Appendix A: WIND HAZARD METHODOLOGY

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Generalized Extreme Value Distribution (GEV)

The general functions of GEV are given here.

Fitting the dataset to a GEV

Data fitting methods used were based on maximum likelihood and probability-weighted moments. Statistical hypothesis tests such as Gumbel tests have also been performed on all datasets with a 95% significance level. If a null hypothesis of a Gumbel Distribution (ie, zero shape parameter) could not be rejected, then the dataset was fitted to a Gumbel Distribution. If a Gumbel Distribution has been rejected in favour of a positive shape parameter, a Weibull Distribution fitting was then performed. Finally, the maximum value of these estimates (different fitting methods and different datasets, yearly and monthly) has been chosen as the estimate for the return period speed. Note that when an estimate from the yearly maxima differed by more than 5% from the estimate derived by the monthly maxima, only the estimate from the monthly maxima has been used. This is because we rank the results from monthly data (ie, a larger sample) as more reliable than the corresponding results from the yearly data (ie, a smaller sample), especially when the data period is short.

The cumulative distribution function (CDF) of GEV

The GEV has a CDF of

$$H(x;\xi,\sigma,\mu) = \begin{cases} e^{-\left(1-\xi\frac{x-\mu}{\sigma}\right)^{1/\xi}} & \xi \neq 0 \\ e^{-e^{-(x-\mu)/\sigma}} & \xi = 0 \end{cases}$$

Equation A.1

where ξ, σ and μ are the shape, scale and location parameters, respectively, and x is the maximum for an epoch.

When $\xi = 0$, the distribution is a Type I GEV or Gumbel Distribution. When $\xi < 0$, the GEV has a long right tail. It is called the Type II (or Frechet) Distribution. When $\xi > 0$, it has a short tail. It is sometimes called the Type III GEV (which is a form of the Weibull Distribution). The Type III GEV has a theoretical upper bound ($\mu+\sigma/\xi$) that may be useful for estimates of extreme values (such as largest possible wind gusts). Many scientists believe that due to physical and meteorological limitations, there is an upper bound to the maximum wind gust.

Return periods

When a threshold of the magnitude of an event is chosen to be sufficiently large, the number of exceedances N_u (where u is the threshold) has an approximate Poisson distribution with parameter λ (the rate of exceedances per year, also called the crossing rate). Hence λT is the number of exceedances in T years. Let λ_U be the number of events exceeding a very high level U. That is,

$$\lambda_U = \lambda T \cdot \Pr\{X > U\} = \lambda T(1 - F(U)) .$$

Equation A.2

Assume U_T is the event with the largest value in T years, and by definition $\lambda_{U_T} = 1$ (i.e. it only happens once in T years). Now

$$\lambda_{U_T} = \lambda T(1 - F(U_T)) = 1$$ **Equation A.3**

so

$$F(U_T) = 1 - \frac{1}{\lambda T}$$ **Equation A.4**

or

$$U_T = F^{-1}\left(1 - \frac{1}{\lambda T}\right)$$ **Equation A.5**

where $F^{-1}(\cdot)$ is the inverse of the CDF of the GEV. The crossing rate λ has the value 1 if yearly maximum data are used in the extreme value analysis, or 12 if the monthly maxima are used.

Quantile estimation

Once its parameters have been estimated, quantile estimates for the GEV can be obtained by inverting Equation A.1 and using Equation A.6:

$$U_T = \begin{cases} \mu + \frac{\sigma}{\xi}\left\{1 - \left[-\ln\left(1 - \frac{1}{\lambda T}\right)\right]^{\xi}\right\} & \xi \neq 0 \\ \mu - \sigma \ln\left[-\ln\left(1 - \frac{1}{\lambda T}\right)\right] & \xi = 0. \end{cases}$$ **Equation A.6**