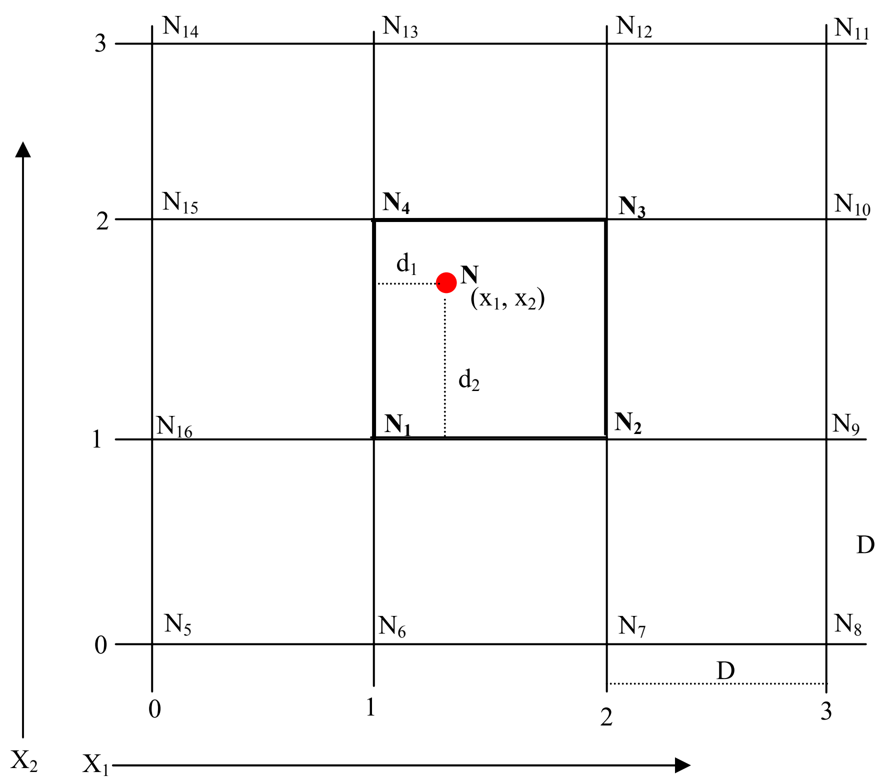
WINTER Interpolation Algorithm

<u>Reference:</u>
"*Numerical Recipes*"
William H. Press, Brian P. Flannery, Saul A. Teukolsky, William T. Vetterling,
Cambridge University Press, 1990 Reprint,
Pages 94-101, 694-695

Winter uses the 16 grid points surrounding the point in question to perform a bi-cubic interpolation.

<u>Assumptions</u>

- Grid is orthogonal
- Grid is regular (cell sizes are all the same and have equal sides).



$d_1 = X_{1(N)} - X_{1(1,1)}$
$d_2 = X_{2(N)} - X_{2(1,1)}$

D = grid size
$T = d_1/D$
$U = d_2/D$

<u>Calculate Cross Derivatives</u>

$Y12(1) = (N_3 - N_7 - N_{15} + N_5) / 4D^2$
$Y12(2) = (N_{10} - N_8 - N_4 + N_6) / 4D^2$
$Y12(3) = (N_{11} - N_9 - N_{13} + N_1) / 4D^2$
$Y12(4) = (N_{12} - N_2 - N_{14} + N_{16}) / 4D^2$

Calculate Derivatives in each direction

Y1(1) = (N_2-N_{16}) / 2D
Y1(2) = (N_9-N_1) / 2D
Y1(3) = (N_{10}-N_4) / 2D
Y1(4) = (N_3-N_{15}) / 2D

Y2(1) = (N_4-N_6) / 2D
Y2(2) = (N_3-N_7) / 2D
Y2(3) = (N_{12}-N_2) / 2D
Y2(4) = (N_{13}-N_1) / 2D

Bi-cubic coefficient matrix C(i,j): i=row, j=column

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	-3	2	0	0	0	0	-3	0	9	-6	2	0	-6	4
2	0	0	0	0	0	0	0	0	3	0	-9	6	-2	0	6	-4
3	0	0	0	0	0	0	0	0	0	0	9	-6	0	0	-6	4
4	0	0	3	-2	0	0	0	0	0	0	-9	6	0	0	6	-4
5	0	0	0	0	1	0	-3	2	-2	0	6	-4	1	0	-3	2
6	0	0	0	0	0	0	0	0	-1	0	3	-2	1	0	-3	2
7	0	0	0	0	0	0	0	0	0	0	-3	2	0	0	3	-2
8	0	0	0	0	0	0	3	-2	0	0	-6	4	0	0	3	-2
9	0	1	-2	1	0	0	0	0	0	-3	6	-3	0	2	-4	2
10	0	0	0	0	0	0	0	0	0	3	-6	3	0	-2	4	-2
11	0	0	0	0	0	0	0	0	0	0	-3	3	0	0	2	-2
12	0	0	-1	1	0	0	0	0	0	0	3	-3	0	0	-2	2
13	0	0	0	0	0	1	-2	1	0	-2	4	-2	0	1	-2	1
14	0	0	0	0	0	0	0	0	0	-1	2	-1	0	1	-2	1
15	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	-1	1
16	0	0	0	0	0	0	-1	1	0	0	2	-2	0	0	-1	1

X_1 = N1
X_2 = N2
X_3 = N3
X_4 = N4
X_5 = Y1(1)*D
X_6 = Y1(2)*D
X_7 = Y1(3)*D
X_8 = Y1(4)*D
X_9 = Y2(1)*D
X_{10} = Y2(2)*D
X_{11} = Y2(3)*D
X_{12} = Y2(4)*D
X_{13} = Y12(1)*D^2
X_{14} = Y12(2)*D^2
X_{15} = Y12(3)*D^2
X_{16} = Y12(4)*D^2

Evaluate $N_{i,j}$ to produce a 16x16 matrix of values (where i is the row index & j is the column index):

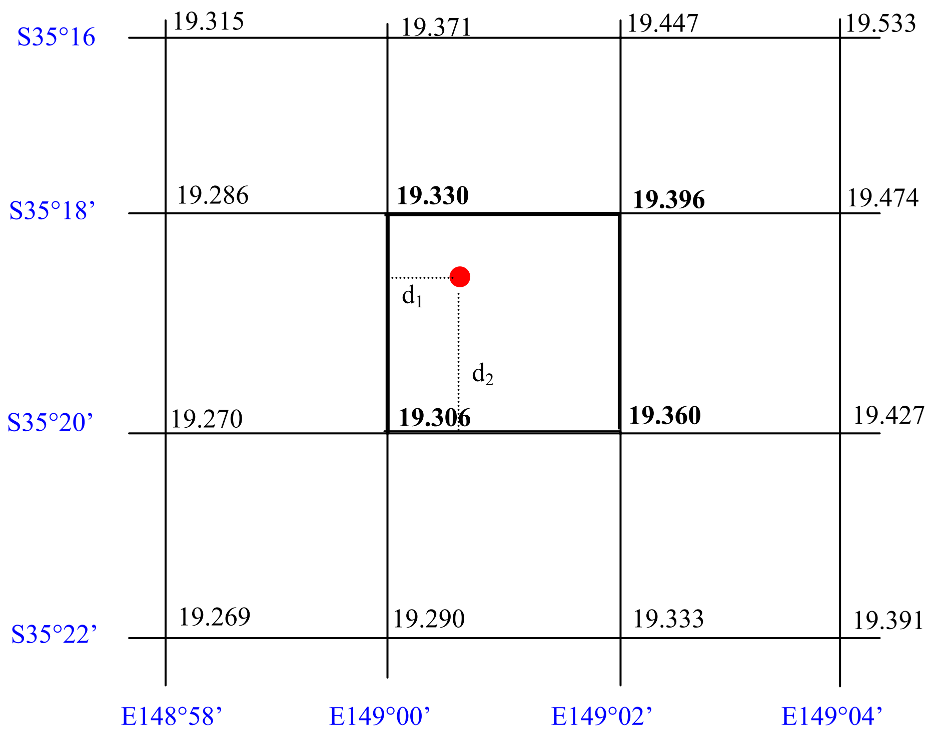
$N_0 = 0$
For i =1 to 16
For j = 1 to 16
$N_{i,j} = N_{i-1} + C_{i,j}*X_i$
End loop
End loop

Compute the interpolated value

$R_1 = 0$
$R_2 = (T * R_1) + ((((C_{16,16}*U) +C_{16,15})*U)+ C_{16,14})*U)+(C_{16,13})$
$R_3 = (T * R_2) + ((((C_{16,12}*U) +C_{16,11})*U)+ C_{16,10})*U)+(C_{16,9})$
$R_4 = (T * R_3) + ((((C_{16,8}*U) +C_{16,7})*U)+ C_{16,6})*U)+(C_{16,5})$

N $= (T * R_4) + ((((C_{16,4}*U) +C_{16,3})*U)+ C_{16,2})*U)+(C_{16,1})$

Test data



Position interpolated: S35° 18′55.93″
E149°00′36.18″

d_1 = 55.93" = 0.9322'
d_2 = 36.18" = 0.6030'

N= 19.334