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International Terrestrial Reference Frame (ITRF) to GDA94 Coordinate Transformations

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1. Overview

This document provides a practical solution to the transformation of International Terrestrial Reference Frame (ITRF) coordinates into GDA94 coordinates. GDA94 is coordinate datum based on ITRF92 at the fixed epoch of 1994.0. ITRF coordinates will in general differ from GDA94 coordinates for two main reasons, namely tectonic motion of the Australian landmass and reference frame differences. Tectonic motion of the Australian landmass is approximately 7cm/year in the NNE direction. Differences between the ITRF92 coordinate reference frame and the ITRF2000 are at the several cm in magnitude. A standard 7-parameter transformation can adequately model these differences at the cm level, provided the 7-parameter transformation parameters are regularly updated to reflect the tectonic motion. A slightly more complex 14-parameter transformation (7-parameters + their rates) can be used as a better long-term practical solution to these coordinate transformations. A 14 parameter transformation parameter allow users to map a 7 parameter transformation to any epoch of interest.

Users of IGS products please note the following. Since 2 December 2001 all International GPS Service (IGS) Products are aligned to ITRF2000. Users transforming coordinates derived from IGS products after 2 December 2001 are provided with additional high quality transformation parameters that referred to as ITRF2000(IGS).

This document provides the 14-Parameter transformations from ITRF2000, ITRF2000(IGS), ITRF97 and ITRF96 to GDA94. This document supersedes versions 31.08.2001 and

02.12.2002 of the same title. **Erratum** in the previous versions 31.08.2001 and 02.12.2002 of this document the transformation parameters r_x , r_z were incorrectly transposed in equations (1) and (2).

2. Important Note

There are two different ways of applying the sign conventions for the rotations. In both cases the sign convention is the same (a positive rotation is an anti-clockwise rotation, when viewed along the positive axis towards the origin) but:

- a) the International Earth Rotation Service (IERS) assumes the rotations to be of the position around the coordinate axes, while
- b) the method historically used in Australia assumes the rotations to be of the coordinate axes.

The only difference in the transformation formula is a change in the signs of the angles (and angle rates) in the rotation matrix. If the sign of the rotation parameters and the formulae used are consistent the correct results will be obtained. In this document the method historically used in Australia (b) is adopted.

3. The 7-Parameter Transformation

The 7-Parameter transformation between an input set of ITRF coordinates and GDA94 can be described by equation (1). Where $\begin{bmatrix} x_{GDA94} & y_{GDA94} & z_{GDA94} \end{bmatrix}^T$ are the transformed GDA94 Earth centred cartesian coordinates (metres), $\begin{bmatrix} x_{ITRF} & y_{ITRF} & z_{ITRF} \end{bmatrix}^T$ are the input ITRF Earth centred Cartesian coordinates (metres), d_x , d_y , d_z are translations (metres), r_x , r_y , r_z are rotations (radians) and S_c is a scale.

$$\begin{bmatrix} x_{GDA94} \\ y_{GDA94} \\ z_{GDA94} \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} + (1 + S_c) \times \begin{bmatrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{bmatrix} \begin{bmatrix} x_{ITRF} \\ y_{ITRF} \\ z_{ITRF} \end{bmatrix} \quad (1)$$

4. The 14-Parameter Transformation

The 14-Parameter transformation between an input set of ITRF coordinates and GDA94 can be described by equation (2). Where $\begin{bmatrix} x_{GDA94} & y_{GDA94} & z_{GDA94} \end{bmatrix}^T$ are the transformed GDA94 Earth centred cartesian coordinates (metres), $\begin{bmatrix} x_{ITRF} & y_{ITRF} & z_{ITRF} \end{bmatrix}^T$ are the input ITRF Earth centred cartesian coordinates (metres), $d_x, d_y, d_z, \dot{d}_x, \dot{d}_y, \dot{d}_z$ are translations and their rates (metres, metres/year), $r_x, r_y, r_z, \dot{r}_x, \dot{r}_y, \dot{r}_z$ are rotations and their rates (radians, radians/year) and s_c, \dot{s}_c is a scale and its rate (/year). The parameter t_0 (years) is the reference epoch which for this document is 2000.0, t (years) is the reference epoch of the input ITRF coordinates.

$$\begin{bmatrix} x_{GDA94} \\ y_{GDA94} \\ z_{GDA94} \end{bmatrix} = \begin{bmatrix} d_x + \dot{d}_x \times (t - t_0) \\ d_y + \dot{d}_y \times (t - t_0) \\ d_z + \dot{d}_z \times (t - t_0) \end{bmatrix} + \{1 + s_c + \dot{s}_c \times (t - t_0)\} \times \tilde{R} \times \begin{bmatrix} x_{ITRF} \\ y_{ITRF} \\ z_{ITRF} \end{bmatrix} \quad (2)$$

$$\tilde{R} = \begin{bmatrix} 1 & \{r_z + \dot{r}_z \times (t - t_0)\} & -\{r_y + \dot{r}_y \times (t - t_0)\} \\ -\{r_z + \dot{r}_z \times (t - t_0)\} & 1 & \{r_x + \dot{r}_x \times (t - t_0)\} \\ \{r_y + \dot{r}_y \times (t - t_0)\} & -\{r_x + \dot{r}_x \times (t - t_0)\} & 1 \end{bmatrix}$$

Appendix A 14 Parameter Transformations

Table A.1 ITRF2000 to GDA94 using 14-Parameter transformation. Note unit changes from equation (1) and (2) for the rotations and scale parameters and their rates.

t (years)	d_x (metres)	d_y (metres)	d_z (metres)	r_x (as)	r_y (as)	r_z (as)	s_c (ppm)
2000.00	-0.0761	-0.0101	0.0444	0.008765	0.009361	0.009325	0.007935
/year	0.0110	-0.0045	-0.0174	0.001034	0.000671	0.001039	-0.000538

Table A.2 ITRF2000(IGS) to GDA94 using 14-Parameter transformation. Note unit changes from equation (1) and (2) for the rotations and scale parameters and their rates.

t (years)	d_x (metres)	d_y (metres)	d_z (metres)	r_x (as)	r_y (as)	r_z (as)	s_c (ppm)
2000.00	-0.0663	-0.0050	0.0426	0.008814	0.009127	0.009042	0.007936
/year	0.0049	0.0039	0.0049	0.001616	0.001200	0.001013	0.000096

Table A.3 ITRF97 to GDA94 using 14-Parameter transformation. Note unit changes from equation (1) and (2) for the rotations and scale parameters and their rates.

t (years)	d_x (metres)	d_y (metres)	d_z (metres)	r_x (as)	r_y (as)	r_z (as)	s_c (ppm)
2000.00	-0.2088	0.0119	0.1855	0.012059	0.013639	0.011825	0.004559
/year	-0.0220	0.0049	0.0169	0.002040	0.001782	0.001697	-0.001090

Table A.4 ITRF96 to GDA94 using 14-Parameter transformation. Note unit changes from equation (1) and (2) for the rotations and scale parameters and their rates.

t (years)	d_x (metres)	d_y (metres)	d_z (metres)	r_x (as)	r_y (as)	r_z (as)	s_c (ppm)
2000.00	-0.0140	0.0431	0.2010	0.012464	0.012013	0.006434	0.024607
/year	0.0411	0.0218	0.0383	0.002542	0.001431	-0.000234	0.005897

Appendix B Sample Calculation

INPUT : ITRF2000 at 1 January 2002 (GRS80 ellipsoid) :-

Cartesian X, Y, Z	-4052052.048	4212836.105	-2545105.587
Longitude, Latitude, Height	133 53 7.8574	-23 40 12.4314	603.287

The computation can be undertaken using equation (2) and the parameters in Table A.1, where $t_0 = 2000.0$ and $t = 2002.0$. Alternatively, using Table A.1 the 7-Parameter transformation parameters can be computed for epoch 2002.0 and equation (1) applied, for example :-

$$d_x = -0.0761 + (2002.0 - 2000.0) \times 0.0110 = -0.0541$$

$$d_y = -0.0101 + (2002.0 - 2000.0) \times -0.0045 = -0.0191$$

$$d_z = 0.0444 + (2002.0 - 2000.0) \times -0.0174 = 0.0096$$

$$r_x = 0.008765 + (2002.0 - 2000.0) \times 0.001034 = 0.010833$$

$$r_y = 0.009361 + (2002.0 - 2000.0) \times 0.000671 = 0.010703$$

$$r_z = 0.009325 + (2002.0 - 2000.0) \times 0.001039 = 0.011403$$

$$s_c = 0.007935 + (2002.0 - 2000.0) \times -0.000538 = 0.006859$$

t (years)	d_x (metres)	d_y (metres)	d_z (metres)	r_x (as)	r_y (as)	r_z (as)	s_c (ppm)
2002.00	-0.0541	-0.0191	0.0096	0.010833	0.010703	0.011403	0.006859

The rotation and scale units can be converted ready for equation (1) or (2) using the following :-

$$1 \text{ as} = 1 \text{ arc second} = 1 \times \frac{\pi}{60 \times 60 \times 180} \text{ radians} = 0.000004848 \text{ radians}$$

$$1 \text{ ppm} = 1 \text{ part per million} = 1 \times \frac{1}{1,000,000} \text{ parts} = 0.000001 \text{ parts}$$

OUTPUT : GDA94 (GRS80 ellipsoid) :-

Cartesian X, Y, Z	-4052051.765	4212836.205	-2545106.027
Longitude, Latitude, Height	133 53 7.8478	-23 40 12.4461	603.350