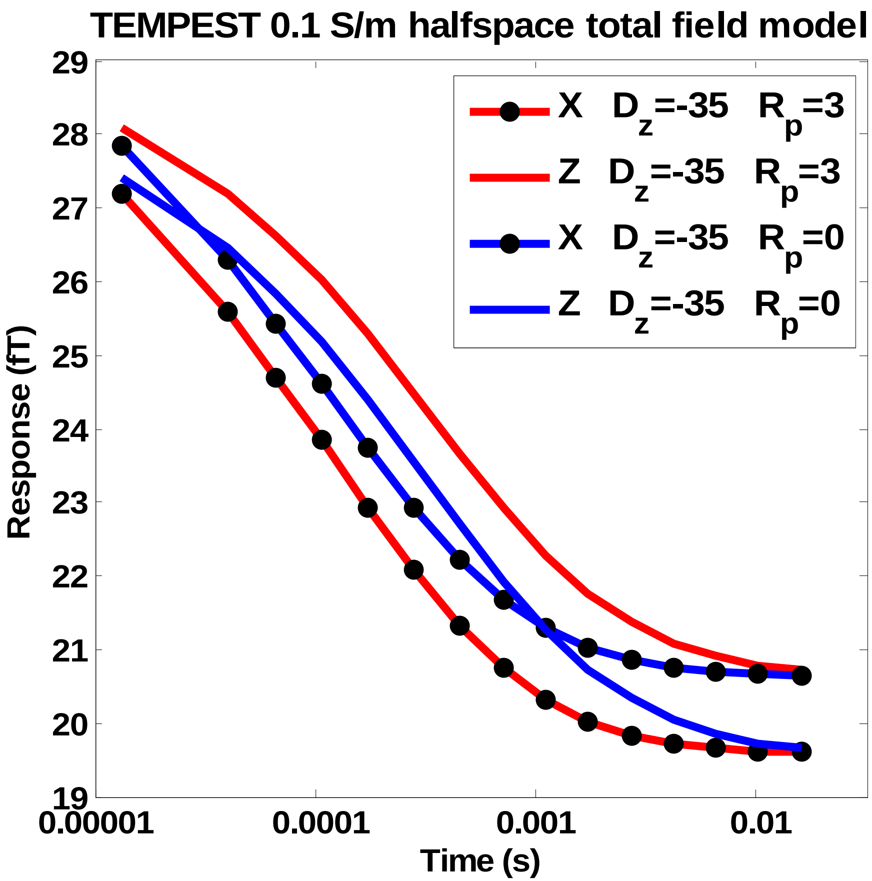
Fixed wing AEM geometry



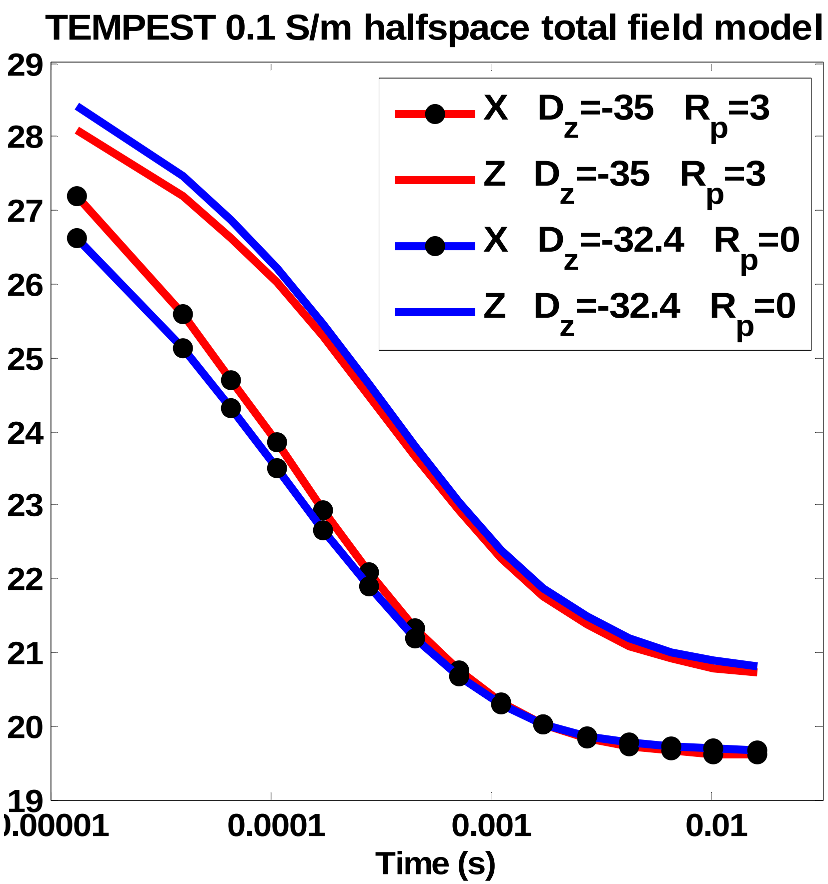
Some parameters are measured (altimeter, gyro), others are assumed to be zero, D_x & D_z are estimated



35 m
32.4 m
estimate
2.6 m
-3°
reality
-3°



TEMPEST 0.1 S/m halfspace total field model
X D_z=-35 R_p=3
Z D_z=-35 R_p=3
X D_z=-35 R_p=0
Z D_z=-35 R_p=0
Response (fT)
29
28
27
26
25
24
23
22
21
20
19
0.00001
0.0001
0.001
0.01
Time (s)



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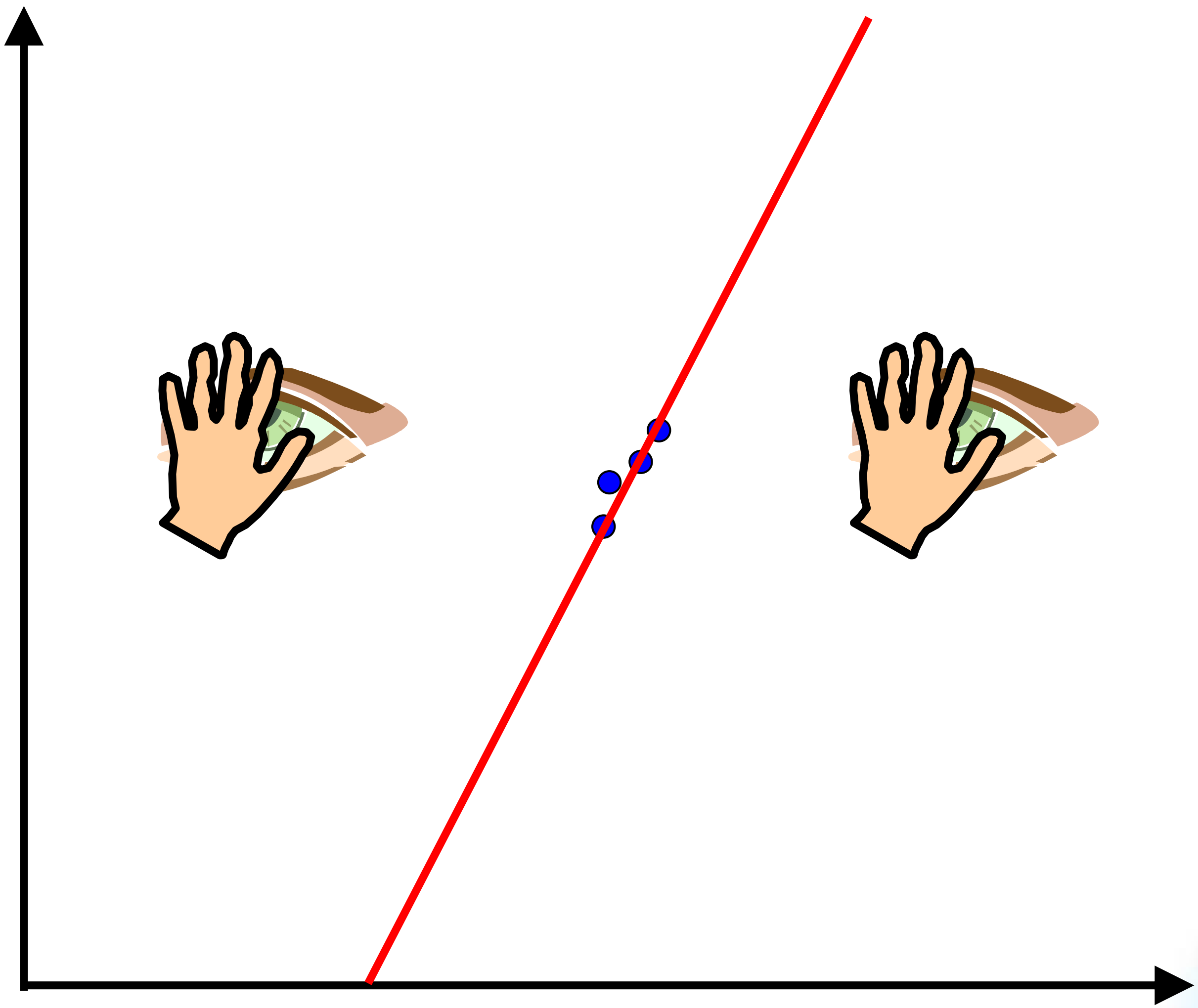
29
28
27
26
25
24
23
22
21
20
19
0.00001
0.0001
0.001
0.01
Time (s)

Sample by sample (stitched) inversion



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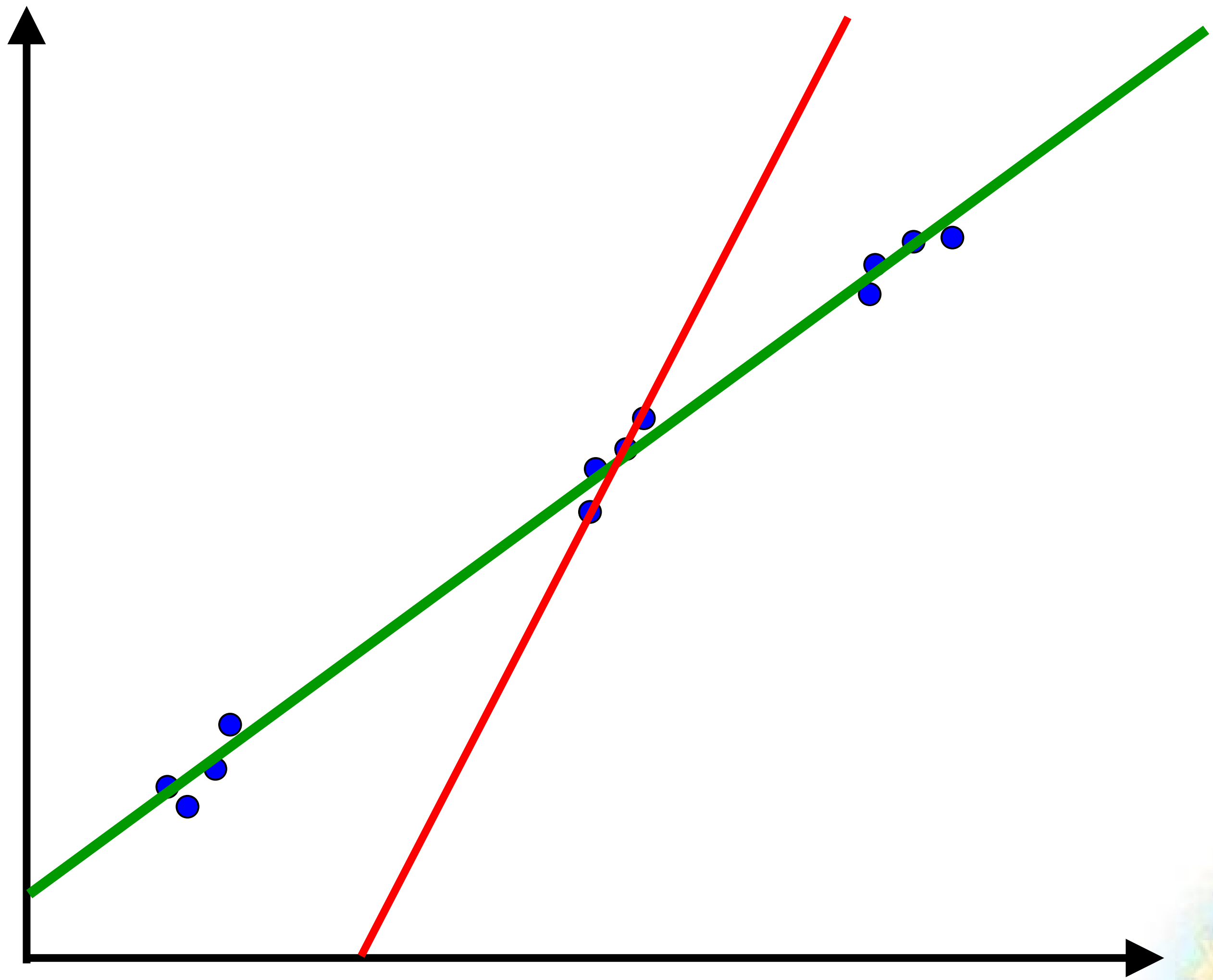
Laterally/spatially constrained inversion



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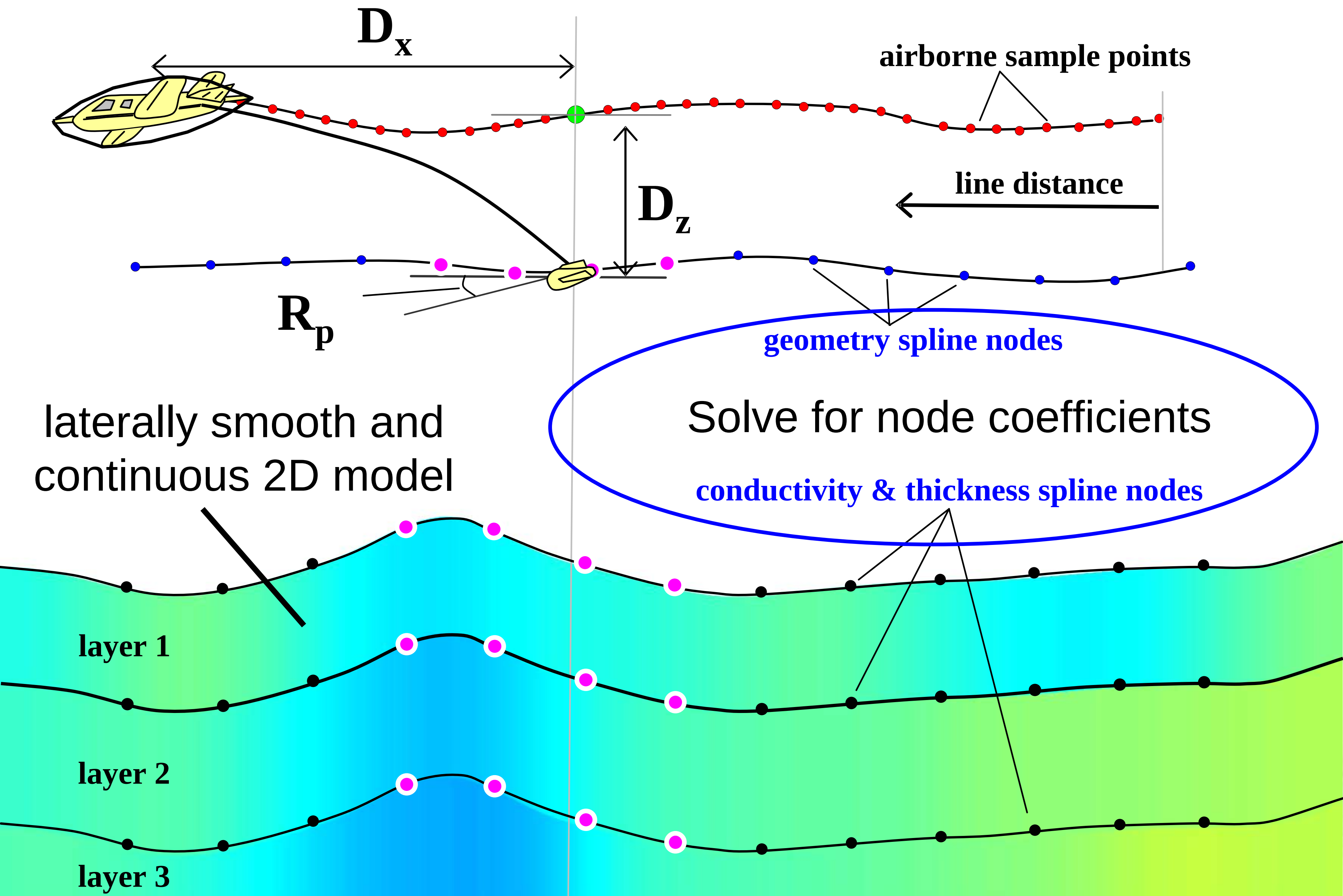
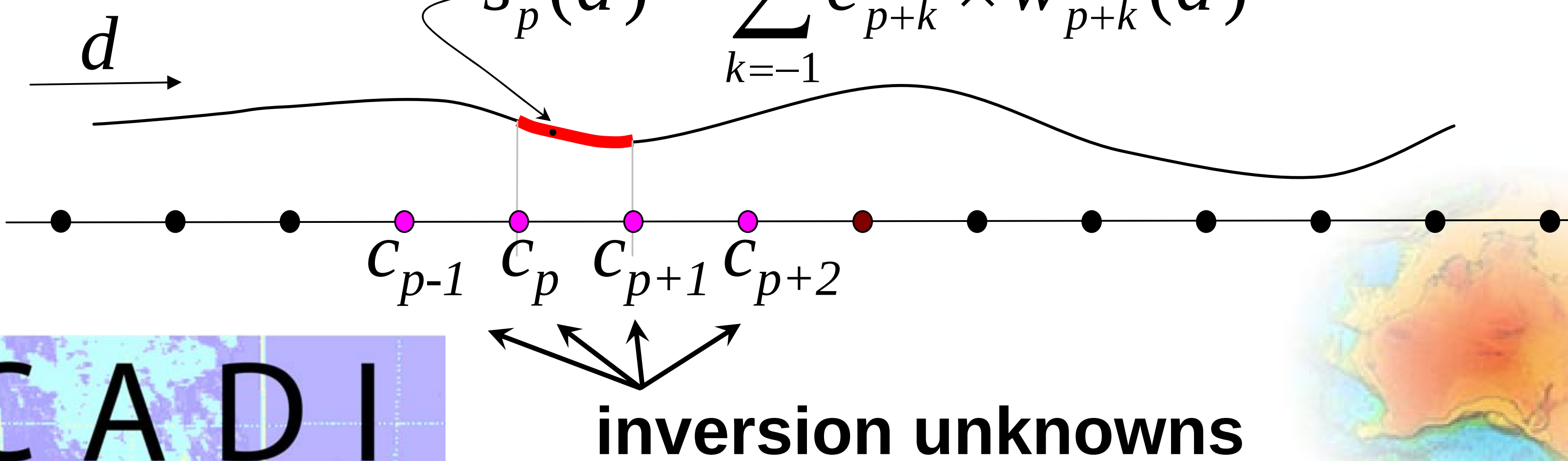
Fixed-wing AEM formulation



Cubic B-Splines

- **Continuous valued functions**
- **Smooth functions (C^2 continuous)**
- **At the point *d*, the spline value *s(d)* is a weighted sum of 4 node coefficients**
- **A spline for each layer conductivity and/or thickness and geometry value**

$$s_p(d) = \sum_{k=-1}^{2} c_{p+k} \times w_{p+k}(d)$$

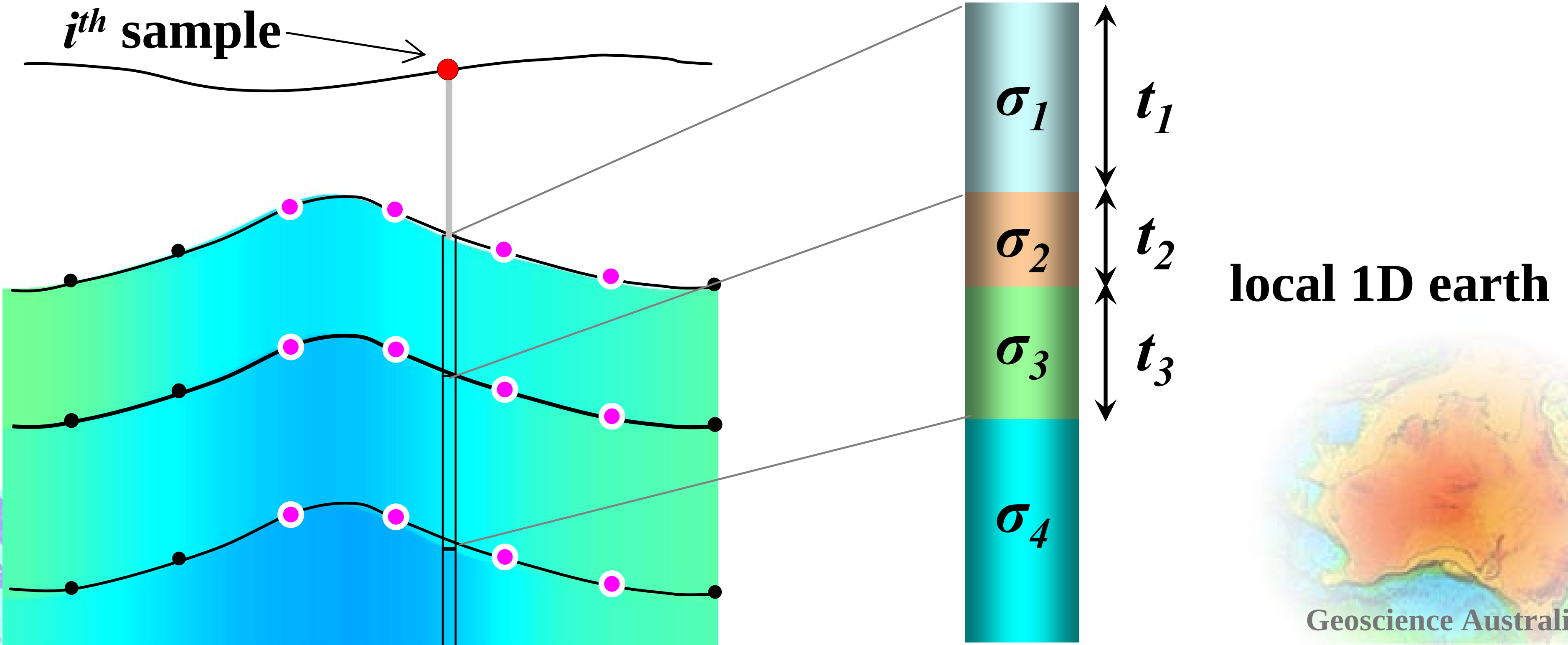


1D Forward problem for i^{th} sample

Compute local geometry values from splines

Compute local 1D earth from splines

$$d_i^{\mathrm{mod}} = f_{1D}(\sigma^l, t^l, T_h, T_r, T_p, T_y, D_x^l, D_y, D_z^l, R_r, R_p^l, R_y)$$



Objective function

$$\phi = \phi_d + \lambda \phi_m$$

data misfit

model regularisation

Data misfit

- **Noise normalised squared error**
- **Sum over N_c channels = windows × components**
- **Sum over N_s samples = samples in line**

$$\phi_d = \frac{1}{N_s \; N_c} \sum_{s}^{N_s} \sum_{c}^{N_c} \left(\frac{d_{s,c}^{\mathrm{mod}} - d_{s,c}^{obs}}{d_{s,c}^{err}} \right)^2$$

Model regularisation

- **Reference model misfit**
  - **difference between inversion model and reference model**
- **Horizontal roughness**
  - **second derivative of logarithm of conductivity between horizontally adjacent conductivity spline nodes**
  - **second derivative of logarithm of thickness between horizontally adjacent thickness spline nodes**
- **Vertical roughness**
  - **second derivative of logarithm of conductivity between adjacent layers**
- **Alpha values control relative importance of terms**

$$\phi_m = \alpha_r \phi_r + \alpha_h \phi_h + \alpha_v \phi_v$$

Objective function minimisation

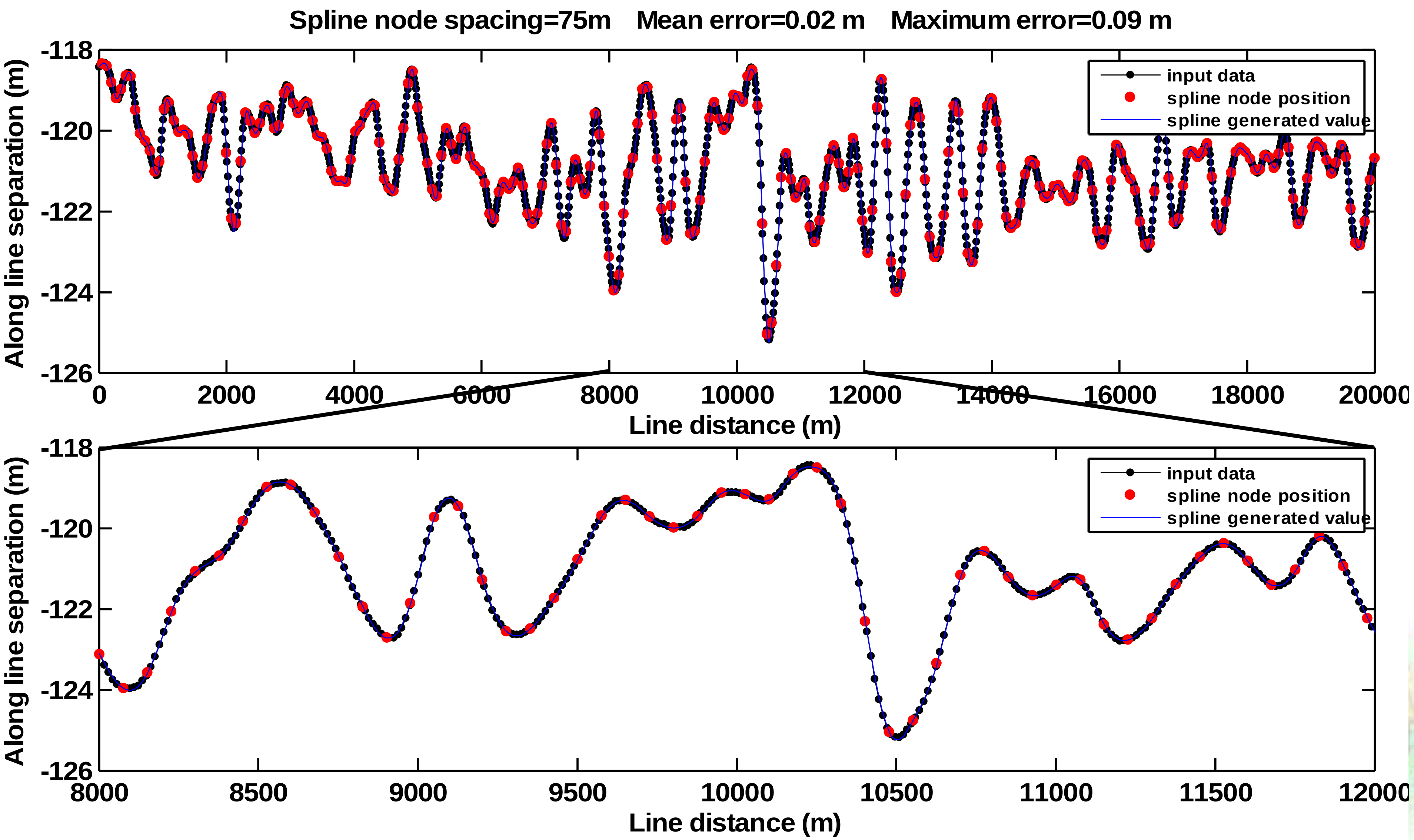
- Iterative non-linear gradient based scheme
- Conjugate gradient method used to solve linearised problem within each iteration
- The α values are chosen and remain fixed
- In each iteration a line search is done on λ to find its value that reduces data misfit by ~30%

$$\mathbf{A}\mathbf{m}_{n+1} = \mathbf{b}$$

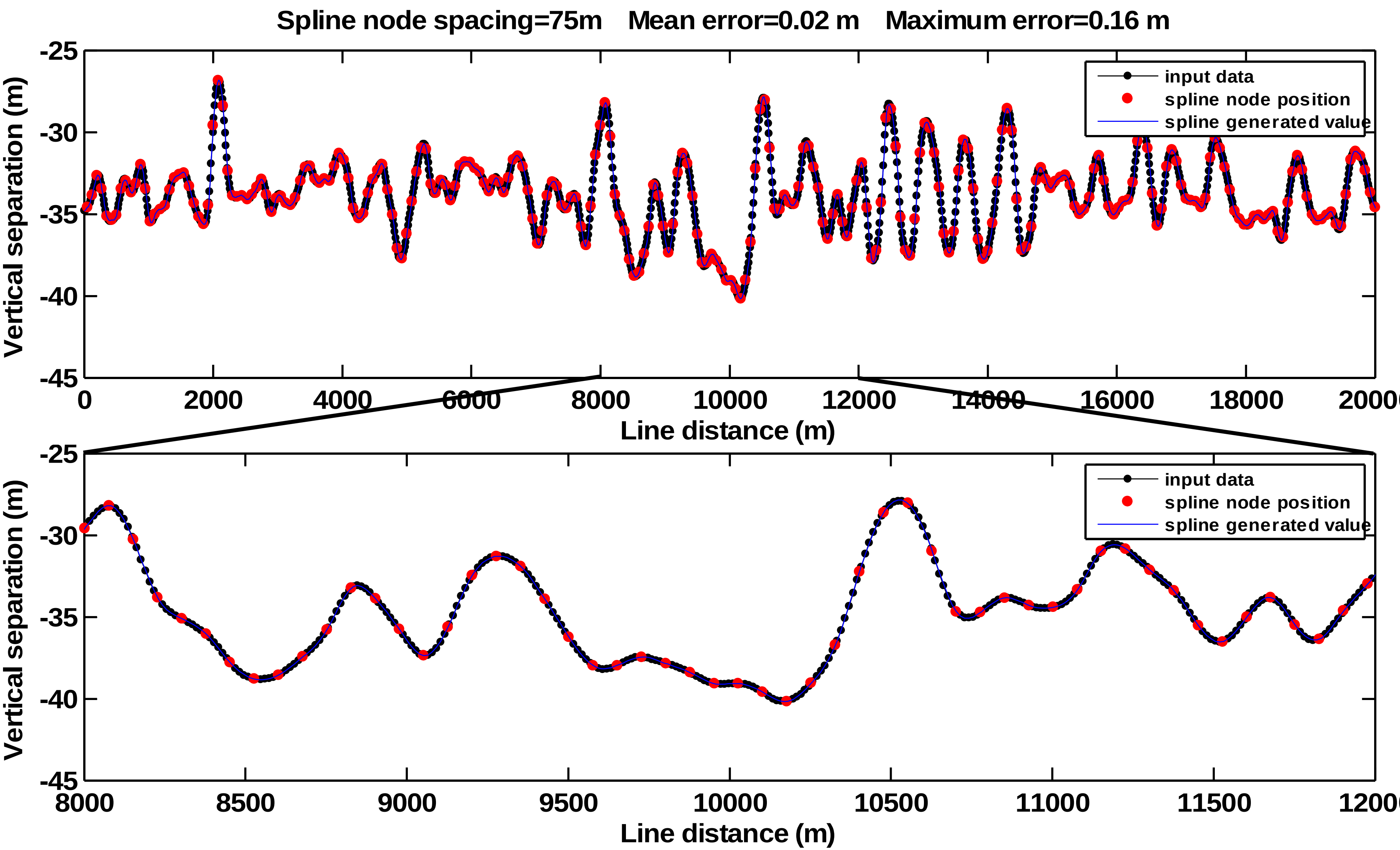
$$\mathbf{A} = \left[\mathbf{J}^{t}\mathbf{W}_{d}\mathbf{J} + \lambda\left(\alpha_{r}\mathbf{S}^{t}\mathbf{W}_{r}\mathbf{S} + \alpha_{h}\mathbf{S}^{t}\mathbf{L}_{h}^{t}\mathbf{L}_{h}\mathbf{S} + \alpha_{v}\mathbf{S}^{t}\mathbf{L}_{v}^{t}\mathbf{L}_{v}\mathbf{S}\right)\right]$$

$$\mathbf{b} = \mathbf{J}^{t}\mathbf{W}_{d}\left[\mathbf{d}^{obs} - \mathbf{d}_{n}^{\mathrm{mod}} + \mathbf{J}\mathbf{m}_{n}\right] + \lambda\mathbf{S}_{t}\mathbf{W}_{r}\mathbf{r}$$

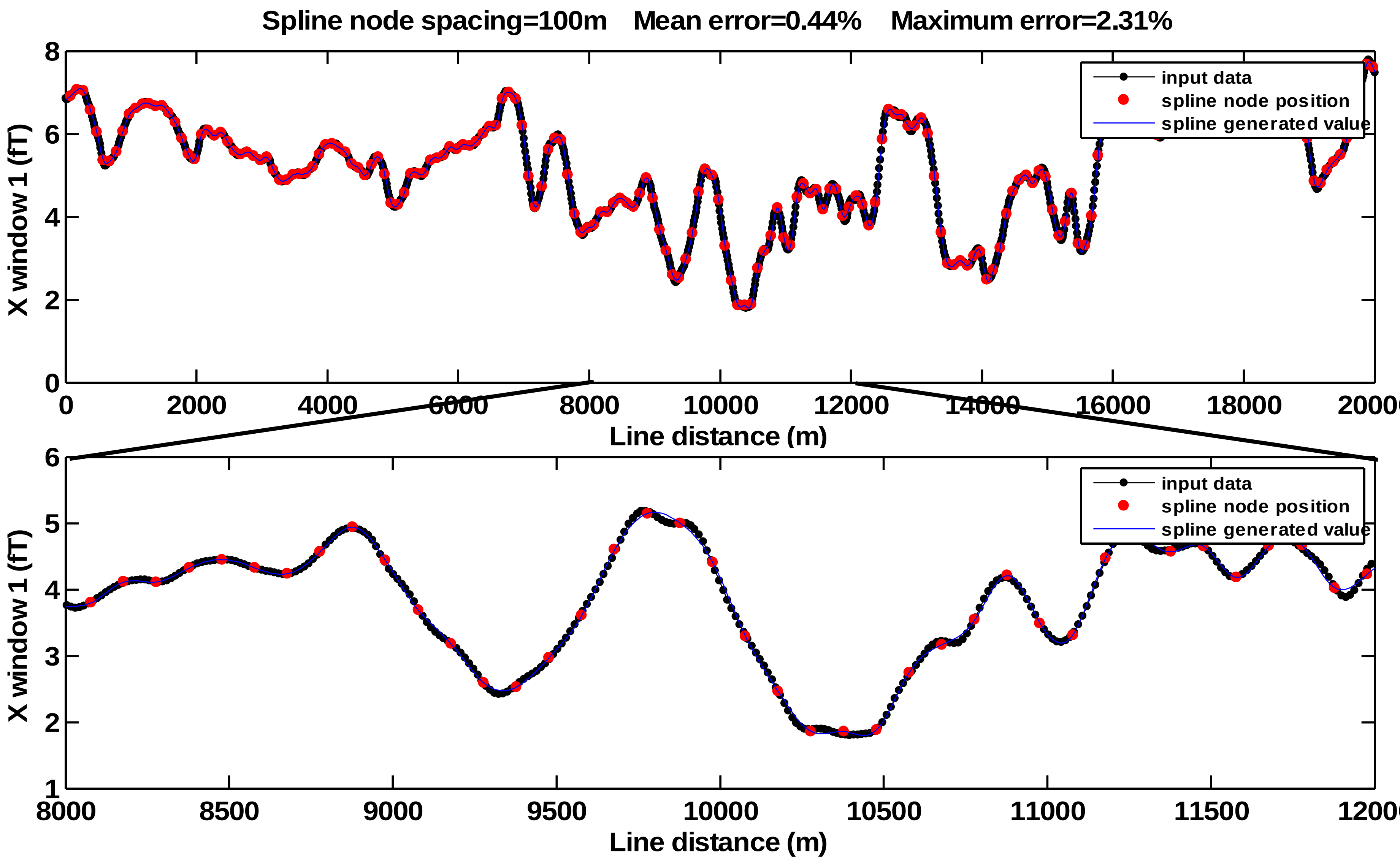
Choosing a spline node spacing (D_x)



Choosing a spline node spacing (D_z)



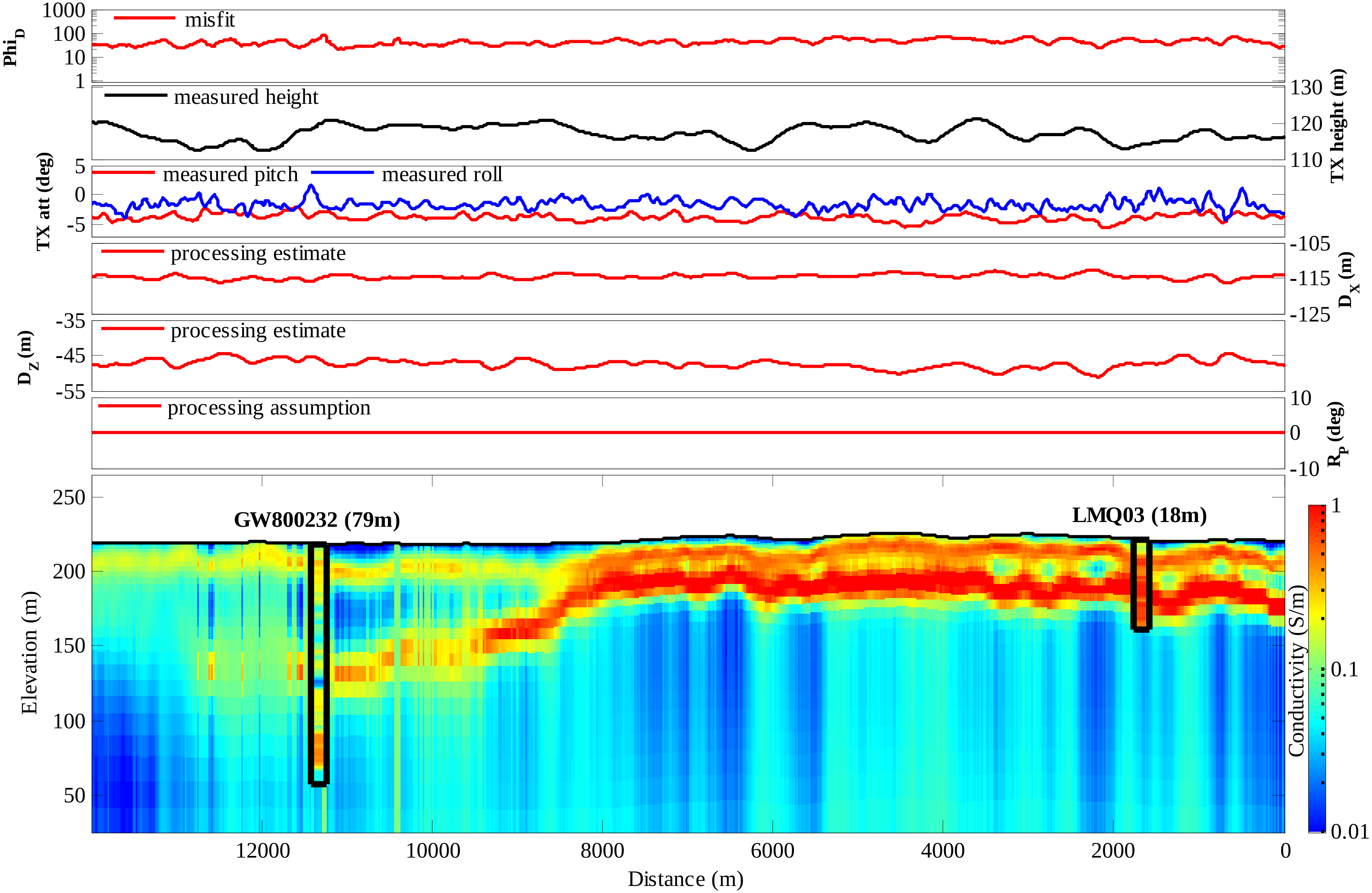
Choosing a spline node spacing (σ)



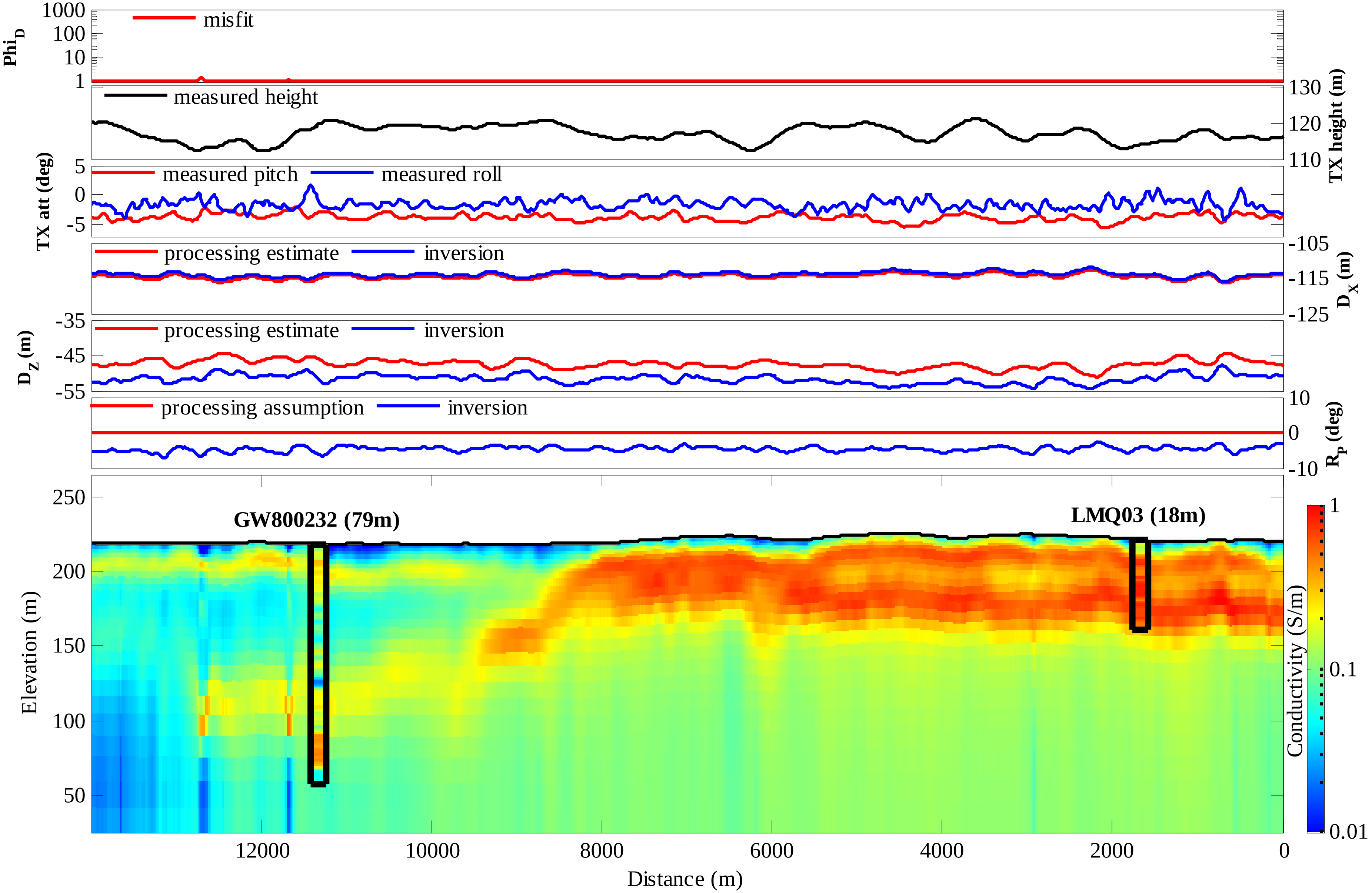
Lower Macquarie Example

- 2007 TEMPEST survey near Dubbo, NSW
- Total field X and Z component data inverted
- Occam's style vertically smooth model with 25 fixed thickness (2, 2.1, …, 17.9m) layers
- 0.1 S/m halfspace conductivity reference
- Processing estimates as geometry reference
- Nodes – conductivity 100m & geometry 75m
- Three comparitive inversions run
  - Sample by sample – not solving for geometry
  - Sample by sample – solving for geometry
  - Line by line holistic – solving for geometry

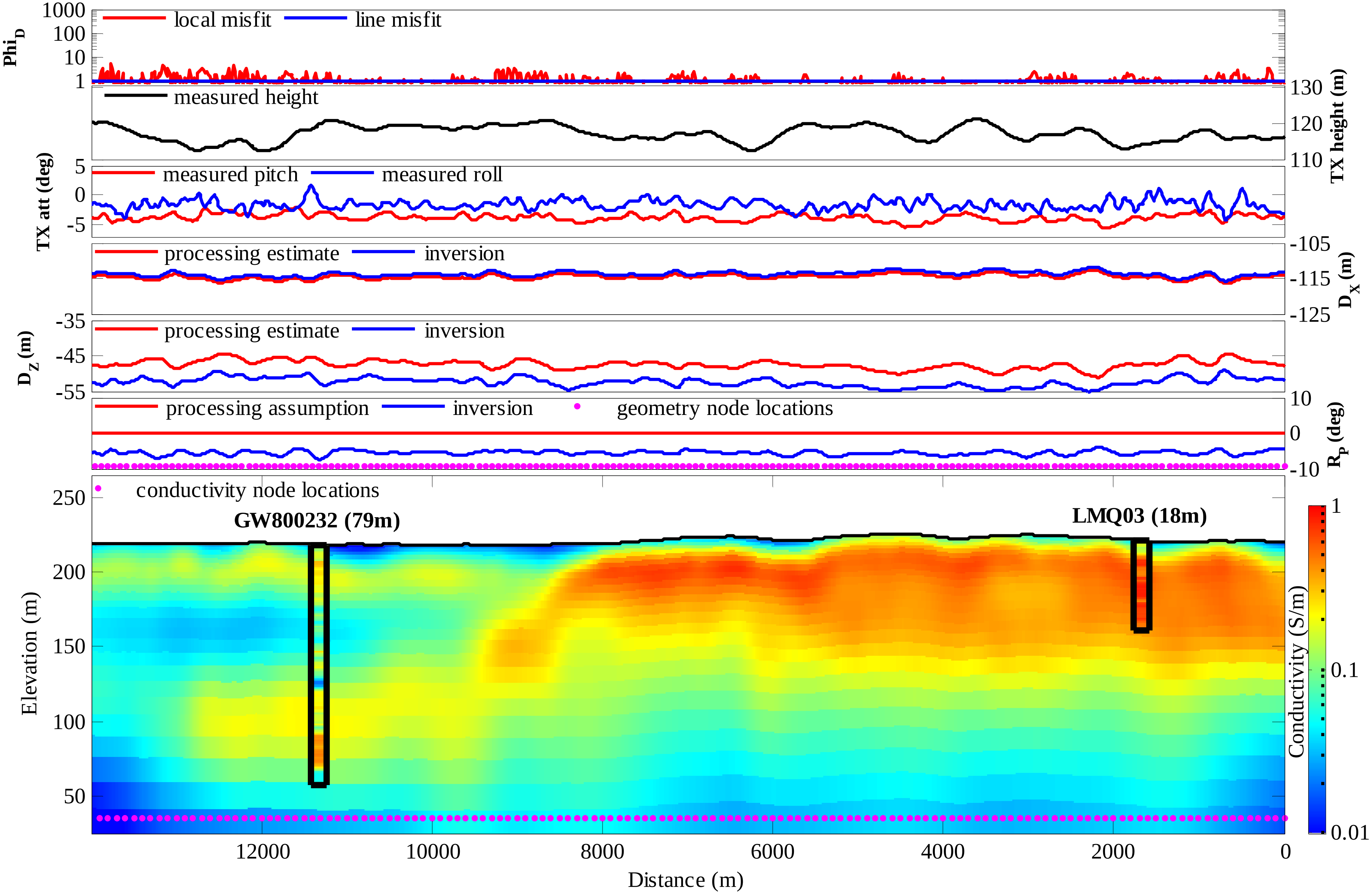
Sample by sample - not solving for geometry

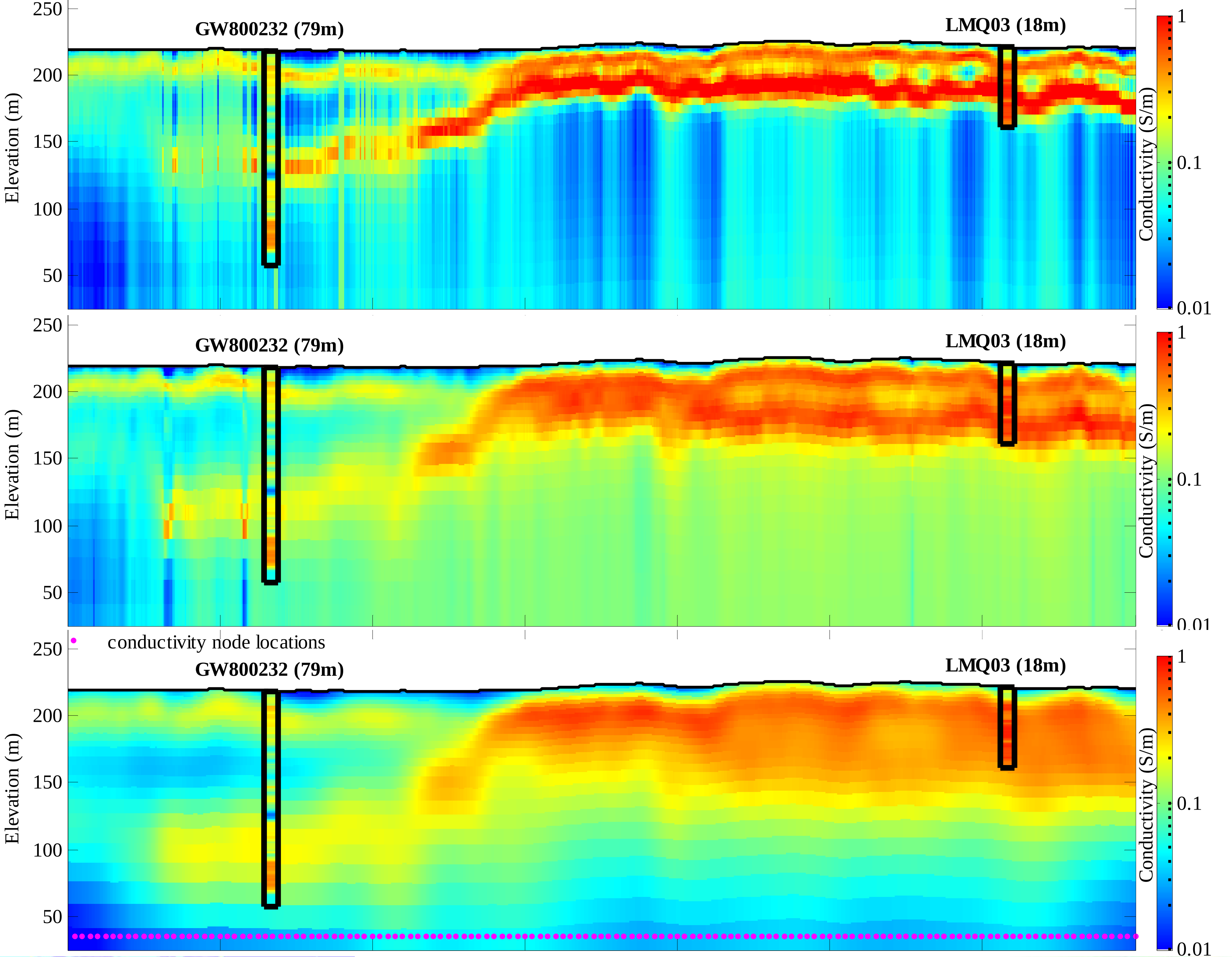


Sample by sample - solving for geometry



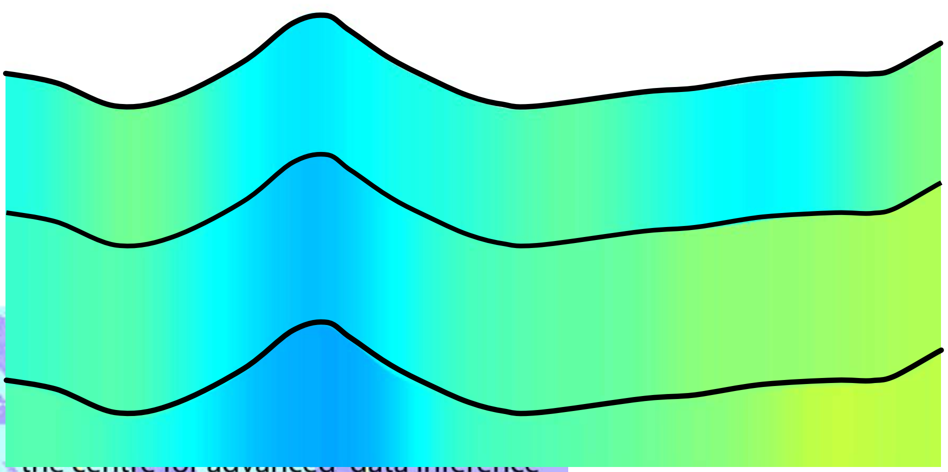
Holistic - solving for geometry

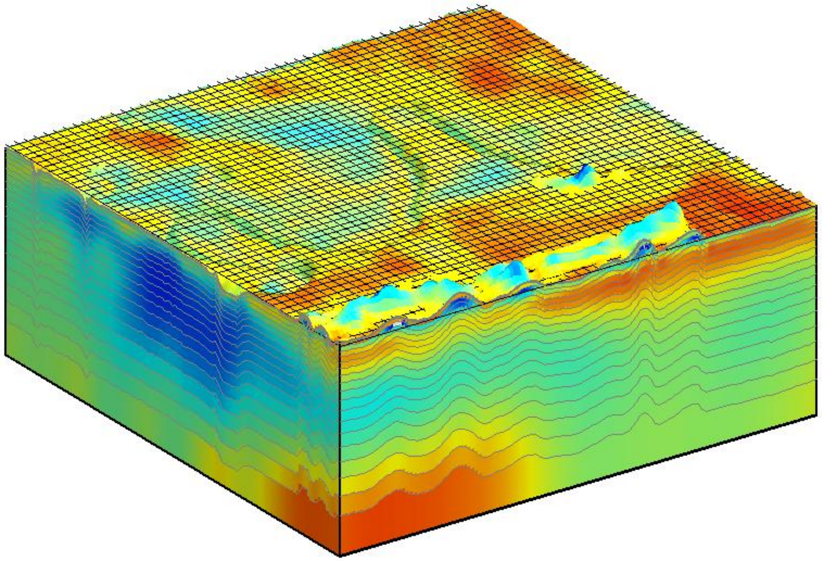



GW800232 (79m)
LMQ03 (18m)
Elevation (m)
Conductivity (S/m)
conductivity node locations
250
200
150
100
50
1
0.1
0.01

Future possibilities

- Include other types of “calibration” parameters – e.g. zero-level drift or gain
- Extend to spatial as in frequency-domain holistic inversion
- Hard constraints – e.g. downhole log & watertable





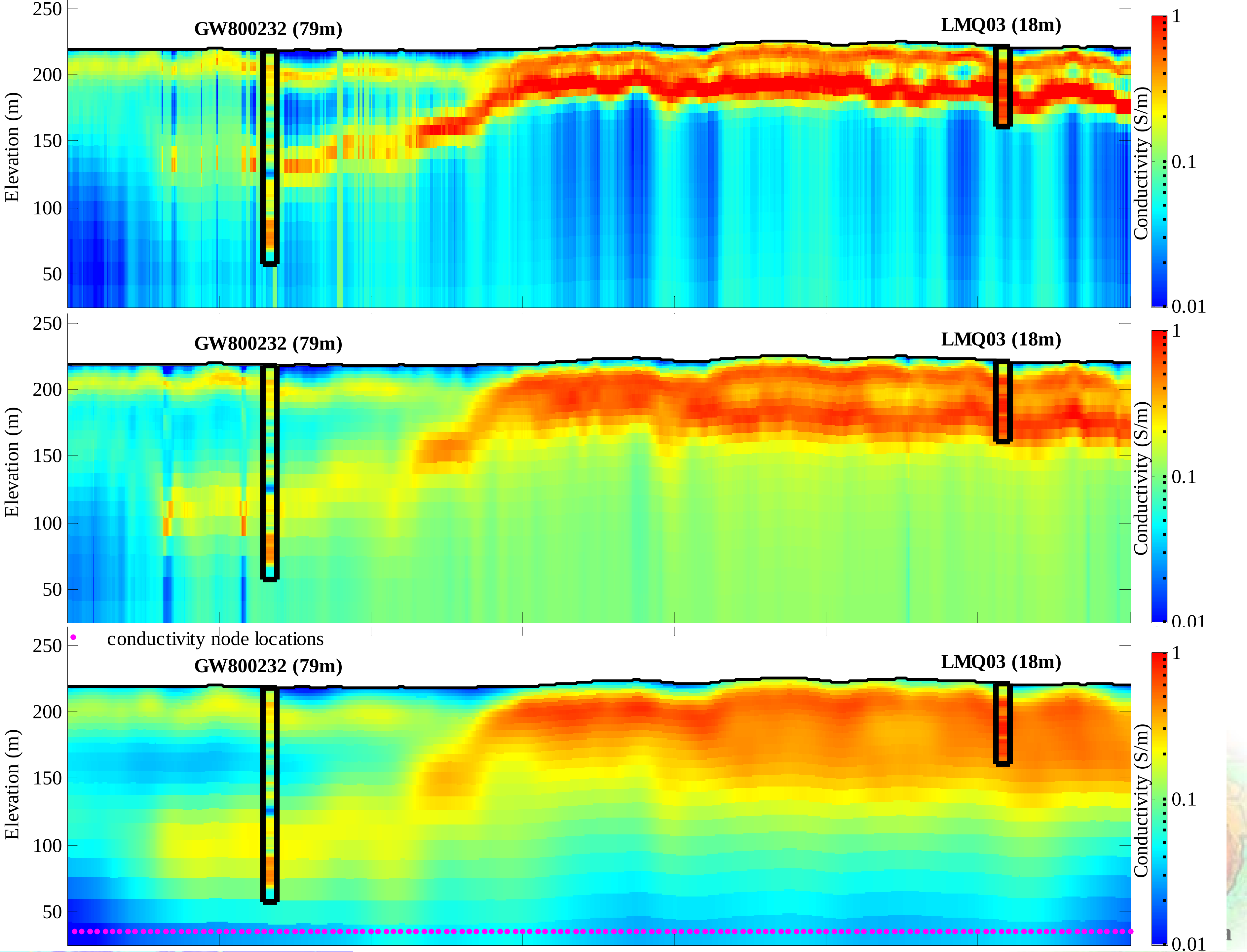
Conclusions

- **AEM data are not always what we think they are – we must do our best to simulate the real AEM system, not the theoretical one**
- **Solving for geometry allows data to be fitted**
- **Inverting whole lines takes advantage of expected spatial continuity of geology and system geometry**

Acknowledgements

- **Geoscience Australia for supporting the PhD research**

- **The Bureau of Rural Science for permission to use the Lower Macquarie downhole logs and AEM data**



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